



1
2025

FIZIKA, MATEMATIKA *va* INFORMATIKA

ILMIY-USLUBIY JURNAL

2001-yildan chiqa boshlagan

Toshkent – 2025

**Bosh muharrir – Xolboy IBRAIMOV pedagogika fanlari
doktori, Akademik**

**Muharrir – Bakhshillo Amrillayevich OLIMOV f.-m.f.n.,
v.v.b., professor**

**Mas’ul kotib – Riskeldi Musamatovich Turgunbayev f.-m.f.n.,
professor**

TAHRIR HAY’ATI A’ZOLARI

IBRAIMOV Xolboy

AYUPOV Shavkat Abdullayevich

OLIMOV Bakhshillo Amrillayevich

AKMALOV Abbos Akromovich

KUVANDIKOV Oblokul

TURSUNMETOV Kamiljan

MAKHMUDOV Yusup Ganiyevich

TURGUNBAYEV Riskeldi Musamatovich

MUSURMONOV Raxmatilla

MAXMUDOV Abdulxalim Xamidovich

MAMARAJABOV Mirsalim Elmirzayevich

XUJANOV Erkin Berdiyevich

ORLOVA Tatyana Alekseyevna

BOZOROV Erkin Xojiyevich

BARAKAYEV Murod

Muassis:

**T.N.Qori Niyoziy nomidagi O‘zbekiston Pedagogika fanlari
ilmiy tadqiqot instituti
71 256 53 57**



MUAVR FORMULASINING BA'ZI TATBIQLARI

M.T. Maxammadaliyev, NamDU dotsenti
Sh.A. Abduraximova, NamDU talabasi
N.K. Razokova, ChDPU katta o'qituvchisi

Ushbu maqolada kompleks analizni elementar matematikaning asosiy bo'limlaridan biri bo'lgan trigonometriyaga tatbiqini keltiramiz. Muavr formulasidan foydalanib trigonometrik funksiyalar uchun daraja pasaytirish formulasini keltirib chiqaramiz

Kalit so'zlar: Kompleks son, Muavr formulasi, Nyuton binomi.

В этой статье мы представляем применение комплексного анализа в тригонометрии, одном из основных разделов элементарной математики. Используя формулу Муавра, выводим формулу понижения степени для тригонометрических функций

Ключевые слова: Комплексное число, формула Муавра, бином Ньютона.

In this article, we present an application of complex analysis to trigonometry, one of the main branches of elementary mathematics. Using Moivre's formula, we derive a formula for reducing the degree of trigonometric functions.

Keywords: Complex number, Moivre's formula, Newton's binomial.

Ma'lumki, trigonometrik funksiyalar elementar matematikaning asosiy hamda maktab o'quvchilari tushunishlari uchun murakkab bo'lgan bo'limlaridan biri xisoblanadi. Biz ushbu maqolada trigonometrik funksiyalar uchun daraja pasaytirish formulasini keltirib chiqaramiz. Buning uchun moduli 1 ga teng bo'lgan kompleks sonni olaylik, ya'ni

$$z = \cos \theta + i \sin \theta. \quad (1)$$

U holda bu kompleks songa teskari kompleks son quyidagicha topiladi:



$$\frac{1}{z} = \cos \theta - i \sin \theta. \quad (2)$$

(1) va (2) formulalardan quyidagini xosil qilamiz

$$z + \frac{1}{z} = 2 \cos \theta. \quad (3)$$

Kompleks analizda ma'lumki, $|z| = 1$ bo'lsa, u holda Muavr formulsiga ko'ra

$$z^n = (\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta. \quad (4)$$

Xuddi shuningdek, Muavr formulasini (2) uchun qo'llasak

$$\frac{1}{z^n} = \cos n\theta - i \sin n\theta. \quad (5)$$

(4) va (5) formulalardan quyidagini olamiz

$$z^n + \frac{1}{z^n} = 2 \cos n\theta. \quad (6)$$

Ma'lumki, haqiqiy analizda ko'phadni darajaga ko'tarish uchun Nyuton binomidan foydalaniladi. (3) uchun Nyuton binomidan foydalanamiz. U holda

