M.N.SOLAYEVA

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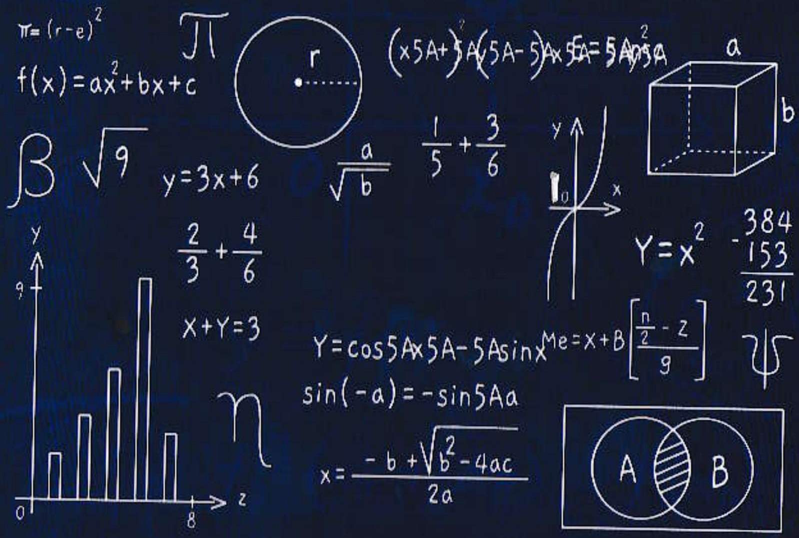
(CALCULAS) FANIDAN O‘QUV QO‘LLANMA



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MATEMATIK ANALIZ (Calculs)

FANIDAN

O'QUV QO'LLANMA

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O‘quv qo‘llanma xozirgi kundagi Oliy ta’lim va o‘rta maxsus ta’lim tizimining talablarini va bundan tashqari bugungi kunning talablarni inobatga olib tuzib chiqilgan. Ushbu qo‘llanmada oliy ta’lim muassasalarining axborot xavfsizligi, kompyuter injineringi va multimediya yo‘nalish talabalari uchun darslikdan tashqari qo‘llanma bo‘lib, o‘zlashtirish bir muncha qiyin bo‘lgan mavzular ham jamlangan.

Ushbu o‘quv qo‘llanmada ketma- ketlik limiti, funksiya va uning limiti tushunchalari, funksiya hosilasi, murakkab funksiyalarning hosilalari, aniqmas integrallar, aniq integrallar va ularning tadbiqlari kabi mavzular oliy ta’lim muassasasi talabalari uchun bir qancha qulaylashtirilgan ya’ni osonlilikdan qiyinlilikka tartibida tuzilgan va oliy ta’lim muassasalarining axborot xavfsizligi, kompyuter injineringi va multimediya yo‘nalish talabalari uchun tushunarli qilib bayon etib berilgan.

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Ushbu o‘quv qo‘llanma Oliy ta’lim, fan va innovatsiyalar vazirligi 2023 yil 29 maydagi 232-sonli buyrug‘iga asosan nashrga ruxsat etildi.

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KIRISH

Mazkur qo‘llanma «Matematik analiz (Calculs)» fani bo‘yicha o‘quv-uslubiy majmuaning tarkibiy qismlaridan biri bo‘lib, unda matematik analizning kirish qismi bo‘lgan sonlar ketma-ketligi, ketma-ketlik limiti, funksiya tushunchalarining kiritilishi, funksiya limiti tushunchasi, funksiya hosilasi va uning tadbiqlari, boshlang‘ich funksiya va aniqmas integrallar, aniq integrallar va ularning tadbiqlari keltirilgan. O‘quv qo‘llanma besh bobdan iborat. Birinchi bobda sonlar ketma-ketliklari va funksiyaning limiti tushunchalari ochib berilgan. Qo‘llanmaning ikkinchi bobi funksiya hosilasini topish va ularning tadbiqlari mavzularini o‘z ichiga olgan. Uchinchi bobda boshlang‘ich funksiya va aniqmas integrallar va ularni hisoblash usullari mavzulari o‘rganilgan. To‘rtinchi bobda esa aniq integrallar, ularni hisoblash usullari, Nyuton-Leybnits formulasi, integralni hisoblash usullari va ularning tadbiqlari talabalar uchun to‘liq o‘rganilgan. Oxirgi, beshinchi bobda barcha boblarda o‘rganilgan mavzularni to‘liq takrorlash uchun misol va masalalar berib o‘tilgan. O‘z navbatida, har bir bob tegishli paragraflarga bo‘lingan bo‘lib, har bir paragraf mavzuga ta’alluqli asosiy ta’riflar, tasdiqlar, teoremalarni o‘z ichiga oladi, shuningdek, ularning har biri an’anaviy misollarni batafsil tahlil yordamida yechish orqali namoyish qilingan. Qo‘llanmada jami 94 ta misol va masalalar yechilgan, 976 ta mustaqil yechish uchun misol va masalalar tavsiya qilingan hamda ularning javoblari berilgan. Hozirgi vaqtda amaliyotda bir necha yaxshi rivojlangan matematik dasturlar (Mathcad, Maple, Mathematica, Mathlab va h.k.) matematik masalalarni kompyuter imkoniyatlaridan foydalanib yechishda samarali natijalar bermoqda. Shu an’anadan chetda qolmaslik uchun, qo‘llanmada ba’zi bo‘limlar bo‘yicha misol va masalalar yechishda «Maple» tizimining qo‘llanilishi va uning qulayliklari namoyish etilgan. Ushbu qo‘llanmani yozishga muallifni undagan narsa, uning hozirgi kundagi oliy ta’lim muassasalarining axborot xavfsizligi, kompyuter injineringi va multimediya yo‘nalishlari uchun matematik analiz fani uchun mo‘ljallangan darslikdagi ba’zi bir misollarning murakkablik darajasi yuqori va o‘zlashtirish bir muncha oson bo‘lgan misollarning kiritilmaganligi. O‘ylaymizki, qo‘llanma o‘z o‘quvchilarini topadi va boshqa mavjud o‘quv adabiyotlari qatorida oliy ta’lim muassasalarining axborot xavfsizligi, kompyuter injineringi va multimediya yo‘nalish talabalari uchun ularga bilimlarini oshirishga ko‘mak beradi.

Bundan tashqari oliy ta'lim muassasalarining ba'zi yo'nalish talabalariga ham yaxshi ko'makchi bo'ladi deb o'ylaymiz. O'quv qo'llanma haqidagi fikr mulohazalar, undagi mavjud kamchiliklar bo'yicha takliflarni muallif mamnuniyat bilan qabul qiladi.

I BOB TO'PLAMLAR VA KETMA-KETLIKLAR NAZARIYASI

Mavzu:Matematik analiz fanining predmeti. To'plamlar va akslantirishlar

Reja:

1. To'plam tushunchasi. To'plamning elementi.
2. To'plamlar ustida amallar.
3. To'plamlarning Dekart ko'paytmalari tushunchalari.
4. To'plamlar ustida amallarning xossalari.

To'plam tushunchasi.

To'plam matematikaning boshlang'ich, ayni paytda muhim tushunchalaridan biri bo'lib, to'plam ta'riflanmaydigan tushunchalardan biridir. Uni ixtiyoriy tabiatli narsalarning (predmetlarning) ma'lum belgilar bo'yicha birlashmasi (majmuasi) sifatida tushuniladi. Masalan, javondagi kitoblar to'plami, bir nuqtadan o'tuvchi to'g'ri chiziqlar to'plami, $x^2-5x+6=0$ tenglamaning ildizlari to'plami deyilishi mumkin.

To'plamni tashkil etgan narsalar uning elementlari deyiladi.

Matematikada to'plamlar lotin alifbosining bosh xarflari bilan, ularning elementlari esa kichik xarflar bilan belgilanadi. Masalan, A,B,C -to'plamlar, a,b,c -to'plamning elementlari.

