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MUAVR FORMULASINING BA’ZI TATBIQLARI

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Ushbu maqolada kompleks analizni elementar matematikaning asosiy bo‘limlaridan biri bo‘lgan trigonometriyaga tatbiqini keltiramiz. Muavr formulasidan foydalanib trigonometrik funksiyalar uchun daraja pasaytirish formulasini keltirib chiqaramiz

Kalit so‘zlar: *Kompleks son, Muavr formulasi, Nyuton binomi.*

В этой статье мы представляем применение комплексного анализа в тригонометрии, одном из основных разделов элементарной математики. Используя формулу Муавра, выводим формулу понижения степени для тригонометрических функций

Ключевые слова: *Комплексное число, формула Муавра, бином Ньютона.*

In this article, we present an application of complex analysis to trigonometry, one of the main branches of elementary mathematics. Using Moivre’s formula, we derive a formula for reducing the degree of trigonometric functions.

Keywords: *Complex number, Moivre’s formula, Newton’s binomial.*

Ma’lumki, trigonometrik funksiyalar elementar matematikaning asosiy hamda maktab o‘quvchilari tushunishlari uchun murakkab bo‘lgan bo‘limlaridan biri xisoblanadi. Biz ushbu maqolada trigonometrik funksiyalar uchun daraja pasaytirish formulasini keltirib chiqaramiz. Buning uchun moduli 1 ga teng bo‘lgan kompleks sonni olaylik, ya’ni

$$z = cos\,\theta + i\sin\theta. \qquad (1)$$

U holda bu kompleks songa teskari kompleks son quyidagicha topiladi:

$$\frac{1}{z} = cos\,\theta - i\sin\theta. \qquad (2)$$

(1) va (2) formulalardan quyidagini xosil qilamiz

$$z + \frac{1}{z} = 2cos\,\theta. \qquad (3)$$

Kompleks analizda ma'lumki, $|z| = 1$ bo'lsa, u holda Muavr formulsiga ko'ra

$$z^n = \left(\cos\theta + i\sin\theta\right)^n = \cos n\theta + i\sin n\theta. \qquad (4)$$

Xuddi shuningdek, Muavr formulasini (2) uchun qo'llasak

$$\frac{1}{z^n} = \cos n\theta - i\sin n\theta. \qquad (5)$$

(4) va (5) formulalardan quyidagini olamiz

$$z^n + \frac{1}{z^n} = 2\cos n\theta. \qquad (6)$$

Ma'lumki, haqiqiy analizda ko'phadni darajaga ko'tarish uchun Nyuton binomidan foydalaniladi. (3) uchun Nyuton binomidan foydalanamiz. U holda

$$\left(2\cos\theta\right)^n = \left(z + \frac{1}{z}\right)^n = z^n + C_n^1 z^{n-1}\left(\frac{1}{z}\right)^1 + C_n^2 z^{n-2}\left(\frac{1}{z}\right)^2 + C_n^3 z^{n-3}\left(\frac{1}{z}\right)^3 + \ldots$$

$$+C_n^r z^{n-r}\left(\frac{1}{z}\right)^r + \ldots + C_n^{n-2} z^2\left(\frac{1}{z}\right)^{n-2} + C_n^{n-1} z^1\left(\frac{1}{z}\right)^{n-1} + C_n^n\left(\frac{1}{z}\right)^n \qquad (7)$$

Kombinatorika nazariyasidan quyidagilar ma'lum:

$$C_n^1 = C_n^{n-1}, \quad C_n^2 = C_n^{n-2}, \quad C_n^3 = C_n^{n-3}, \quad \ldots, \quad C_n^r = C_n^{n-r}.$$

Oxirgi tengliklardan foydalanib, (7) dan quyidagi ifodani tuzamiz

$$\left(2\cos\theta\right)^n=\left(z+\frac{1}{z}\right)^n=\left(z^n+\frac{1}{z^n}\right)+C_n^1\left(z^{n-2}+\frac{1}{z^{n-2}}\right)+C_n^2\left(z^{n-4}+\frac{1}{z^{n-4}}\right)+\ldots$$

Natijada (6) ga ko‘ra

$$2^{n-1}\cos^n\theta=\cos n\theta+C_n^1\cos\left(n-2\right)\theta+C_n^2\cos\left(n-4\right)\theta+\ldots \qquad (8)$$

Bu yerda n ni juft va toq hollarini alohida qarab chiqamiz.

I hol. n soni toq bo‘lsin. Bu holda (7) dagi hadlarimiz soni juft bo‘lib, o‘zaro juftlik hosil qila oladi. Oxirgi juftlikda (7) ning ikkita o‘rta hadlari $C_n^{\frac{n-1}{2}}$ va $C_n^{\frac{n+1}{2}}$ bo‘lib. Ularning yig‘indisi $C_n^{\frac{n-1}{2}}\left(z+\frac{1}{z}\right)=C_n^{\frac{n-1}{2}}2\cos\theta$ ga teng. Natijada (8) quyidagi ko‘rinishga keladi

$$2^{n-1}\cos^n\theta=\cos n\theta+C_n^1\cos\left(n-2\right)\theta+C_n^2\cos\left(n-4\right)\theta+\ldots+C_n^{\frac{n-1}{2}}\cos\theta.$$

Misol 1. $\cos^5\theta$ ni θ burchak bo‘yicha yoying.