$$\begin{aligned} (2 \cos \theta)^n &= \left(z + \frac{1}{z} \right)^n = z^n + C_n^1 z^{n-1} \left(\frac{1}{z} \right)^1 + C_n^2 z^{n-2} \left(\frac{1}{z} \right)^2 + C_n^3 z^{n-3} \left(\frac{1}{z} \right)^3 + \dots \\ &+ C_n^r z^{n-r} \left(\frac{1}{z} \right)^r + \dots + C_n^{n-2} z^2 \left(\frac{1}{z} \right)^{n-2} + C_n^{n-1} z^1 \left(\frac{1}{z} \right)^{n-1} + C_n^n \left(\frac{1}{z} \right)^n \end{aligned} \quad (7)$$

Kombinatorika nazariyasidan quyidagilar ma'lum:

$$C_n^1 = C_n^{n-1}, \quad C_n^2 = C_n^{n-2}, \quad C_n^3 = C_n^{n-3}, \quad \dots, \quad C_n^r = C_n^{n-r}.$$



Oxirgi tengliklardan foydalanib, (7) dan quyidagi ifodani tuzamiz

$$(2\cos\theta)^n = \left(z + \frac{1}{z}\right)^n = \left(z^n + \frac{1}{z^n}\right) + C_n^1 \left(z^{n-2} + \frac{1}{z^{n-2}}\right) + C_n^2 \left(z^{n-4} + \frac{1}{z^{n-4}}\right) + \dots$$

Natijada (6) ga ko'ra

$$2^{n-1} \cos^n \theta = \cos n\theta + C_n^1 \cos(n-2)\theta + C_n^2 \cos(n-4)\theta + \dots \quad (8)$$

Bu yerda n ni juft va toq hollarini alohida qarab chiqamiz.

I hol. n soni toq bo'lsin. Bu holda (7) dagi hadlarimiz soni juft bo'lib, o'zaro juftlik hosil qila oladi. Oxirgi juftlikda (7) ning

ikkita o'rta hadlari $C_n^{\frac{n-1}{2}}$ va $C_n^{\frac{n+1}{2}}$ bo'lib. Ularning yig'indisi

$C_n^{\frac{n-1}{2}} \left(z + \frac{1}{z}\right) = C_n^{\frac{n-1}{2}} 2\cos\theta$ ga teng. Natijada (8) quyidagi ko'rinishga keladi

$$2^{n-1} \cos^n \theta = \cos n\theta + C_n^1 \cos(n-2)\theta + C_n^2 \cos(n-4)\theta + \dots + C_n^{\frac{n-1}{2}} \cos\theta.$$

Misol 1. $\cos^5 \theta$ ni θ burchak bo'yicha yoying.

Yechish: n toq bo'lgani uchun yuqoridagi formuladan quyidagini olamiz

$$(2\cos\theta)^5 = \left(z + \frac{1}{z}\right)^5 = \left(z^5 + \frac{1}{z^5}\right) + C_5^1 \left(z^3 + \frac{1}{z^3}\right) + C_5^2 \left(z + \frac{1}{z}\right).$$

Natijada (6) tenglikka ko'ra

$$\cos^5 \theta = \frac{1}{2^4} (\cos 5\theta + 5\cos 3\theta + 10\cos \theta).$$

II hol. n soni juft bo'lsin. Bu holda (7) dagi hadlarimiz soni toq bo'lib, o'zaro juftlik hosil qilganimizdan so'ng juftlik hosil qilmaydigan bitta o'rta had qoladi. Bu had z ga bo'lg'liq bo'lmay bo'lmay $C_n^{\frac{n}{2}}$ ga teng

bo'ladi. Demak, n juft bo'lganda (8) ifodaning oxirgi qo'shiluvchisi $C_n^{\frac{n}{2}}$ bo'ladi. U holda

$$2^{n-1} \cos^n \theta = \cos n\theta + C_n^1 \cos(n-2)\theta + C_n^2 \cos(n-4)\theta + \dots + C_n^{\frac{n}{2}}.$$

Misol 2. $\cos^6 \theta$ ni θ burchak bo'yicha yoying.