Ba'zan to'plamlar ularning elementlarini ko'rsatish bilan yoziladi:

Masalan:

$$A=\{2,4,6,8,10,12\},$$
$$N=\{1,2,3,\dots,n,\dots\},$$
$$Z=\{\dots,-2,-1,0,1,2,\dots\}.$$

Agar a biror A to'plamning elementi bo'lsa, $a\in A$ kabi yoziladi va "a element A to'plamga tegishli" deb o'qiladi. Agar a shu to'plamga tegishli bo'lmasa, uni $a\notin A$ kabi yoziladi va « a element A to'plamga tegishli emas » deb o'qiladi. Masalan, yuqoridagi A to'plamda $10\in A$, $15\notin A$.

Agar A chekli sondagi elementlardan tashkil topgan bo'lsa, u chekli to'plam, aks holda cheksiz to'plam deyiladi. Masalan, $A = \{2,4,6,8,10,12\}$ chekli to'plam, bir nuqtadan o'tuvchi barcha to'g'ri chiziqlar to'plami esa cheksiz to'plam bo'ladi.

Agar A to'plam elementlari orasida P xususiyatli elementlar bo'lmasa, u holda

$$\{x \in A \mid P\}$$

bitta ham elementga ega bo'lmagan to'plam bo'lib, uni bo'sh to'plam deyiladi. Bo'sh to'plam $\varnothing$ kabi belgilanadi.

Masalan, $x^2 + x + 1 = 0$ tenglamaning haqiqiy ildizlaridan iborat A bo'sh to'plam bo'ladi:

$$\varnothing = \{x \in A \mid x^2 + x + 1 = 0\}.$$

0≤x<7 tengsizlikni qanoatlantiruvchi butun sonlar to'plamini bevosita elementlarini ko'rsatish va xarakteristik xossa orqali yozing.

Yechish:

a) *A={0,1,2,3,4,5,6,}*

b)A={ x | x ∈Z, o≤x<7 }

N-natural sonlar, Z butun sonlar, R ratsional.

To'plamlar ustida amallar va ularning xossalari.

Ta'rif. A va B to'plamlar berilgan bo'lib, A to'plamning barcha elementlari B to'plamga tegishli bo'lsa, A **to'plam B ning qismi (qismiy to'plam) deyiladi va** $A \subset B$ (yoki $B \supset A$) kabi yoziladi.

Ta'rif. A va B to'plamlari berilgan bo'lib, A va B to'plamlarning barcha elementlaridan tashkil topgan C to'plamga A **va** B **to'plamlarning yig'indisi yoki birlashmasi deyiladi va** C=A   B kabi yoziladi.

Ta'rif. A va B to'plamlari berilgan bo'lib, A va B to'plamlarning barcha umumiy elementlaridan tashkil topgan C to'plamga A **va** B **to'plamlarning kesishmasi deyiladi va** A ∩ B=C kabi yoziladi

Masalan: $A = \{x,\ x \in R,\ 2 < x < 5\}$, $B = \{x,\ x \in R,\ 3 \le x + 3 < 7\}$ to'plamlar berilgan bo'lsin, u holda ushbu to'plamlarning birlashmasi va kesishmasini toping.

Yechish: berilgan to'plamlarning to'la sonli oraliqlarini topamiz va yuqoridagi ta'riflardan to'plamlarning birlashmasi va kesishmasini topamiz.

$$A = \{x,\ x \in R,\ 2 < x < 5\},\ B = \{x,\ x \in R,\ 3 \le x + 3 < 7\} \Rightarrow$$
$$\Rightarrow A = \{x,\ x \in R,\ 2 < x < 5\},\ B = \{x,\ x \in R,\ 0 \le x < 4\} \Rightarrow$$
$$\Rightarrow A \quad B = \{x,\ x \in R,\ 0 \le x < 5\}\ va\ A \cap B = \{x,\ x \in R,\ 2 < x < 4\}$$

To'plamlarning kesishmasining xossalari

1°. B⊂ A bo'lsa, A ∩ B= B

2°. A ∩ B= B∩ A (kommutativlik xossa).

3°A ∩(B ∩ C) =(A ∩ B) ∩ C = A ∩B ∩ C (assotsiativlik xossa).

4°. A ∩ ∅ = ∅

5°.A∩A=A.

6°. A   (B∩ C) =(A   *B)*∩(A   C)

7°A∩ (B   C) =(A∩*B)*   (A∩C)

To'plamlarning birlashmasining xossalari

1°. B⊂ A⇒A   B=A.

2°. A   B=B   A

3°. A   (B   C) =(A   B)   C=A   B   C

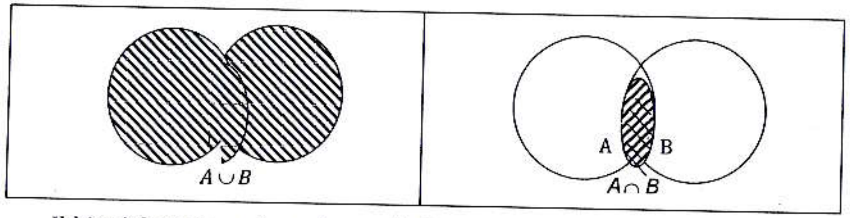
4°. A   ∅ =A.

5°. A   A=A.

Ta'rif: A to'plamning B to'plamga tegishli bo'lmagan elementlaridan tashkil topgan to'plam, A va B to'plamlar ayirmasi (yoki B to'plamning A to'plamgacha) bo'lgan to'ldiruvchisi deyilib, $A \setminus B$shaklda ifoda etiladi.

Ta'rif: A to'plamning B ga kirmagan va B to'plamning A ga kirmagan elementlaridan tashkil topgan to'plam A va B to'plamlarning simmetrik ayirmasi deyilib, A ∆B shaklda belgilanadi, ya'ni $A \Delta B = (A \setminus B) \cup (B \setminus A)$

To'plamlar ustida kiritilgan amallarni geometrik shakl ko'rinishida ifoda etaylik.



Ikki to'plamning ayirmasi xossalari

1°. $A\cap B=\varnothing \Rightarrow A\setminus B=A$.

2°. $B\subset A\Rightarrow A\setminus B=B'_A$

3°. $A=B\Rightarrow A\setminus B=\varnothing$

4°. $A\setminus(B\quad C)=(A\setminus B)\cap(A\setminus C)=A\setminus B\setminus C$.

5°. $A\setminus(B\cap C)=(A\setminus B)\quad(A\setminus C)$.

6°. $(A\cap B)'=A'\quad B'$.

7°. $(A\quad B)'=(A'\cap B')$.

To'plamlarning dekart (to'g'ri) ko'paytmasi.

Ta'rif. *A va B to'plamlarning dekart ko'paytmasi deb,* 1-elementi A to'plamdan, 2-elementi B to'plamdan olingan $(a; b)$ ko'rinishdagi barcha tartiblangan juftliklar to'plamiga aytiladi. Dekart ko'paytma $A\times B$ ko'rinishda belgilanadi: $A\times B=\{(a;\ b)\ |\ a\in A \text{ va } b\in B\}$

Masalan:

1) $A=\{2;3;4;5\}$, $B=\{a;b;c\}$ **bo'lsa,** $A\times B=\{(2;a),(2;b),(2;c),(3;a),(3;b),(3;c),(4;a),(4;b),(4;c),(5;a),(5;b),(5;c)\}$ **bo'ladi.**

2) $A=\{4,5,7\}$ va $B=\{-1,2,3,4\}$ to'plamlar berilgan bo'lsin. U holda A va B to'plamlarning to'g'ri ko'paytmasi quyidagicha bo'ladi:

$$A*B=\{(4;-1),(4;2),(4;3),(4;4),(5;-1),(5;2),(5;3),(5;4),(7;-1),(7;2),(7;3),(7;4)\}$$

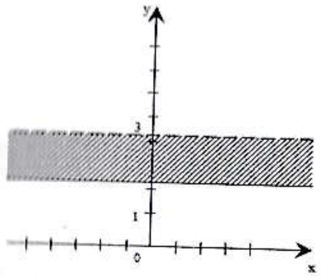
Agar biz to'g'ri ko'paytma elementi (x,y) dagi x ni biror nuqtani abssissasi, y ni esa ordinatasi desak, u holda bu to'g'ri ko'paytma tekislikdagi nuqtalar to'plamini ifodalaydi.

