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Mundarija

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On the spaces with an order unit: a new point of view

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Abstract

In the present paper in a partially ordered linear space we consider a base that generates a convex Hausdorff topology in it. Further using elements of this base we obtain a metric and a norm on the given space. We show the achieved space is a space with an order unit. The way obtaining a partially ordered topological linear space gives the possibility to investigate the spaces with an order unit with a new point of view.

Keywords: Metric; norm; order unit; A-subspace.

MSC 2020: 46A03, 46A40, 46B40

1. Introduction

Ordered vector spaces made their debut at the beginning of the twentieth century. They were developed in parallel with functional analysis and operator theory. The theory of ordered vector spaces plays a prominent role in functional analysis [1], [2]. It also contributes to a wide variety of applications and is an indispensable tool for studying a variety of problems in engineering and economics [3], [4].

Recently, it has been appeared the order-preserving analysis [5] which can be interpreted as a generalization of the Functional Analysis. Properties of order-preserving functionals on a space with an order unit was investigated in [6]. τ-smooth order-preserving functionals were considered in [7]. In [8] the authors established a variant of the Hahn-Banach theorem. Uniform boundedness principle for order-preserving operators was explored in [9]. The notion of an order-preserving functional finite degree was introduced in [10], and was got analogical to the linear case properties. An open mapping theorem for order-preserving operators was established in [11]. And a variant of the Banach-Steinhaus theorem for order-oreserving operators was obtained in [12].

In the present paper we offer a way obtaining to get a partially ordered topological linear space which gives possibility to investigate the spaces with an order unit with a new point of view. We introduce a base in a partially ordered linear space which generates a topology in it. Using this base we construct a metric and a norm in the space. In this case, the methods in [13] [14], were widely used. A notion of an A-subspace was considered and constructed examples showing difference between subspace and A-subspace. Note that A-subspaces widely used in the order-preserving analysis.

2. Preliminaries

Let L be an ordered real linear space. We denote by L^+ the set of non-negative elements of L. If for every $a \in L$ there exists such a number $\lambda \in \mathbb{R}^+$ that $-\lambda e \leq a \leq \lambda e$ then the element $e \in L^+$ is called the order unit of the space L. Moreover, if the order $\leq$ in the space L is the Archimedean order, then a correspondence $a \mapsto \|a\| = \inf\{\lambda > 0\colon\ -\lambda e \leq a \leq \lambda e\}$ forms a norm. If the space L is a Banach space with respect to this norm, then (L, e) is called a space with an order unit [15], [11].

We will offer a way to build a space with an order unit. Consider arbitrary points x_1, x_2 in a partially ordered linear space L over the field $\mathbb{R}$ of real numbers. A set $[x_1, x_2] = \{\alpha x_1 + (1-\alpha)x_2\colon\ \alpha \in [0, 1]\}$ is called a segment connecting x_1 and x_2 [8], [9]. A point $x \in [x_1, x_2]$ is called an interior point of the segment $[x_1, x_2]$ if $x_1 \neq x \neq x_2$.

Definition 2.1 [5] Let K be a cone and $x \in K$. A point x is called an interior point of K, if for every segment $[x_1,\, x_2] \subset L$ containing x as interior point the segment $[x_1,\, x_2] \cap K$ also contains it as an interior point. The set of all interior points of the cone K is called the interior of the cone and denoted by $Int(K)$.

Let L be a partially ordered linear space over the field $\mathbb{R}$ of real numbers, the element $\theta \in L$ be the zero of the space L and a subset K be a cone in L satisfying the condition $K - K = L$. We assign some interior point $x_0 \in K$ and, taking the number $\delta > 0$ define the following set [16]:

$$\langle \theta,\, \delta \rangle = \{x \in L\colon\ (\delta x_0 \pm x) \in Int(K)\}. \tag{1}$$

For each $z \in\ L$ we form the following set

$$\langle z,\, \delta \rangle = \{y \in L\colon\ (\delta x_0 \pm (y - z)) \in Int(K)\}. \tag{2}$$

Let us build the following the family of the sets of the form (2)

$$\mathfrak{B} = \{\langle z,\, \delta \rangle\colon\ z \in L,\, \delta > 0\} = \bigcup_{z \in L} \{\langle z,\, \delta \rangle\colon\ \delta > 0\}. \tag{3}$$

In the work [16] the authors showed that the family $\mathfrak{B}$ forms a neighborhood system of some topology in L. It has proven that the ordered linear space L is a Hausdorff topological space according to this topology, and is a linear topological space with respect to the usual algebraic operations.