Yechish: n juft bo'lgani uchun (8) formulani quyidagicha qo'llaymiz

$$(2 \cos \theta)^6 = \left(z + \frac{1}{z}\right)^6 = \left(z^6 + \frac{1}{z^6}\right) + C_6^1 \left(z^4 + \frac{1}{z^4}\right) + C_6^2 \left(z^2 + \frac{1}{z^2}\right) + C_6^3.$$

Natijada (6) tenglikka ko'ra

$$\cos^6 \theta = \frac{1}{2^5} (\cos 6\theta + 6 \cdot \cos 4\theta + 15 \cdot \cos 2\theta + 10)$$

Endi sinus uchun daraja pasaytirish formulasini keltirib chiqaramiz.

(1) va (2) formulalardan quyidagini hosil qilamiz:

$$z - \frac{1}{z} = 2i \sin \theta. \quad (9)$$

(4) va (5) formulalardan esa

$$z^n - \frac{1}{z^n} = 2i \sin n\theta. \quad (10)$$

Agar (9) uchun Nyuton binomidan foydalansak, u holda quyidagi ko'rinishga keladi.

$$(2i \sin \theta)^n = \left(z - \frac{1}{z}\right)^n = z^n + C_n^1 z^{n-1} \left(-\frac{1}{z}\right) + C_n^2 z^{n-2} \left(-\frac{1}{z}\right)^2 + C_n^3 z^{n-3} \left(-\frac{1}{z}\right)^3 + \dots \quad (11)$$

$$+ C_n^r z^{n-r} \left(-\frac{1}{z}\right)^r + \dots + C_n^{n-2} z^2 \left(-\frac{1}{z}\right)^{n-2} + C_n^{n-1} z^1 \left(-\frac{1}{z}\right)^{n-1} + C_n^n \left(-\frac{1}{z}\right)^n.$$



Endi n natural sonini juft va toq hollarini ko'ramiz.

I hol. n soni toq bo'lsin. Bu holda (11) dagi hadlarimiz soni juft bo'lib, o'zaro juftlik hosil qila oladi. Agar

$$\left(-\frac{1}{z}\right)^n = -\frac{1}{z^n}, \quad \left(-\frac{1}{z}\right)^{n-1} = \frac{1}{z^{n-1}}, \quad i^n = i(-1)^{\frac{n-1}{2}}$$

tengliklardan foydalansak, u holda (11) dan quyidagi ifodani tuzamiz:

$$\begin{aligned} 2^n \cdot i(-1)^{\frac{n-1}{2}} \cdot \sin^n \theta &= z^n - C_n^1 z^{n-2} + C_n^2 z^{n-4} - \dots - C_n^{\frac{n-1}{2}} \left(-\frac{1}{z}\right)^{n-4} + C_n^1 \left(\frac{1}{z}\right)^{n-2} - \left(\frac{1}{z}\right)^n = \\ &= \left(z^n - \frac{1}{z^n}\right) - C_n^1 \left(z^{n-2} - \frac{1}{z^{n-2}}\right) + C_n^2 \left(z^{n-4} - \frac{1}{z^{n-4}}\right) - \dots \end{aligned}$$

Agar yuqoridagi tenglikka (10) formulani qo'llasak

$$2^{n-1} (-1)^{\frac{n-1}{2}} \sin^n \theta = \sin n\theta - C_n^1 \sin(n-2)\theta + C_n^2 \sin(n-4)\theta - \dots_{n+1} \quad (12)$$

Oxirgi juftlikda (11) ning ikkita o'rta hadi $C_n^{\frac{n-1}{2}}$ va $C_n^{\frac{n-1}{2}}$ bo'lib Ularning yig'indisi

$$\begin{aligned} \frac{1}{2i} \left[C_n^{\frac{n-1}{2}} \cdot z \cdot (-1)^{\frac{n-1}{2}} + C_n^{\frac{n+1}{2}} \cdot \frac{1}{z} \cdot (-1)^{\frac{n+1}{2}} \right] &= \frac{1}{2i} C_n^{\frac{n-1}{2}} \cdot (-1)^{\frac{n-1}{2}} \left(z - \frac{1}{z} \right) = \\ &= \frac{1}{2i} C_n^{\frac{n-1}{2}} \cdot (-1)^{\frac{n-1}{2}} \cdot 2i \sin \theta = C_n^{\frac{n-1}{2}} \cdot (-1)^{\frac{n-1}{2}} \cdot \sin \theta. \end{aligned}$$