Boshqacha aytganda haqiqiy sonlar to'plami R ni R ga to'g'ri ko'paytmasi $R\times R$ ni tasvirlaydi.

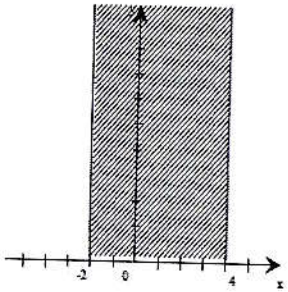
Sonli to'plamlar dekart ko'paytmasini koordinata tekisligida tasvirlash qulay. Masalan, $A=\{2;3;4\}$, $B=\{4;5\}$ bo'lsin, u holda $A\times B=\{(2;4),(2;5),(3;4),(3;5);(4;4),(4;5)\}$ bo'ladi.

Koordinata tekisligida shunday koordinatali nuqtalarni tasvirlaymizki, bunda A to'plam Ox o'qida va B to'plam Oy o'qida olinadi.

A=(2;3); B=R



A=[-2;4]; B=R



Dekart ko'paytmaning xossalari:

1. $A\times B\neq B\times A$.

2. $A\times(B\cup C)=(A\times B)\cup(A\times C)$.

3. $A\times(B\cap C)=(A\times B)\cap(A\times C)$.

Mustaqil yechish uchun misollar.

1-misol: Quyida beligan tenglamalarning yechimlar to‘plamini toping va bu qiymatlarni to‘plam shaklida yozing.

1.1 $x^4-10x^2+9=0$

1.2 $x^4-13x^2+36=0$

1.3 $x^4-5x^2+4=0$

1.4 $x^4-2x^2+1=0$

1.5 $x^4-10x^2+25=0$

1.6 $x^4-5x^2+6=0$

1.7 $x^4-6x^2+5=0$

1.8 $x^4-37x^2+36=0$

1.9 $x^4-50x^2+49=0$

1.10 $x^4-10x^2+21=0$

1.11 $4x^4-37x^2+9=0$

1.12 $5x^4-16x^2+3=0$

1.13 $4x^4-17x^2+4=0$

1.14 $\frac{x}{x-5}+\frac{x-2}{x-6}=0$

1.15 $\frac{x+1}{x-2}+\frac{1-x}{x+3}=0$

1.16 $\frac{x^2+1}{x-4}+\frac{x^2-1}{x+3}=0$

1.17 $\frac{1}{x-2}+\frac{2}{x-1}=2$

1.18 $\frac{x^2+1}{x}+\frac{x}{x^2+1}=2,3$

1.19 $\frac{x+1}{x-2}+\frac{x-2}{x+1}=0$

1.20 $\frac{4}{x^2+4}+\frac{5}{x^2+5}=2$

1.21 $\sqrt{3x+4}+\sqrt{x-4}=2\sqrt{x}$

1.22 $\sqrt{15-x}+\sqrt{3-x}=6$

1.23 $\sqrt{3x+7}-\sqrt{x+1}=2$

1.24 $\sqrt{x+1}-\sqrt{9-x}=\sqrt{2x-12}$

1.25 $\sqrt{10-x^2}+\sqrt{x^2+3}=5$

1.26 $\sqrt{x+1}+\sqrt{4x+13}=\sqrt{3x+12}$

1.27 $\sqrt{2x+5}+\sqrt{5x+6}=\sqrt{12x+25}$

1.28 $x^2-4x-6=\sqrt{2x^2-8x+12}$

1.29 $\sqrt{x^2+32}-2\sqrt[4]{x^2+32}=3$

1.30 $x^2+\sqrt{x^2+20}=22$

2-misol: Quyidagi tengsizliklar yechimlari to‘plamini toping va ularni to‘plam ko‘rinishida yozing.

2.1 $(x+1)(3-x)(x-2)^2>0$

2.2 $\sqrt{3x-x^2}<4-x$

2.3 $\frac{1}{3x-2-x^2}-\frac{3}{7x-4-3x^2}>0$

2.4 $\frac{1}{x+2}<\frac{3}{x-3}$

2.5 $\frac{x^2-10x+9}{x^2-10x+25}>0$

2.6 $|2x^2-9x+15|>20$

2.7 $0,5^{2+4+6+...+2x}>0,5^{72}$

2.8 $\sqrt{x^2-x-12}<x$

2.9 $2^{x^2+5x}>16$

2.10 $2^{x+2}-2^{x+3}-2^{x+4}>5^{x+1}-5^{x+2}$

2.11 $\frac{x^3-x^2+x-1}{x+8}\le 0$

2.12 $\frac{x^4+x^2+1}{x^2-4x-5}<0$

2.13 $\frac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)}>1$

2.14 $\frac{1}{3^x+5}<\frac{1}{3^{x+1}-1}$

2.15 $\frac{|x-3|}{x^2-5x+6}>2$

2.16 $\frac{1}{x+2}<\frac{3}{x-3}$

2.17 $|x^2-5x|<6$

2.18 $5x-20<x^2<8x$

2.19 $\frac{1}{2-x}+\frac{5}{2+x}<1$

2.20 $\log_{\frac{1}{3}}\frac{3x-1}{x+2}<1$

2.21 $\log_{\pi}(x+27)-\log_{\pi}(16-2x)<\log_{\pi}x$

2.22 $\log_2\log_{\frac{1}{3}}\log_5 x>0$

2.23 $\log_x[\log_9(3^x-9)]<1$

2.24 $3^{\sqrt{x}}+3^{\sqrt{x}-1}-3^{\sqrt{x}-2}<11$

2.25 $0,5^x<0,25^{x^2}$

2.26 $\log^2_{0,5}x+\log_{0,5}x-2<0$

2.27 $\log_3 x+\log_{\sqrt{3}}x+\log_{\frac{1}{3}}x<6$

2.28 $\text{T}^{\log_3\log_{x^2}\log_{x^2}x^4}>0$

2.29 $0,5^{2\sqrt{x}}+2>3\cdot 0,5^{\sqrt{x}}$

2.30 $\sqrt{x^3+3x+4}>-2$

3-misol: Quyidagi trigonometrik tenglamalar yechimlarini toping va ikki to'plam shaklida yozib ushbu to'plarning yigindidisi, ayirmasi va kesishmasi toping.

3.1 $7+4\sin x\cos x+1,5(tgx+ctgx)=0$
$\frac{4ctgx}{1+cyg^2x}+\sin^2 2x+1=0$

3.2 $\frac{\sin^2 2x-4\sin^2 x}{\sin^2 2x+4\sin^2 x-4}+1=2tg^2x$
$\sin x\cdot\sin(60^0-x)\sin(60^0+x)+1=0$

3.3 $tg3x-tgx-4\sin x=0$
$ctgx-\sin x=2\sin^2\frac{x}{2}$

3.4 $\sin 2x=\cos^4\frac{x}{2}-\sin^4\frac{x}{2}$
$(1+\cos 4x)\sin 2x=\cos^2 2x$

3.5 $tg2x\cos 3x+\sin 3x+\sqrt{2}\sin 5x=0$
$ctg(\frac{\pi}{2}+x)-tg^2x=\frac{(\cos 2x-1)}{\cos^2 x}$

3.6 $\cos\frac{x}{2}\cos\frac{3x}{2}-\sin x\sin 3x-\sin 2x\sin 3x=$
$tgx+tg2x-tg3x=0$

3.7 $\sin(15^0+x)+\sin(45^0-x)=1$
$\frac{1}{\cos x}+ctg3x=ctg\frac{3x}{2}$

3.8 $\sin x\sin 3x+\sin 4x\sin 8x=0$
$2tg^3x-2tg^2x+3tgx-3=0$

3.9 $2+tgxctg\frac{x}{2}+ctgxtg\frac{x}{2}=0$
$\sin 2x+\sin 8x=\sqrt{2}\cos 3x$

3.10 $\sin 3x\cos 3x=\sin 2x$
$\cos 2x-5\sin x-3=0$

Quyidagi trigonometric tengsizliklar yechimlarini toping va ikki to'plam shaklida yozib ushbu to'plarning yigindidisi, ayirmasi va kesishmasi toping.