Yechish: n toq bo‘lgani uchun yuqoridagi formuladan quyidagini olamiz

$$\left(2\cos\theta\right)^5=\left(z+\frac{1}{z}\right)^5=\left(z^5+\frac{1}{z^5}\right)+C_5^1\left(z^3+\frac{1}{z^3}\right)+C_5^2\left(z+\frac{1}{z}\right).$$

Natijada (6) tenglikka ko‘ra

$$\cos^5\theta=\frac{1}{2^4}\left(\cos5\theta+5\cos3\theta+10\cos\theta\right).$$

II hol. n soni juft bo‘lsin. Bu holda (7) dagi hadlarimiz soni toq bo‘lib, o‘zaro juftlik hosil qilganimizdan so‘ng juftlik hosil qilmaydigan bitta o‘rta had qoladi. Bu had *z* ga bo‘g‘liq bo‘lmay bo‘lmay $C_n^{\frac{n}{2}}$ ga teng bo'ladi. Demak, n juft bo'lganda (8) ifodaning oxirgi qo'shiluvchisi $C_n^{\frac{n}{2}}$ bo'ladi. U holda

$$2^{n-1}\cos^n\theta = \cos n\theta + C_n^1\cos(n-2)\theta + C_n^2\cos(n-4)\theta + \ldots + C_n^{\frac{n}{2}}.$$

Misol 2. $\cos^6\theta$ ni θ burchak bo'yicha yoying.

Yechish: n juft bo'lgani uchun (8) formulani quyidagicha qo'llaymiz

$$(2\cos\theta)^6 = \left(z+\frac{1}{z}\right)^6 = \left(z^6+\frac{1}{z^6}\right) + C_6^1\left(z^4+\frac{1}{z^4}\right) + C_6^2\left(z^2+\frac{1}{z^2}\right) + C_6^3.$$

Natijada (6) tenglikka ko'ra

$$\cos^6\theta = \frac{1}{2^5}\left(\cos 6\theta + 6\cdot\cos 4\theta + 15\cdot\cos 2\theta + 10\right)$$

Endi sinus uchun daraja pasaytirish formulasini keltirib chiqaramiz. (1) va (2) formulalardan quyidagini hosil qilamiz:

$$z - \frac{1}{z} = 2i\sin\theta. \tag{9}$$

(4) va (5) formulalardan esa

$$z^n - \frac{1}{z^n} = 2i\sin n\theta. \tag{10}$$

Agar (9) uchun Nyuton binomidan foydalansak, u holda quyidagi ko'rinishga keladi.

$$(2i\sin\theta)^n = \left(z-\frac{1}{z}\right)^n = z^n + C_n^1 z^{n-1}\left(-\frac{1}{z}\right)^1 + C_n^2 z^{n-2}\left(-\frac{1}{z}\right)^2 + C_n^3 z^{n-3}\left(-\frac{1}{z}\right)^3 + \ldots$$

(11)

$$+C_n^r z^{n-r}\left(-\frac{1}{z}\right)^r + \ldots + C_n^{n-2} z^2\left(-\frac{1}{z}\right)^{n-2} + C_n^{n-1} z^1\left(-\frac{1}{z}\right)^{n-1} + C_n^n\left(-\frac{1}{z}\right)^n.$$

Endi n natural sonini juft va toq hollarini ko‘ramiz.

I hol. n soni toq bo‘lsin. Bu holda (11) dagi hadlarimiz soni juft bo‘lib, o‘zaro juftlik hosil qila oladi. Agar

$$\left(-\frac{1}{z}\right)^{n}=-\frac{1}{z^{n}} \quad , \left(-\frac{1}{z}\right)^{n-1}=\frac{1}{z^{n-1}} \quad , \quad i^{n}=i\left(-1\right)^{\frac{n-1}{2}}$$

tengliklardan foydalansak, u holda (11) dan quyidagi ifodani tuzamiz:

$$2^{n}\cdot i\left(-1\right)^{\frac{n-1}{2}}\cdot\sin^{n}\theta=z^{n}-C_{n}^{1}z^{n-2}+C_{n}^{2}z^{n-4}-....-C_{n}^{2}\left(-\frac{1}{z}\right)^{n-4}+C_{n}^{1}\left(\frac{1}{z}\right)^{n-2}-\left(\frac{1}{z}\right)^{n}=$$

$$=\left(z^{n}-\frac{1}{z^{n}}\right)-C_{n}^{1}\left(z^{n-2}-\frac{1}{z^{n-2}}\right)+C_{n}^{2}\left(z^{n-4}-\frac{1}{z^{n-4}}\right)-...$$

Agar yuqoridagi tenglikka (10) formulani qo‘llasak

$$2^{n-1}\left(-1\right)^{\frac{n-1}{2}}\sin^{n}\theta=\sin n\theta-C_{n}^{1}\sin\left(n-2\right)\theta+C_{n}^{2}\sin\left(n-4\right)\theta-... \qquad (12)$$