3. Main part

At first, we show that the linear topological space L is metrizable.

Consider a corresponding $\rho\colon\ L \times L \to \mathbb{R}$ which defines by the following equality

$$\rho(x,\, y) = \inf\{\lambda > 0\colon\ y - x \in \langle \theta,\, \lambda \rangle\}. \tag{4}$$

Theorem 3.1. *The function $\rho\colon\ L \times L \to \mathbb{R}$ is a metric on L.*

Proof. To show that L is a metric space, we need to check that the function ρ satisfies the metric conditions.

$(M1)$ Identity axiom. Let $x,\, y \in L$. Identity axiom requires that $\rho(x,\, y) = 0$ if and only if $x = y$. Indeed, the equality $\rho(x,\, y) = 0$ by the definition means that for every $\lambda > 0$ one has $y - x \in \langle \theta,\, \lambda \rangle$. But the exact lower bound of such λ is zero. That is why $y - x = 0$ which yields $y = x$.

Clearly, the equality $x = y$ immediately implies that $\rho(x,\, y) = 0$.

$(M2)$ Non-negativity axiom. Now we consider the case $x \neq y$. Since L is a Hausdorff topological space, in this case there exist $\lambda_1,\, \lambda_2 > 0$ such that, relations $x \notin \langle y,\, \lambda_2 \rangle$, and $y \notin \langle x,\, \lambda_1 \rangle)$ are fulfilled. The latter relations imply $x - y \notin \langle \theta,\, \lambda_2 \rangle$ and $y - x \notin \langle \theta,\, \lambda_1 \rangle$. This ensures that the following inequality holds

$$\rho(x,\, y) \geq \min\{\lambda_1,\, \lambda_2\} > \inf\{\lambda > 0\colon\ y - x \in \langle \theta,\, \lambda \rangle\} \geq 0.$$

Thus, $\rho(x,\, y) > 0 \Leftrightarrow x \neq y$. So, $\rho(x,\, y) \geq 0$.

$(M3)$ Symmetry axiom. According to the definition of the function ρ and the neighborhood $\langle \theta,\, \delta \rangle$ we have:

$$\Big(y - x \in \langle \theta,\, \delta \rangle\Big) \Leftrightarrow \Big(\delta x_0 \pm (y - x) \in Int(K)\Big) \Leftrightarrow \Big(\delta x_0 \pm (x - y) \in Int(K) \Leftrightarrow (x - y) \in \langle \theta,\, \delta \rangle\Big).$$

So, $\rho(x,\, y) = \rho(y,\, x)$.

$(M4)$ Triangle axiom. Suppose $y - x \in \langle \theta,\, \delta_1 \rangle$ and $z - y \in \langle \theta,\, \delta_2 \rangle$. Then

$$z - x = (y - x) + (z - y) \in \langle \theta,\, \delta_1 + \delta_2 \rangle.$$

Thus, there is $\delta > 0$ satisfying relations $\delta \leq \delta_1 + \delta_2$ and $z - x \in \langle \theta,\, \delta \rangle$. Thus, we have the following expression:

$$\begin{aligned} \rho(x,\, y) + \rho(y,\, z) &= \inf\{\lambda_1 > 0\colon\ y - x \in \langle \theta,\, \lambda_1 \rangle\} + \inf\{\lambda_2 > 0\colon\ z - y \in \langle \theta,\, \lambda_2 \rangle\} \geq \\ &\geq \inf\{\lambda_1 + \lambda_2\colon\ \lambda_i > 0,\, i = 1,\, 2,\, y - x \in \langle \theta,\, \lambda_1 \rangle \ \text{and}\ \ z - y \in \langle \theta,\, \lambda_2 \rangle\} \geq \\ &\geq \inf\{\lambda > 0\colon\ z - x \in \langle \theta,\, \lambda \rangle\} = \\ &= \rho(x,\, z), \end{aligned}$$

that is, $\rho(x,\, z) \leq \rho(x,\, y) + \rho(z,\, y)$.