Demak, n soni toq bo'lsa (12) ning oxirgi qo'shiluvchisi $C_n^{\frac{n-1}{2}} (-1)^{\frac{n-1}{2}} \sin \theta$ ga teng bo'ladi. Natijada (12) quyidagi ko'rinishga keladi

$$\begin{aligned} 2^{n-1} (-1)^{\frac{n-1}{2}} \sin^n \theta &= \sin n\theta - C_n^1 \sin(n-2)\theta + \\ &+ C_n^2 \sin(n-4)\theta - \dots + (-1)^{\frac{n-1}{2}} C_n^{\frac{n-1}{2}} \sin \theta. \end{aligned}$$

Misol 3. $\sin^7 \theta$ ni θ burchak bo'yicha yoying.

Yechish: n toq bo'lgani uchun (12) formulani quyidagicha qo'llaymiz:

$$(2i \sin \theta)^7 = \left(z - \frac{1}{z}\right)^7 = \left(z^7 - \frac{1}{z^7}\right) - C_7^1 \left(z^5 - \frac{1}{z^5}\right) + C_7^2 \left(z^3 - \frac{1}{z^3}\right) - C_7^3 \left(z - \frac{1}{z}\right).$$

Natijada (10) tenglikka ko'ra

$$\sin^7 \theta = -\frac{1}{64} [\sin 7\theta - 7 \sin 5\theta + 21 \sin 3\theta - 35 \sin \theta].$$

II hol. n soni juft bo'lsin. Bu holda (11) dagi hadlarimiz soni toq bo'lib, o'zaro juftlik hosil qilganimizdan so'ng juftlik hosil qilmaydigan bitta o'rta had qoladi. Bu had z ga bo'g'liq bo'lmay bo'lmay C_n^2 ga tengdir.

$$\left(-\frac{1}{z}\right)^n = \frac{1}{z^n}, \quad \left(-\frac{1}{z}\right)^{n-1} = -\frac{1}{z^{n-1}}, \quad i^n = (i^2)^{\frac{n}{2}} = (-1)^{\frac{n}{2}}$$

tengliklardan foydalanib, (12) ga qo'yamiz:

$$\begin{aligned} (-1)^{\frac{n}{2}} \cdot \sin \theta &= z^n - C_n^1 z^{n-2} + C_n^2 z^{n-4} + C_n^3 z^{n-6} + \dots + C_n^2 \left(\frac{1}{z}\right)^{n-2} - C_n^1 \left(\frac{1}{z}\right)^{n-1} + \left(\frac{1}{z}\right)^n = \\ &= \left(z^n + \frac{1}{z^n}\right) - C_n^1 \left(z^{n-2} + \frac{1}{z^{n-2}}\right) + C_n^2 \left(z^{n-4} + \frac{1}{z^{n-4}}\right) - \dots \end{aligned}$$

Agar oxirgi tenglikka (6) formulani qo'llasak, u holda quyidagiga ega bo'lamiz.

$$\begin{aligned} 2^{n-1} (-1)^{\frac{n}{2}} \sin^n \theta &= \cos n\theta - C_n^1 \cos(n-2)\theta + \\ &+ C_n^2 \cos(n-4)\theta - \dots + (-1)^{\frac{n}{2}} C_n^{\frac{n}{2}}. \end{aligned}$$

Misol 4. $\sin^6 \theta$ ni θ burchak bo'yicha yoying.