3.11 $\sin x>0$, $\cos x>0$

3.12 $\sin x<0$, $\cos x<0$

3.13 $\sin x>0$, $\cos x<0$

3.14 $\sin x<0$, $\cos x>0$

3.15 $tgx>0$, $ctgx>0$

3.16 $tgx<0$, $ctgx<0$

3.17 $tgx>0$, $ctgx<0$

3.18 $tgx<0$, $ctgx>0$

3.19 $\sin x>0$, $tgx<0$

3.20 $\sin x<0$, $tgx>0$

4-misol: X universal to'plamning ixtiyoriy A, B va C qism to'plamlari uchun quyidagi munosabatlarni isbotlang:

4.1 A\(B∪C)=(A\B)∩(A\C);

4.2 (A∪B)\(A∩B)=(A\B) ∪ (B\A);

4.3 A\(B∩C)=(A\B) ∪ (A\C);

4.4 A\(A\B)=A∩B;

4.5 A\B=A\(A∩B);

4.6 A∩ (B\C)=(A∩B)/C;

4.7 A∩ (B\C)=(A∩B)\(A∩C);

4.8 A∪ (B\A)=A∪B;

4.9 A\B=(A∪B)\B ;

4.10 (A\B)\C=A\(B∪C);

4.11 A\(B\C)=(A\B) ∪ (A∩C);

(A\B) ∩C=(A∩C)\(B∩C).

4.13 N natural sonlar to'plami va Z butun sonlar to'plami birlashmasini toping.

G ratsional sonlar to'plami, R haqiqiy sonlar to'plami bo'lsa $G\cap R$ ni toping.

Ratsional va irratsional sonlar to'plami birlashmasini toping.

A to'g'ri to'rtburchaklar to'plami, B romblar to'plami bo'lsa, $A\cap B$ ni toping.

A juft sonlar to‘plami Z butun sonlar to‘plami bo‘lsa, ularning kesishmasini toping.

A juft sonlar to‘plami B toq sonlar to‘plami bo‘lsa, A va B larning kesishmasini toping.

$\{0;\ 1,2\}$ bo‘lsa hamma qism to‘plamlar to‘plamini toping.

A juft sonlar to‘plami, B toq sonlar to‘plami, C tub sonlar to‘plami bo‘lsa, $A \cup B$, $A \cap B$, $A \cap C$ toping.

5-misol: Quyidagi berilgan tengsizliklar yechimlar oraliqlarini toping va koordinatalar sisitemasida Dekart ko‘paytmasini tasvirlang.

5.1 $0<3x+1<16$, $x^2+3x+2<0$

5.2 $0<2x+1<15$, $x^2-3x+2<0$

5.3 $-2<5x-1<16$, $x^2+5x+6<0$

5.4 $-3<x+1<1$, $x^2-5x+6<0$

5.5 $-3<2x+1<1$, $x^2-5x-6<0$

5.6 $-5<2x+1<5$, $x^2-5x+4<0$

5.7 $-5<3x+7<5$, $x^2+5x+4<0$

5.8 $-13<5x+2<17$, $x^2-7x+6<0$

5.9 $-3<5x+2<7$, $x^2+7x+6<0$

5.10 $-13<10x+2<17$, $x^2+7x+12<0$

5.11 $-23<10x+2<27$, $x^2-7x+12<0$

5.12 $-4<2x+2<0$, $x^2-7x+6<0$

5.13 $-4<x+2<0$, $x^2-8x+7<0$

5.14 $-4<3x+2<0$, $x^2+8x+7<0$

5.15 $-4<4x+2<6$, $x^2-9x+8<0$

5.16 $-4<3x+2<6$, $x^2+9x+8<0$

5.17 $-4<x+12<6$, $x^2-9x+20<0$

5.18 $-4<x+1<6$, $x^2+9x+20<0$

5.19 $-4<x+1<6$, $x^2+9x+18<0$

5.20 $-4<x+1<6$, $x^2-9x+18<0$

6-misol: Quyidagi berilgan tenglamalarning barcha haqiqiy yechimlarini toping va koordinatalar sisitemasida Dekart ko‘paytmasini tasvirlang.

6.1 $x^4-10x^2+9=0$, $\sqrt{3x+4}+\sqrt{x-4}=2\sqrt{x}$

6.2 $x^4-13x^2+36=0$, $\sqrt{15-x}+\sqrt{3-x}=6$

6.3 $x^4-5x^2+4=0$, $\sqrt{3x+7}-\sqrt{x+1}=2$

6.4 $x^4-2x^2+1=0$, $\sqrt{x+1}-\sqrt{9-x}=\sqrt{2x-12}$

6.5 $x^4-10x^2+25=0$, $\sqrt{10-x^2}+\sqrt{x^2+3}=5$

6.6 $x^4-5x^2+6=0$, $\sqrt{x+1}+\sqrt{4x+13}=\sqrt{3x+12}$

6.7 $x^4-6x^2+5=0$, $\sqrt{2x+5}+\sqrt{5x+6}=\sqrt{12x+25}$

6.8 $x^4-37x^2+36=0$, $x^2-4x-6=\sqrt{2x^2-8x+12}$

6.9 $x^4-50x^2+49=0$, $\sqrt{x^2+32}-2\sqrt[4]{x^2+32}=3$

6.10 $x^4-10x^2+21=0$, $x^2-4x-6=\sqrt{2x^2-8x+12}$

6.11 $4x^4-37x^2+9=0$, $x^2-4x-3=0$

6.12 $5x^4-16x^2+3=0$, $x^2-4x+3=0$

6.13 $4x^4-17x^2+4=0$, $x^2+4x-3=0$

6.14 $\frac{x}{x-5}+\frac{x-2}{x-6}=0$, $x^2+4x+3=0$

6.15 $\frac{x+1}{x-2}+\frac{1-x}{x+3}=0$, $x^2-14x-40=0$

6.16 $\frac{x^2+1}{x-4}+\frac{x^2-1}{x+3}=0$, $x^2-14x+40=0$

6.17 $\frac{1}{x-2}+\frac{2}{x-1}=2$, $x^2+14x-40=0$

6.18 $\frac{x^2+1}{x}+\frac{x}{x^2+1}=2,3$, $x^2+14x+40=0$

6.19 $\frac{x+1}{x-2}+\frac{x-2}{x+1}=0$

$x^2-14x-24=0$

6.20 $\frac{4}{x^2+4}+\frac{5}{x^2+5}=2$

$x^2-14x+24=0$

Javoblar:

1.1) $\{-3,-1,1,3\}$,

1.2) $\{-3,-2,2,3\}$,

1.3) $\{-2,-1,1,2\}$,

1.4) $\{-1,1\}$,

1.5) $\{-\sqrt{5},\sqrt{5}\}$,

1.6) $\{-\sqrt{3},-\sqrt{2},\sqrt{2},\sqrt{3}\}$,

1.7) $\{-\sqrt{5},-1,1,\sqrt{5}\}$,

1.8) $\{-6,-1,1,6\}$,

1.9) $\{-7,-1,1,7\}$

1.10) $\{-\sqrt{7},-\sqrt{3},\sqrt{3},\sqrt{7}\}$,

1.11) $\{-3,-\frac{1}{2},\frac{1}{2},3\}$,

1.12) $\{-\sqrt{3},-\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}},\sqrt{3}\}$,