The proof of Theorem 3.1 is complete. □

Theorem 3.2. *Let L be a real linear space and let K be its cone such that $K - K = L$. Moreover, let $x_0 \in Int(K)$ and $\delta > 0$. A function $\|\cdot\|\colon L \to \mathbb{R}$ defined by the equality $\|x\| = \inf\{\lambda > 0\colon x \in \langle\theta, \lambda\rangle\}$ satisfies all conditions of the norm.*

Proof. $(N1)$ Non-negativity. Take any $x \in L$. By the definition $\|x\|$ is the lower bound of positive numbers λ for which $x \in \langle\theta, \lambda\rangle\}$. So, it is natural that $\|x\| \geq 0$.

Let $\|x\| = 0$. Then for all $\lambda > 0$ it should be $x \in \langle\theta, \lambda\rangle$. Suppose $x \neq \theta$. Then by virtue of L is a Hausdorff space there exists $\lambda' > 0$ such that $x \notin \langle\theta, \lambda'\rangle$. But this generates a contradiction. So, $x = \theta$.

In the case when $x = \theta$, it is clear that for each $\lambda > 0$ necessarily will be $x \in \langle\theta, \lambda\rangle$. The exact lower bound of such positive λ numbers is equal to 0, i. e. $\|x\| = 0$.

$(N2)$ Homogeneity. For every $x \in L$ and $\alpha \in \mathbb{R}$, it is necessary to prove that $\|\alpha x\| = |\alpha| \cdot \|x\|$. Note that for each $\delta > \|x\|$ we have $x \in \langle\theta, \delta\rangle$. According to Lemma 2.3 of the work [16], we get $\alpha x \in \alpha\langle\theta, \delta\rangle = \langle\theta, |\alpha|\delta\rangle$. The exact lower bound of numbers in the form $|\alpha|\delta$ according to the variable δ is $|\alpha|\|x\|$, i. e. $\|\alpha x\| = |\alpha|\|x\|$.

$(N3)$ Triangle axiom. For an arbitrary $\varepsilon > 0$ it should be $x \in \langle\theta, \|x\| + \varepsilon\rangle$, $y \in \langle\theta, \|y\| + \varepsilon\rangle$. Consequently,

$$x + y \in \langle\theta, \|x\| + \|y\| + 2\varepsilon\rangle.$$

The last inequality gives that

$$\|x\| + \|y\| + 2\varepsilon > \|x + y\|.$$

The arbitrariness of ε provides $\|x\| + \|y\| \geq \|x + y\|$ while ε tends to 0.

The proof of Theorem 3.2 is complete. □

Theorem 3.3. *Let L be a real linear space and let K be its cone such that $K - K = L$. Then the linear topological space L, which base has the shape (3) is a space with an order unit. Moreover, each point $x_0 \in Int(K)$ is its (strong) order unit.*

Proof. We need to check the conditions below.
$(OU1)$ Determine the order. We assume that the relationship $x \leq y$ between the points $x, y \in L$ is fulfilled if and only if $y - x \in K$.

• Reflexivity: For any $x \in L$, $x \leq x$. Indeed, $x - x = \theta \in K$.

• Antisymmetry: Let $x \leq y$ and $y \leq x$ for points $x, y \in L$. Then $y - x \in K$ and $x - y = -(y - x) \in K$, that is, $y - x \in (-K)$. From the property $K \cap (-K) = \{\theta\}$ follows that $y - x = \theta$. This means that $y = x$.

• Transitivity: Let $x \leq y$ and $y \leq z$ for points $x, y, z \in L$. In that case, $y - x \in K$ and $z - y \in K$. And as a result of adding them, we get the relationship $z - x \in K$. This means that $x \leq z$.

$(OU2)$ Determination of the unit element: Fix any point $x_0 \in Int(K)$ and show it is a strong unit. Any point $x \in L$ has a number $\delta > 0$ such that $x \in \langle\theta, \delta\rangle$. It follows that $\delta x_0 \pm x \in Int(K)$. Therefore, $\delta x_0 - x \in Int(K)$ and $\delta x_0 + x \in Int(K)$, from here $\delta x_0 - x \in K$ and $\delta x_0 - (-x) \in K$, or $x \leq \delta x_0$ and $-x \leq \delta x_0$. The last two inequalities can be written in the form of $-\delta x_0 \leq x \leq \delta x_0$ inequality. So, the point $x_0 \in Int(K)$ is a strong unit.