Oxirgi juftlikda (11) ning ikkita o‘rta hadi $C_{n}^{\frac{n-1}{2}}$ va $C_{n}^{\frac{n+1}{2}}$ bo‘lib Ularning yig‘indisi

$$\frac{1}{2i}\left[C_{n}^{\frac{n-1}{2}}\cdot z\cdot\left(-1\right)^{\frac{n-1}{2}}+C_{n}^{\frac{n+1}{2}}\cdot\frac{1}{z}\cdot\left(-1\right)^{\frac{n+1}{2}}\right]=\frac{1}{2i}C_{n}^{\frac{n-1}{2}}\cdot\left(-1\right)^{\frac{n-1}{2}}\left(z-\frac{1}{z}\right)=$$

$$=\frac{1}{2i}C_{n}^{\frac{n-1}{2}}\cdot\left(-1\right)^{\frac{n-1}{2}}\cdot 2i\sin\theta=C_{n}^{\frac{n-1}{2}}\cdot\left(-1\right)^{\frac{n-1}{2}}\cdot\sin\theta.$$

Demak, n soni toq bo‘lsa (12) ning oxirgi qo‘shiluvchisi $C_{n}^{\frac{n-1}{2}}\left(-1\right)^{\frac{n-1}{2}}\sin\theta$ga teng bo‘ladi. Natijada (12) quyidagi ko‘rinishga keladi

$$2^{n-1}\left(-1\right)^{\frac{n-1}{2}}\sin^{n}\theta=\sin n\theta-C_{n}^{1}\sin\left(n-2\right)\theta+$$
$$+C_{n}^{2}\sin\left(n-4\right)\theta-...+\left(-1\right)^{\frac{n-1}{2}}C_{n}^{\frac{n-1}{2}}\sin\theta.$$

Misol 3. $\sin^7\theta$ ni θ burchak bo'yicha yoying.

Yechish: n toq bo'lgani uchun (12) formulani quyidagicha qo'llaymiz:

$$(2i\sin\theta)^7=\left(z-\frac{1}{z}\right)^7=\left(z^7-\frac{1}{z^7}\right)-C_7^1\left(z^5-\frac{1}{z^5}\right)+C_7^2\left(z^3-\frac{1}{z^3}\right)-C_7^3\left(z-\frac{1}{z}\right).$$

Natijada (10) tenglikka ko'ra

$$\sin^7\theta=-\frac{1}{64}[\sin 7\theta-7\sin 5\theta+21\sin 3\theta-35\sin\theta].$$

II hol. n soni juft bo'lsin. Bu holda (11) dagi hadlarimiz soni toq bo'lib, o'zaro juftlik hosil qilganimizdan so'ng juftlik hosil qilmaydigan bitta o'rta had qoladi. Bu had z ga bo'g'liq bo'lmay bo'lmay $C_n^{\frac{n}{2}}$ ga tengdir.

$$\left(-\frac{1}{z}\right)^n=\frac{1}{z^n},\quad \left(-\frac{1}{z}\right)^{n-1}=-\frac{1}{z^{n-1}},\quad i^n=\left(i^2\right)^{\frac{n}{2}}=(-1)^{\frac{n}{2}}$$

tengliklardan foydalanib, (12) ga qo'yamiz:

$$(-1)^{\frac{n}{2}}\cdot\sin\theta=z^n-C_n^1z^{n-2}+C_n^2z^{n-4}+C_n^3z^{n-6}+....+C_n^2\left(\frac{1}{z}\right)^{n-4}-C_n^1\left(\frac{1}{z}\right)^{n-2}+\left(\frac{1}{z}\right)^n=$$

$$=\left(z^n+\frac{1}{z^n}\right)-C_n^1\left(z^{n-2}+\frac{1}{z^{n-2}}\right)+C_n^2\left(z^{n-4}+\frac{1}{z^{n-4}}\right)-...$$

Agar oxirgi tenglikka (6) formulani qo'llasak, u holda quyidagiga ega bo'lamiz.

$$2^{n-1}(-1)^{\frac{n}{2}}\sin^n\theta=\cos n\theta-C_n^1\cos(n-2)\theta+$$

$$+C_n^2\cos(n-4)\theta-...+(-1)^{\frac{n}{2}}C_n^{\frac{n}{2}}.$$

Misol 4. $\sin^6\theta$ ni θ burchak bo'yicha yoying.

Yechish: *n* juft bo'lgani uchun (12) formulani quyidagicha qo'llaymiz:

$$\left(2i\sin\theta\right)^6=\left(z-\frac{1}{z}\right)^6=\left(z^6+\frac{1}{z^6}\right)-C_6^1\left(z^4+\frac{1}{z^4}\right)+C_6^2\left(z^2+\frac{1}{z^2}\right)-C_6^3$$

Natijada (6) tenglikka ko'ra

$$\sin^6\theta=-\frac{1}{32}(\cos 6\theta-6\cos 4\theta+15\cos 2\theta-10)$$

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MUNDARIJA

ILMIY-OMMABOP BO‘LIM

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