1.13) $\{-2,-\frac{1}{2},\frac{1}{2},2\}$,

1.14) $\{\frac{13-\sqrt{89}}{4},\frac{13+\sqrt{89}}{4}\}$,

1.15) $\{-\frac{1}{7}\}$,

1.17) $\{\frac{3}{2},3\}$,

1.20) $\{0\}$,

1.21) $\{4\}$

1.22) $\{-1\}$,

1.23) $\{-1,3\}$,

1.24) $\{7,8\}$,

1.25) $\{-\sqrt{6},-1,1,\sqrt{6}\}$,

2.1) $(-1,2)\cup(2,3)$,

2.2) $[0,3]$,

2.3) $(\frac{4}{3},2)$,

2.4) $(-\frac{9}{2},2)\cup(3,\infty)$,

2.5) $(-\infty,1)\cup(9,\infty)$,

2.6) $(-\infty,-\frac{1}{2})\cup(5,\infty)$,

2.7) $\{1,2,3,4,5,6,7\}$,

2.8) $[4,\infty)$,

2.9) $(-\infty,\frac{-5-\sqrt{41}}{2})\cup(\frac{-5+\sqrt{41}}{2},\infty)$,

2.10) $(0,\infty)$,

2.11) $(-8,1]$,

2.12) $(-1,5)$,

2.13) $(-\infty,-3)\cup(-2,-1)$,

2.14) $(-1,1)$,

2.15) $(\frac{3}{2},2)$,

2.16) $(-\frac{9}{2},-2)\cup(3,\infty)$,

2.17) $(-1,2)\cup(3,6)$,

2.18) $(0,8)$,

2.19) $(-\infty,-2)\cup(2,\infty)$,

2.20) $(-\infty,-2)\cup(\frac{5}{8},\infty)$,

2.21) $(3,\frac{9}{2})$,

2.22) $(0,\sqrt[3]{5})$

Mavzu: Yaqinlashuvchi ketma-ketliklar ularning xossalari.

Reja

1. Sonli ketma-ketlik haqida tushuncha. Ketma-ketlikning berilish usullari.
2. Chegaralangan ketma-ketliklar, monoton ketma-ketliklar.
3. Ketma-ketlik limitining ta'rifi. Yaqinlashuvchi ketma-ketliklarning xossalari.

Sonlar ketma-ketligi haqida tushuncha.

Ketma-ketlik, bu har bir natural songa biror haqiqiy sonni mos qo'yuvchi moslikka sonlar ketma-ketligi deyiladi hamda $\{a_n\},\{b_n\},\ldots,\{x_n\}$ kabi belgilanadi. Odatda ko'p hollarda ketma-ketlikni $\{x_n\}$ kabi belgilashdan ko'p foydalaniladi. Masalan $x_n = \frac{1}{n}$, $x_n = n$, $x_n = (n+1)^{2n}$...

Chegaralangan ketma-ketliklar. Monoton ketma-ketliklar.

Endi quyida ketma-ketliklarning chegaralangan va chegaralanmaganliklari bilan tanishib o'tamiz.

Biror $\{x_n\}$ ketma-ketlik berilgan bo'lsin.

Ta'rif: Agar shunday o'zgarmas M soni mavjud bo'lsaki, $\{x_n\}$ ketma-ketlikning ixtiyoriy hadi uchun $x_n \le M$ tengsizlik bajarilsa, $\{x_n\}$ ketma-ketlik yuqoridan chegaralangan deyilidi.

Ushbu ta'rifni matematik tilda quyidagicha yozishimiz mumkin, $\exists M, \forall n \in N : x_n \le M$ tensizlik bajarilsa berilgan ketma-ketlik yuqoridan M soni bilan chegaralangan deyiladi va M soniga ketma-ketlikning yuqori chegarasi deyiladi.

Ta'rif: Agar shunday o'zgarmas m soni mavjud bo'lsaki, $\{x_n\}$ ketma-ketlikning ixtiyoriy hadi uchun $x_n \ge m$ tengsizlik bajarilsa, $\{x_n\}$ ketma-ketlik quyidan chegaralangan deyilidi.

Ushbu ta'rifni matematik tilda quyidagicha yozishimiz mumkin, $\exists m, \forall n \in N : x_n \ge m$ tensizlik bajarilsa berilgan ketma-ketlik quyiidan m soni bilan chegaralangan deyiladi va m soniga ketma-ketlikning quyi chegarasi deyiladi.

Ta'rif: Agar $\{x_n\}$ ketma-ketlikning ham yuqoridan ham quyidan chegaralangan bo'lsa u holda ketma-ketlik chegaralangan deyiladi.

Ushbu ta'rifni matematik tilda quyidagicha yozishimiz mumkin, $\exists M, m, \forall n \in N : m \le x_n \le M$ qo'shtensizlik bajarilsa berilgan ketma-ketlik chegaralangan deyiladi.

Yuqoridan, quyidan chegaralangan va chegaralangan ketma-ketliklarga bir nechta misollar ko'rib chiqamiz.

1-misol: $x_n = 2n+1$ ketma-ketlik yuqoridan chegaralanganmi yoki quyidanmi?

Yechish: ushbu ketma-ketlik quyidan chegaralangan bo'lib yuqorida aytib o'tganimiz arifmetik progressiya bo'lib, n ning birga teng qiymatida ketma-ketlik o'zining 3 ga teng bo'lgan qiymatini qabul qiladi va quyi chegarasi 3 ga teng bo'ladi.

2-misol: $x_n = \frac{4n+3}{2n+1}$ ketma-ketlik yuqoridan chegaralanganmi yoki quyidanmi?

Yechish: Ushbu ketma-ketlikni chegaralanganlikka tekshirish uchun soddalashtirish bajaramiz, $x_n = \frac{4n+3}{2n+1} = \frac{4n+2+1}{2n+1} = 2 + \frac{1}{2n+1}$ kabi ikki qismga ajratamiz va yig'indining ikkinchi qismi musbat bo'lib, ketma-ketlik 2dan qat'iy katta va $\frac{1}{2n+1} \le \frac{1}{3}$ tengsizlik ham o'rinli bo'lib, $2 < x_n = 2 + \frac{1}{2n+1} \le 2\frac{1}{3}$ qo'shtengsizlik o'rinli va bu esa ketma-ketlikni ham yuqoridan ham quyidan chegaralanganligini bildiradi va yuqoridagi ta'rifdan ketma-ketlik chegaralangandir.

$\{x_n\}$ ketma-ketlik berilgan bo'lsin. Agar $\{x_n\}$ ketma-ketlik uchun

Ta'rif: $\{x_n\}$ ketma-ketlik uchun, ixtiyoriy n natural sonlar uchun $x_n \le x_{n+1}$ ($x_n < x_{n+1}$)- tengsizlik bajarilsa ketma-ketlik o'suvchi (qat'iy o'suvchi) deyiladi.

Ushbu ta'rifni matematik ko'rinishda quyidagicha ifodalash mumkin.

$\forall n \in N$ **да** $x_n \le x_{n+1}$ **бўлса,** $\{x_n\}$ **- o'suvchi**

$\forall n \in N$ **да** $x_n < x_{n+1}$ **бўлса,** $\{x_n\}$ **- qat'iy o'suvchi**

Ta'rif: $\{x_n\}$ ketma-ketlik uchun, ixtiyoriy n natural sonlar uchun $x_n \ge x_{n+1}$ ($x_n > x_{n+1}$)- tengsizlik bajarilsa ketma-ketlik kamayuvchi (qat'iy kamayuvchi) deyiladi.

Ushbu ta'rifni ham qisqachi quyidagicha ifodalash mumkin.

$\forall n \in N$ **да** $x_n \ge x_{n+1}$ **бўлса,** $\{x_n\}$ **- kamayuvchi**

$\forall n \in N$ **да** $x_n > x_{n+1}$ **бўлса,** $\{x_n\}$ **- qat'iy kamayuvchi**

ketma-ketlik deyiladi.

O'suvchi hamda kamayuvchi ketma-ketliklar umumiy nom bilan monoton ketma-ketliklar deyiladi.

Masalan: $x_n = \dfrac{n+1}{n}$ ketma-ketlikni monotonlikka tekshiring.