The proof of Theorem 3.3 is complete. □

Determination of the norm with respect to the unit element. We define the norm with respect to the unit element by the classical way:

$$\|x\| = \inf\{\lambda > 0\colon \ -\lambda x_0 \leq x \leq \lambda x_0\}. \tag{5}$$

Definition 3.1 [7]Let L be a space with an order unit, x_0 its any order unit, and B a subset in L. The set B is called an A-subspace in L with respect to x_0, if $\theta \in B$ and $x \in B$ provides $(x + \lambda x_0) \in B$ for every $\lambda \in \mathbb{R}$. The A-subspace of the space L with respect to the order unit x_0 is called A^{x_0}-subspace.

Lemma 3.1. *The subspace L_1 of the space L with an order unit is an A^{x_0}-subspace if and only if L_1 contains x_0.*

Proof. Let a subspace L_1 of space L with the order unit x_0 be a subspace A^{x_0}-subspace. Since L_1 is a subspace $\theta \in L_1$. Moreover, for every pair of points $x, y \in L_1$ and any numbers $\alpha, \beta \in \mathbb{R}$ one has $\alpha x + \beta y \in L_1$. Since

L_1 is an A^{x_0}-subspace we have $x + \lambda x_0 \in L_1$ for arbitrary $x \in L_1$ and $\lambda \in \mathbb{R}$. In particular, $x_0 \in L_1$ in the case $x = 0$ and $\lambda = 1$.

Now let a subspace L_1 of L contains x_0. Clearly, $\theta \in L_1$. For any $x \in L_1$ and each $\lambda \in \mathbb{R}$ we have $x+\lambda x_0 \in L_1$. Consequently, L_1 is an A^{x_0}-subspace.

Lemma 3.1 has been proved. □

Example 3.1. In the Euclidean plane $\mathbb{R}^2$ with a point-wise order the set $K = \{(u, v) \in \mathbb{R}^2 \colon\ u \geq 0 \text{ and } v \geq 0\}$ is a cone in it. It is not difficult to understand that $Int(K) = \{(u, v) \in \mathbb{R}^2 \colon\ u > 0 \text{ and } v > 0\}$. An arbitrary taken point in $Int(K)$ can be obtained as an order unit. For instance, the point $x_0 = (2, 1)$ can be assigned as a unit order. The set $L_1 = \{(u, v) \in \mathbb{R}^2 \colon\ v = 3u\}$ is a subspace and $x_0 \notin L_1$. Hence, the subspace L_1 is not a $A^{(2,\,1)}$-subspace.

If $x_0 = \left(\frac{1}{3},\, 1\right)$ is taken as an order unit, then $x_0 \in L_1 = \{(u, v) \in \mathbb{R}^2 \colon\ v = 3u\}$ and L_1 is an $A^{\left(\frac{1}{3},\,1\right)}$-subspace.

In general, an A-subspace is not necessarily a subspace of a space with an order unit.

Example 3.2. In the above example, let $x_0 = (2, 1)$ be an unit order. Then

$$L_1 = \left\{(u, v) \in \mathbb{R}^2 \colon\ v = \tfrac{1}{2}u \vee\ , v = \tfrac{1}{2}u + 1\ \vee v = \tfrac{1}{2}u - 2\right\}$$

is an $A^{(2,\,1)}$-subspace, but not a subspace.

The whole space L is its an A^{x_0}-subspace for any order unit $x_0 \in Int(K)$. Besides, the set $\{\lambda x_0 \colon\ \lambda \in \mathbb{R}\}$ is also an A^{x_0}-subspace. These two A^{x_0}-subspaces we call trivial A^{x_0}-subspaces [6]. However, unlike linear subspaces, the set $\{\theta\}$ is not an A^{x_0}-subspace for any order unit $x_0 \in Int(K)$.

Lemma 3.2. *The intersection of an arbitrary family of A^{x_0}-subspaces in a space L with an order unit is an A^{x_0}-subspace.*

Proof. Assume that $\{L_\alpha \colon\ \alpha \in I\}$ is a family of A^{x_0}-subspaces in a space L with an order unit. Then, since $\theta \in L_\alpha$ for each $\alpha \in I$, we have $\theta \in \bigcap\limits_{\alpha\in I} L_\alpha$. Let $x \in \bigcap\limits_{\alpha\in I} L_\alpha$ be an arbitrary point and $\lambda \in \mathbb{R}$ any number. Since $x + \lambda x_0 \in L_\alpha$ for all $\alpha \in I$, we have $(x + \lambda x_0) \in \bigcap\limits_{\alpha\in I} L_\alpha$.