Yechish: n juft bo'lgani uchun (12) formulani quyidagicha qo'llaymiz:

$$(2i \sin \theta)^6 = \left(z - \frac{1}{z}\right)^6 = \left(z^6 + \frac{1}{z^6}\right) - C_6^1 \left(z^4 + \frac{1}{z^4}\right) + C_6^2 \left(z^2 + \frac{1}{z^2}\right) - C_6^3$$

Natijada (6) tenglikka ko'ra

$$\sin^6 \theta = -\frac{1}{32}(\cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10)$$

Adabiyotlar:

1. Berg Ch. Complex Analysis. Department of Mathematical Sciences Universitetsparken. Copenhagen. 192 p., 2012.
2. Kishan H. Trigonometry. Atlantic publishing and Distributors. Delhi 2005



MUNDARIJA

ILMIY-OMMABOP BO‘LIM

Raxmatullayeva Gulira’no Valijon qizi. Molekulyar fizikadan qiziqarli va noodatiy topshiriqlar orqali o‘quvchilarning fanga bo‘lgan qiziqishini oshirish metodikasi	3
N. Dilmuradov, E.O.Sharipov, X.U.Chuyanov. Hosila yordamida modellash tirish.....	11
K. N. Po‘latova. Geometrik masalalar: ijodiy tafakkurni shakllantirishning asosiy omili	20
N.R. Zaynalov, A.A. Shakarov, B.Sh. Ibrohimov. Xitoy qoldiqlar teoremasiga doir misollar.....	36
Zaripov Olimjan Kuvandiq o‘g‘li. Oliy ta’lim muassasalarida fanlararo integratsiyani mobil ta’lim texnologiyalari asosida o‘qitish metodikasini takomillashtirish	42
X.A. Abdimurodova, X.N. Izzatilloeva, M.D. Yo‘ldoshyeva. Kasr tartibli hosila qatnashgan differensial tenglamalarni yechish	51
M.T. Maxammadaliyev, Sh.A. Abduraximova, N.K. Razokova. Muavr formulasining ba’zi tatbiqlari.....	60
M.K. Yuldashyev. Informatika fanini o‘qitishning ilmiy-metodik asoslari va zamonaviy tendensiyalari.....	67

MATEMATIKA JOZIBASI

Najmiddinova Durdona Oybek qizi. Astronomiyadan masalalar yechish texnologiyalari	74
N.X.Qurbonov. Uchburchak tengsizligi yordamida yechiladigan ayrim masalalarning yechish usullari.....	82

ILG‘OR TAJRIBA VA O‘QITISH METODIKASI

У. Ш. Убайдуллаев. Краевая задача с разрывными условиями склеивания для уравнения смешанного типа типа с оператором дробного порядка в смысле Капуто в прямоугольной области.....	91
S.A.Alibekov. Raqamli transformatsiya: qishloq xo‘jaligida axborot texnologiyalarining samaradorligi va istiqbollari.....	98
T.A. Орлова, М.Ш. Фатхуллаева. Игровые методы, применяемые в обучение учащихся 7 классов по физике в общеобразовательных школах	105



E.B. Maxmanov. Fizikadan ta'lim natijalarini baholashga asoslangan darslarni loyihalash va o'tkazish.....	112
C.C. Эгамбердиев. Инновационная методика проведения современного урока по математике для учеников начальных классов.....	127

OLIMPIADA VA MASALALAR YECHISH BO'LIMI

<i>Masalalar va yechimlar</i>	135
-------------------------------------	-----

TALAB, TAKLIF VA TAHLIL

Masodiqova Dilruxsor Ravshanjon qizi. Inklyuzivlik tamoyillarini hisobga olgan holda pedagogik jarayonni tashkil etish.....	148
M. B.Dusmuratov. Fizik kattalikning ekstremal qiymatlarini hosila yordamida aniqlashga oid masalalar yechish	152
M. O. Toxirova. Maktabda nyuton qonunlari mavzusini o'qitishda fanlararo bog'lanish	162
A.F.Qorayev. Refleksiv videotreninglar asosida ta'lim jarayonini samarali tashkil etish	167
R.N. To'rayev, T.Yu. To'rayev, M. Mamayusupov. Elektron ta'lim muhitida mustaqil ta'limni takomillashtirish bosqichlari.....	176
K.F.Yusupova. Tabiiy fanlarni o'qitishda integrativ materiallarni tanlash tamoyillari.....	184
M.I. Djumayev. Raqamli ta'lim muhitida xalqaro standartlashtirish mohiyati.....	192
S.S. Jumanazarov. O'qituvchilarining malakasini oshirishda veb-kvest ta'lim texnologiyasidan foydalanish.....	200