Yechish: Yuqoridagi ta'riflardan kelib chiqib, ketma-ketlik uchun $\forall n \in N$ да $x_n < x_{n+1}$, $x_n > x_{n+1}$ ikkita tengsizlikdan birortasining bajarilish yoki bajarilmasligini tekshiramiz. Buning uchun $x_n - x_{n+1}$ ayirmani ko'rib chiqamiz, agar ayirma musbat bo'lsa ketma-ketlik kamayuvchi, agar ayirma manfiy bo'lsa ketma-ketlik o'suvchi bo'ladi.

$$x_n - x_{n+1} = \frac{n+1}{n} - \frac{n+2}{n+1} = \frac{n^2+2n+1-n^2-2n}{n(n+1)} = \frac{1}{n(n+1)}$$ bo'lib ayirmaning qiymati musbat va bundan ko'rinadiki berilgan ketma-ketlik kamayuvchi.

Ketma-ketlikning limiti ta'rifi. Yaqinlashuvchi ketma-ketliklarning xossalari.

Faraz qilaylik $\{x_n\}$ ketma-ketlik va $a \in R$ soni berilgan bo'lsin.

Ta'rif: Ihtiyoriy $\varepsilon > 0$ son uchun shunday n_0 nomer topilib ε ga bog'liq bo'lgan, $n > n_0$ larda $|x_n - a| < \varepsilon$ tengsizlik bajarilsa, u holda a soniga ketma-ketlikning limiti deyiladi va $\lim_{n\to\infty} x_n = a$ kabi belgilanadi.

Ushbu ta'rifning ham matematik yozuvdagi shaklini quyidagicha yozamiz. $\forall \varepsilon > 0, \exists n_0(\varepsilon), n > n_0 : |x_n - a| < \varepsilon$ tengsizlik bajarilsa a soniga $\{x_n\}$ ketma-ketlikning limiti deyiladi.

Ketma-ketlik limiti bu ketma-ketlik hadlari soni cheksizlikka intilganda, ketma-ketlikning o'zi qanday songa intiladi? savoliga javob beradi. Ketma-ketlikning barcha hadlari yig'indisini topish bu murakkab masala hisoblanadi. Shuningdek ketma-ketlikning cheksiz ko'p hadlari qaysi oraliqda, yoki qaysi son atrofida o'zgarishini topish mumkin. Agar ketma-ketlikning cheksiz ko'p hadlari biror aniq sonning atrofida o'zgarsa, u holda bu ketma-ketlik yaqinlashuvchi ketma- ketlik deyiladi. Hamma joyda limiti mavjud va chekli bo'lgan ketma- ketlik yaqinlashuvchi ketma-ketlik deyiladi.

Masalan: $x_n = \dfrac{1}{n}$ oddiy ketma-ketliklardan biri bo'lgan ushbu ketma-ketlikning limiti nolga teng ekanligini ta'rif yordamida isbotlab ko'rsatib o'tamiz. Ketma-ketlik n cheksizlikka intilgan sari ketma-ketlikning hadlari juda kichiklashib boradi va nolga intiladi, ammo nolga teng bo'la olmaydi. Haqiqatda nolga intilishini isbotlaymiz, buning uchun yuqorida aytilganidek ta'rifdan foydalanamiz. $\forall \varepsilon > 0$ son uchun shunday n_0 nomer topilib $n > n_0$ larda $\dfrac{1}{n} < \varepsilon$ tengsizlik bajarilishini ko'rsatishimiz lozim. Tengsizlikni ishlaydigan bo'lsak $\dfrac{1}{\varepsilon} < n$, $n > \dfrac{1}{\varepsilon}$ tengsizlik kelib chiqadi. Bu tengsizlikda $\dfrac{1}{\varepsilon}$ ni n_0 sifatida qabul qilsak, u holda haqiqatda ketma-ketlik limiti nolga intilishini aniqlashimiz mumkin. Endi ketma- ketlikning hadlarini ko'rib chiqamiz, ketma- ketlikning hadlari $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ ... lardan iborat. Ko'rinib turibdiki ketma ketlik birinchi hadi 1 ga teng, lekin limiti nolga teng. Tabiiy savol tug'iladi. Ketma-ketlikning bitta hadi 1 ga teng lekin limiti nolga teng nima sababdan? Yuqorida ta'kidlaganimizdek ketma-ketlik limiti bu ketma-ketlikning cheksiz ko'p hadlari qaysi sonning atrofida joylashgan bo'ladi? Savoliga javob beradi. Demak, yuqoridagi ketma-ketlik cheksiz ko'p hadlari nol sonining atrofida joylashgan bo'ladi.

Shuningdek, cheksiz kamayuvchi geometrik progressiya uchun cheksiz kamayuvchi geometrik progressiyaning barcha hadlari yig'indisi formulasi qanday kelib chiqqanligi ketma-ketlik limitiga bog'liq ekanligini ko'rsatishimiz ham mumkin.

Aytaylik $b_1, b_2 \ldots b_n$ - cheksiz kamayuvchi geometrik progressiya berilgan bo'lsin, bu progressiyaning hadlari yig'indisini topamiz. $S_n = \dfrac{b_1(q^n - 1)}{q - 1}$ bu yerda S_n -progressiyaning yig'indisi, b_1 - progressiyaning birinchi hadi, q - progressiyaning mahraji. Progressiya cheksiz kamayuvchi bo'lgan holda progressiya mahraji birdan kichik bo'ladi. Progressiyaning barcha hadlari yig'indisini topish uchun yig'indi formulasini hadlar sonini cheksizlikka intiltirib limitga o'tamiz. Bundan esa

$$\lim_{n\to\infty} \frac{b_1(q^n - 1)}{q - 1} = \frac{b_1 \cdot (-1)}{q - 1} = \frac{b_1}{1 - q}$$

ekanligi kelib chiqadi.

3-misol. $\lim\limits_{n\to\infty} \dfrac{2n - 5}{n} = 2$ tenglik to'g'ri ekanligini ta'rif yordamida isbotlang.

Yechish: Ixtiyoriy $\varepsilon > 0$ soni uchun shunday musbat (soniga bog'liq bo'lgan) mavjud bo'lib, barcha $n > n_0$ lar uchun

$$|x_n - a| < \varepsilon \quad \left|\frac{2n - 5}{n} - 2\right| = \left|\frac{2n - 5 - 2n}{n}\right| = \left|\frac{-5}{n}\right| = \frac{5}{n} < \varepsilon \Rightarrow n > \frac{5}{\varepsilon}$$

ekanligidan n_0 nomer sifatida $\dfrac{5}{\varepsilon}$ ni oladigan bo'lsa, u holda $\lim\limits_{n\to\infty} \dfrac{2n - 5}{n} = 2$ tenglik to'g'ri ekanligi kelib chiqadi.

4-misol. $\lim\limits_{n\to\infty} \dfrac{3n^2 - n + 2}{5n^2 + 2}$ limitni hisoblang.

Yechish: Ushbu limitni hisoblash uchun kasrning sur'at va maxrajidan o'zgaruvchining eng katta darajasini qavsdan tashqariga chiqazib soddalashtiramiz.

$$\lim_{n\to\infty} \frac{3n^2 - n + 2}{5n^2 + 2} = \lim_{n\to\infty} \frac{n^2(3 - \frac{1}{n} + \frac{2}{n^2})}{n^2(5 + \frac{2}{n^2})} = \lim_{n\to\infty} \frac{3 - \frac{1}{n} + \frac{2}{n^2}}{5 + \frac{2}{n^2}} = \frac{3}{5}.$$

5-misol. $\lim\limits_{n\to\infty} \dfrac{(n+1)! + (n+2)!}{(n+3)!}$ limitni hisoblang. (bu yerda $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$ ga teng).

Yechish: Ushbu limitni hisoblash uchun oldin kasrni soddalashtiramiz.

$$\frac{(n+1)! + (n+2)!}{(n+3)!} = \frac{1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n+1) + 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n+1)(n+2)}{1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n+1)(n+2)(n+3)} =$$

$$\frac{1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n+1)(1 + n + 2)}{1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n+1)(n+2)(n+3)} = \frac{n+3}{n^2 + 5n + 6}$$

demak $\lim\limits_{n\to\infty} \dfrac{(n+1)! + (n+2)!}{(n+3)!} = \lim\limits_{n\to\infty} \dfrac{n+3}{n^2 + 5n + 6} = \lim\limits_{n\to\infty} \dfrac{1 + \frac{3}{n}}{n + 5 + \frac{6}{n}} = 0$.