Lemma 3.2 has been proved. □

The intersection of all A^{x_0}-subspaces containing a given set $X \subset L$ is the minimal A^{x_0}-subspace containing the set X. This A^{x_0}-subspace is called a weakly additive span [3] of the set X (with respect to the unit order x_0) and is denoted as $A^{x_0}(X)$.

Lemma 3.3. *The union of an arbitrary family of A^{x_0}-subspaces in a space L with an order unit is an A^{x_0}-subspace.*

Proof. Assume that $\{L_\alpha \colon\ \alpha \in I\}$ is a family of A^{x_0}-subspaces in a space L with an order unit. Then, since $\theta \in L_\alpha$ for each $\alpha \in I$ it follows that $\theta \in \bigcup\limits_{\alpha\in I} L_\alpha$. For any $x \in \bigcup\limits_{\alpha\in I} L_\alpha$ and $\lambda \in \mathbb{R}$ we have $(x + \lambda x_0) \in \bigcup\limits_{\alpha\in I} L_\alpha$ because of existing $\alpha \in I$ such that $x \in L_\alpha$ and $(x + \lambda x_0) \in L_\alpha$.

Lemma 3.3 has been proved. □

Let L be a linearly ordered space, and let $a, b \in L$. The set $[a, b]_o = \{x \colon\ a \leq x \leq b\}$ is called an order-interval. Clearly, if $a \nleq b$ then $[a, b]_o = \varnothing$.

Definition 3.2. If $[a, b] \subseteq A$ for any taken elements $a, b \in A$, then $A \subseteq L$ is said to be convex with respect to the order in L

Lemma 3.4. *For any number $\delta > 0$, the neighborhood $\langle\theta, \delta\rangle$ is a convex set with respect to the order in L.*

Proof. Suppose $x_1 \leq x_2$ for elements $x_1, x_2 \in \langle\theta, \delta\rangle$. Consider an arbitrary element x such that $x_1 \leq x \leq x_2$. We have $\delta x_0 + x \in Int(K)$ since $-x \leq -x_1$ and $\delta x_0 - x \in Int(K)$ because of $x \leq x_2$, that is, $\delta x_0 \pm x \in Int(K)$. Therefore, $x \in \langle\theta, \delta\rangle$. Since the element x is arbitrary, $[x_1, x_2]_o \subset \langle\theta, \delta\rangle$.

Lemma 3.4 has been proved. □

In essence, Lemma 3.4 can be clarified as follows:

Lemma 3.5. *For any number $\delta > 0$ and any point $x_0 \in Int(K)$ the equality $Cl(\langle\theta, \delta\rangle) = [-\delta x_0, \delta x_0]_o$ holds.*

Theorem 3.4 *The topology generated by the base (3) is a convex topology on L.*

Proof. First of all, let us show that $\langle\theta,\,\delta\rangle$ is convex, $\delta>0$. For each pair $x,\,y\in\langle\theta,\,\delta\rangle$ and arbitrary number $\alpha\in[0,\,1]$ from $\delta x_0\pm x\in Int(K)$ and $\delta x_0\pm y\in Int(K)$ it follows $\delta x_0\pm\alpha x\in Int(K)$ and $\delta x_0\pm(1-\alpha)y\in Int(K)$. That is why $\delta x_0\pm(\alpha x+(1-\alpha)y)\in Int(K)$. So, $\alpha x+(1-\alpha)y\in\langle\theta,\,\delta\rangle$, i. e. the neighbourhood of the zero is convex. The convexity of $\langle\theta,\,\delta\rangle$ and since $\langle z,\,\delta\rangle=\langle\theta,\,\delta\rangle+z$ we conclude that a neighbourhood $\langle z,\,\delta\rangle$ is convex.