6-misol. $\lim\limits_{n\to\infty} (\sqrt{n+1} - \sqrt{n-1})$ limitni hisoblang.

Yechish: ushbu limitda $\lim\limits_{n\to\infty} (\sqrt{n+1} - \sqrt{n-1}) = \infty - \infty$ ko'rinishga kelib qoladi. Bu aniqmaslikka tengdir. Bu aniqmaslikni ochish uchun limit ostidagi ifodani qo'shmasiga ko'paytirib bo'lamiz. U holda quyidagi ifoda kelib chiqadi:

$$\lim_{n\to\infty} \frac{n + 1 - n + 1}{\sqrt{n+1} + \sqrt{n-1}} = \lim_{n\to\infty} \frac{2}{\sqrt{n+1} + \sqrt{n-1}} = 0$$

ekanligi kelib chiqadi.

7-misol. $\lim\limits_{n\to\infty} \left(\dfrac{5 + 3n}{2n + 4} - \dfrac{n}{2n - 3}\right)$ limitni hisoblang.

Yechish: $\lim\limits_{n\to\infty} \left(\dfrac{5 + 3n}{2n + 4} - \dfrac{n}{2n - 3}\right) = \lim\limits_{n\to\infty} \dfrac{(5 + 3n)(2n - 3) - n(2n + 4)}{(2n + 4)(2n - 3)} =$

$$\lim_{n\to\infty}\frac{6n^2+n-15-2n^2-4n}{(2n+4)(2n-3)}=\lim_{n\to\infty}\frac{4n^2-3n-15}{(2n+4)(2n-3)}=\lim_{n\to\infty}\frac{4n^2-3n-15}{4n^2+2n-12}=$$

$$\lim_{n\to\infty}\frac{4-\frac{3}{n}-\frac{15}{n^2}}{4+\frac{2}{n}-\frac{12}{n^2}}=1$$

8-misol. $\lim\limits_{n\to\infty}\frac{2\cdot 6^n-8}{6^n+3}$ limitni hisoblang.

Yechish: Ushbu misolda kasr $\frac{\infty}{\infty}$ ko'rinishiga keladi, bu esa aniqmaslik ekanligi ma'lum va bu aniqmaslikdan qutilish uchun kasr surat va maxrajlardan bir hil ko'paytuvchilarni ya'ni cheksizlikka intiluvchi hadni kavsdan tashqariga chiqaramiz. $\lim\limits_{n\to\infty}\frac{2\cdot 6^n-8}{6^n+3}=\lim\limits_{n\to\infty}\frac{6^n(2-\frac{8}{6^n})}{6^n(1+\frac{3}{6^n})}$ endi kasrni qisqartiramiz va $\lim\limits_{n\to\infty}\frac{8}{6^n}=0$ ekanligidan va $\lim\limits_{n\to\infty}\frac{3}{6^n}=0$ tengligidan

$$\lim_{n\to\infty}\frac{6^n(2-\frac{8}{6^n})}{6^n(1+\frac{3}{6^n})}=\frac{2}{1}=2 \text{ hosil qilamiz.}$$

9-misol. $\lim\limits_{n\to\infty}\frac{3^{n+1}+4^n}{3^n+4^n}$ limitni hisoblang.

Yechish: Ushbu limitni hisoblash uchun, yuqoridagi kabi natijasi birdan kichik bo'ladigan qilib, kasr surat va maxrajidan bir hil sonlarni chiqaramiz.

$\lim\limits_{n\to\infty}\frac{3^{n+1}+4^n}{3^n+4^n}=\lim\limits_{n\to\infty}\frac{\frac{3^{n+1}}{4^n}+1}{\frac{3^n}{4^n}+1}=1$ ga teng bo'ladi, $\frac{3^n}{4^n}$ ifoda birdan kichik va nolga intilishini inobatga olib, haqiqatda limitning qiymati birga teng ekanligi kelib chiqadi.

10- misol. $\lim\limits_{n\to\infty}(\frac{1+2+3+...+n}{2n+1}-\frac{n}{4})$ limitni hisoblang.

Yechish: ushbu limitni hisoblash uchun ifodani soddalashtiramiz. $\frac{1+2+3+...+n}{2n+1}-\frac{n}{4}$ bu ifodani soddalashtirishda arifmetik progressiyaning yig'indisi formulasidan foydalanamiz.

$$\frac{1+2+3+...+n}{2n+1}-\frac{n}{4}=\frac{\frac{1+n}{2}\cdot n}{2n+1}-\frac{n}{4}=\frac{n(n+1)}{2(2n+1)}-\frac{n}{4}=\frac{2n(n+1)-(2n+1)n}{4(2n+1)}=$$

$$\frac{2n^2+2n-2n^2-n}{4(2n+1)}=\frac{n}{8n+4} \text{ ekanligidan}$$

$$\lim_{n\to\infty}(\frac{1+2+3+...+n}{2n+1}-\frac{n}{4})=\lim_{n\to\infty}\frac{n}{8n+4}=\lim_{n\to\infty}\frac{1}{8+\frac{4}{n}}=\frac{1}{8}.$$

Yaqinlashuvchi ketma-ketliklar va ularning xossalari.

Teorema: $\{x_n\}$ ketma-ketlik yaqinlashuvchi bo'lsa, u holda bu ketma-ketlik chegaralangan bo'ladi.

Teorema: Agar $\{x_n\}$ ketma-ketlik yaqinlashuvchi va $\lim\limits_{n\to\infty}x_n=A$ bo'lib, $A>p$ $(A<q)$ tengsizlik bajarilsa, u holda shunday $n_0\in N$ topiladiki, $\forall n>n_0$ larda $x_n>p$ $(x_n<q)$ bo'ladi.

Teorema: Agar $\{x_n\}$ va $\{y_n\}$ ketma-ketliklar yaqinlashuvchi bo'lib,

1) $\lim\limits_{n\to\infty}x_n=A,\ \lim\limits_{n\to\infty}y_n=B$

$\forall n\in N$ uchun $x_n\le y_n$ $(x_n\ge y_n)$, bo'lsa u holda $A\le B$ $(A\ge B)$ bo'ladi.

Teorema (Ikki mirshab teoremasi): Agar $\{x_n\}$ va $\{z_n\}$ ketma-ketliklar yaqinlashuvchi bo'lib,

2) $\lim\limits_{n\to\infty}x_n=A,\ \lim\limits_{n\to\infty}z_n=A$

3) $\forall n\in N$ uchun $x_n\le y_n\le z_n$, bo'lsa

u holda $\{y_n\}$ ketma-ketlik ham yaqinlashuvchi va $\lim\limits_{n\to\infty}y_n=A$ bo'ladi.

Yuqoridagi bir nechta teoremalarni isbotsiz keltirib o'tdik, bu teoremalarning isbotlarini talabalarga mustaqil vazifa sifatida qoldiramiz.

Mustaqil yechish uchun misollar

1-misol: quyidagi ketma-ketliklarni chegaralanganligini ko'rsating.

1.1 $x_n = \dfrac{n+1}{n^2+2n}$

1.2 $x_n = \dfrac{n-3}{n^2+n}$

1.3 $x_n = \dfrac{n^2+1}{n^2+2n}$

1.4. $x_n = \dfrac{n^3+1}{n^2+5n}$

1.5. $x_n = \dfrac{n+1}{n}$

1.6 $x_n = \dfrac{2^n}{3^{2n}}$

1.7 $x_n = \dfrac{5^n}{2^{2n}}$

1.8 $x_n = \dfrac{2^n}{3^{n-3}}$

1.9 $x_n = \dfrac{2n+3}{3n-1}$

1.10 $x_n = \dfrac{n^3+2n^2+n}{2n^3+n+1}$

2-misol: Quyidagi ketma-ketliklarni monotonlikka tekshiring.