The proof of Theorem 3.4 is complete. □

Lemma 3.6. *If for a nonnegative vector x_0 in the partially ordered topological vector space L the set $Int([-x_0,\,x_0]_o)$ is an open neighborhood of the zero, then the point x_0 is an inner point in the cone K.*

Proof. Let $x_0\in L$ be a positive vector. Since $Int([-x_0,\,x_0]_o)$ is a neighborhood of the zero there exists a vector $u\in Int([-x_0,\,x_0]_o)$ such that $u\in Int(K)$. Otherwise we have $Int([-x_0,\,x_0]_o)\cap Int(K)=\varnothing$. Consequently, $Int([-x_0,\,x_0]_o)\cap K=\varnothing$. The last equality holds if and only if $x_0\in K\setminus Int(K)$. Then $Int([-x_0,\,x_0]_o)=\varnothing$ and $Int([-x_0,\,x_0]_o)$ does not contains the zero, i. e. it is not an open neighbourhood the zero in L. So, $x_0\in Int(K)$.

Lemma 3.6 has been proved. □

Proposition 3.1. *For every pair of numbers δ_1, δ_2 with $\delta_1<\delta_2$ and for any $x\in L$ we have $\langle x,\,\delta_1\rangle\subset\langle x,\,\delta_2\rangle$.*

Lemma 3.7. *For any open neighborhood V of the zero in a partially ordered linear topological space L there is a sequence $\{V_n\}$ satisfying the conditions $V_1=V$ and $V_{n+1}+V_{n+1}\subseteq V_n$, $n=1,\,2,\,\ldots$.*

Proof. Taking an arbitrary number $\delta>0$, and putting $\delta_1=\delta$, $\delta_n=\frac{1}{3^{n-1}}\delta$, $n=1,\,2,\,\ldots$, we have

$$\langle\theta,\,\delta_{n+1}\rangle+\langle\theta,\,\delta_{n+1}\rangle=\langle\theta,\,\tfrac{2}{3}\delta_n\rangle\subset\langle\theta,\,\delta_n\rangle.$$

Lemma 3.7 has been proved. □

Theorem 3.5. *A partially ordered topological linear space L with the topology generated by the base (3) is complete.*

Proof. To show that this topological vector space is complete, it is necessary to show that any Cauchy sequence converges. Let $\{x_n\}\subset L$ be a Cauchy sequence. Then, according to the definition of the Cauchy sequence, for each $\varepsilon>0$ it has to exist a number $N\in\mathbb{N}$ for numbers $m,\,k>N$ for which $x_m-x_k\in\langle\theta,\,\varepsilon\rangle$. Tending ε to 0, that implies $N\to\infty$, we have

$$\lim_{\substack{N\to\infty,\\ m>N,\,k>N}}(x_m-x_k)=\theta.$$

In the other words, $\lim\limits_{\substack{N\to\infty,\\ m>N}}x_m=\lim\limits_{\substack{N\to\infty,\\ k>N}}x_k$, or, more simpler $\lim\limits_{m\to\infty}x_m=\lim\limits_{k\to\infty}x_k$. The last equality shows that the given sequence $\{x_n\}$ converges. Let $x'=\lim\limits_{n\to\infty}x_n$. It remains, that $x_0\in L$. But it follows from the convexity of basic neighbourhoods of the zero.

The proof of Theorem 3.5 is complete. □

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Tartib birlikli fazolar: yangicha qarash

Aktamov Feruz

Ushbu maqolada qisman tartiblangan chiziqli fazoda qavariq Hausdorf topologiyasini hosil qiluvchi baza qaralgan. Keyinchalik, ushbu bazaning elementlaridan foydalanib, berilgan fazoga metrika va norma kiritiladi. Maqolada hosil qilingan bu fazo tartib birlikli fazo ekanligi ko'rsatiladi. Qisman tartiblangan topologik chiziqli fazoni hosil qilishning bu usuli tartib birlikli fazolarni yangicha talqinda tadqiq qilish imkonini beradi.

Kalit so'zlar: Metrika; norma; tartib birlik; A-qismfazo.

Пространства с порядковой единицей: новый взгляд

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В настоящей работе в частично упорядоченном линейном пространстве рассматривается база, порождающая в ней выпуклую топологию Хаусдорфа. Далее, используя элементы этой базы, получаем метрику и норму в данном пространстве. Мы показываем, что полученное пространство – это пространство с порядковой единицей. Такой способ получения частично упорядоченного топологического линейного пространства дает возможность исследовать пространства с единицей порядка с новой точки зрения.

Ключевые слова: Метрика; норма; порядковая единица; A-подпространство.

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