2.1 $x_n = n^2+1$

2.2 $x_n = \dfrac{1}{n+1}$

2.3 $x_n = \dfrac{1}{3^n}$

2.4 $x_n = \left(\dfrac{2}{3}\right)^n$

2.5 $x_n = 2^{2n+1}$

2.6 $x_n = \dfrac{n}{3n+1}$

2.7 $x_n = \dfrac{2^n+1}{3^{2n}}$

2.8 $x_n = \dfrac{n^2+1}{n^3}$

2.9 $x_n = e^n$

2.10 $x_n = \dfrac{n}{\pi^n}$

3-misol: a soni quyidagi ketma-ketliklarning limiti ekanligini isbotlang.

3.1 $x_n = \dfrac{n+3}{5n-1}$, $a = \dfrac{1}{5}$

3.2 $x_n = \dfrac{n-3}{n-1}$, $a = 1$

3.3 $x_n = \dfrac{n+3}{5n^2-1}$, $a = 0$

3.4 $x_n = \dfrac{n^2+3}{7n^2+2n-1}$, $a = \dfrac{1}{7}$

3.5 $x_n = \dfrac{\cos n\pi}{5n-1}$, $a = 0$

3.6 $x_n = \dfrac{\sin\frac{3\pi n}{2}}{n+8}$, $a = 0$

3.7 $x_n = \dfrac{3}{2^n}$, $a = 0$

3.8 $x_n = \dfrac{3^n}{3^n+15}$, $a = 1$

3.9 $x_n = \dfrac{3^n}{2^{2n}}$, $a = 0$

3.10 $x_n = \left(\dfrac{e}{\pi}\right)^{n+1}$, $a = 0$

4-misol: Berilgan limitlarni hisoblang.

4.1 a) $\lim\limits_{n\to\infty}\dfrac{4-n^2}{3-n^2}$; b) $\lim\limits_{n\to\infty}\dfrac{n+(-1)^n}{n-(-1)^n}$;

4.2 a) $\lim\limits_{n\to\infty}\dfrac{6n+4}{7-9n}$; b) $\lim\limits_{n\to\infty}\dfrac{4n^3-5n^2+10n}{21n^3+7n-8}$;

4.3 a) $\lim\limits_{n\to\infty}\dfrac{3n^2-n+2}{5n^2+2}$; b) $\lim\limits_{n\to\infty}\dfrac{2n^2+n+4n^3}{1-n+2n^3}$;

4.4 a) $\lim\limits_{n\to\infty}\dfrac{3n^2-n-2}{5n^2-2}$; b) $\lim\limits_{n\to\infty}\dfrac{n^4+5n^2-1}{10n^3-3n+2}$;

4.5 a) $\lim\limits_{n\to\infty}\dfrac{n^2-2n+2}{2n+1+2n^2}$; b) $\lim\limits_{n\to\infty}\dfrac{7n^2-1}{5n^3+4^2-2n+1}$;

4.6 a) $\lim\limits_{n\to\infty}\dfrac{(n+1)^2}{n^2+n}$; b) $\lim\limits_{n\to\infty}\dfrac{1000n^3+100n}{2n^4}$;

4.7 a) $\lim\limits_{n\to\infty}\dfrac{4-n^2}{3-n^2}$; b) $\lim\limits_{n\to\infty}(\sqrt{n^2+n}-\sqrt{n})$;

4.8 a) $\lim\limits_{n\to\infty}\dfrac{6n+4}{7-9n}$; b) $\lim\limits_{n\to\infty}(\sqrt[3]{n^3+n^2}-\sqrt[3]{n^2-n}$;

4.9 a) $\lim\limits_{n\to\infty}\frac{(n+1)!+(n+2)!}{(n+3)!}$; b) $\lim\limits_{n\to\infty}(\sqrt{n^2+n}-\sqrt{n^2-n})$;

4.10 a) $\lim\limits_{n\to\infty}\frac{5(n+1)!}{4n!+(n-1)!}$; b) $\lim\limits_{n\to\infty}\frac{\sqrt{n^3}}{\sqrt{n}+1}$;

4.11 a) $\lim\limits_{n\to\infty}\frac{(2n+3)n!}{(n+2)!+(n+1)!}$; b) $\lim\limits_{n\to\infty}\frac{\sqrt{n^2+3n}}{n+1}$;

4.12 a) $\lim\limits_{n\to\infty}\frac{n!}{(n-1)!-(n+1)!}$; b) $\lim\limits_{n\to\infty}\frac{2n+1}{\sqrt[3]{n^2+n+4}}$;

4.13 a) $\lim\limits_{n\to\infty}\frac{(7n+1)n!}{9(n+1)!+7n!}$; b) $\lim\limits_{n\to\infty}(\frac{2}{n^2}+\frac{4}{n^2}+...+\frac{2n+2}{n^2})$;

4.14 a) $\lim\limits_{n\to\infty}(\frac{1+2+3+...+n}{4n^2+1})$; b) $\lim\limits_{x\to\infty}\frac{(n+1)^3-(n-1)^3}{n^2+1}$;

4.15 a) $\lim\limits_{n\to\infty}\frac{n!-(n+2)!}{(n+3)n!+(n+1)!}$; b) $\lim\limits_{n\to\infty}(\frac{3}{n+2}-\frac{5}{2n+1})$;

4.16 a) $\lim\limits_{n\to\infty}\frac{n(n-3)!+(n-2)!}{(n-1)!-(n-2)!}$; b) $\lim\limits_{n\to\infty}(\frac{1}{n^2}+\frac{3}{n^2}+...+\frac{2n-1}{n^2})$;

4.17 a) $\lim\limits_{n\to\infty}\frac{(n-1)!+3n!}{(n+1)(n-1)!-(n-2)!}$; b) $\lim\limits_{n\to\infty}(\frac{5+3n}{2n+4}-\frac{n}{2n-3})$;

4.18 a) $\lim\limits_{n\to\infty}\frac{2(n+1)!-(n+2)!}{(n+3)!-(n+1)!}$; b) $\lim\limits_{n\to\infty}(\frac{2n^3+5}{4n+1}-\frac{n^3+4}{2n+3})$;

4.19 a) $\lim\limits_{n\to\infty}\frac{(3n+1)(n-1)!-3(n+1)!}{7(n+1)!+4n!}$; b) $\lim\limits_{n\to\infty}(\frac{n^3}{2n^2-1}-\frac{n^2}{2n+1})$;

4.20 a) $\lim\limits_{n\to\infty}\frac{(n-1)!+(n-3)!}{(2n^2+1)(n-3)!+(n-2)!}$; b) $\lim\limits_{n\to\infty}(\frac{n^2+1}{n+2}-\frac{n^2-1}{n+3})$;

4.21 a) $\lim\limits_{n\to\infty}(\frac{1+2+3+...+n}{n^2+1})$; b) $\lim\limits_{n\to\infty}\frac{5^n-1}{5^n+1}$;

4.22 a) $\lim\limits_{n\to\infty}\frac{(n+3)!+(n+2)!}{(2n^2+3)(n+1)!-(n+2)!}$; b) $\lim\limits_{n\to\infty}\frac{2^{n+1}+3^{n+1}}{2^n+3^n}$;

4.23 a) $\lim\limits_{n\to\infty}\frac{2n!+(n+1)!}{(3n+1)n!}$; b) $\lim\limits_{n\to\infty}(\frac{1+2+3+...+n}{n+2}-\frac{n}{2})$;

Javoblar:

4.1) a) 1, **б)** 1,

4.2) a) $-\frac{2}{3}$, **б)** $\frac{4}{21}$,

4.3) a) $\frac{3}{5}$, **б)** 2,

4.4) a) $\frac{3}{5}$, **б)** ∞,

4.5) a) $\frac{1}{2}$, **б)** 0,

4.6) a) 1, **б)** 0,

4.7) a) 1, **б)** 1,

4.8) a) $-\frac{2}{3}$, **б)** ∞,

4.9) a) 0, **б)** 1,

4.10) a) ∞, **б)** ∞,

4.11) a) 0, **б)** 1,

4.12) a) 0, **б)** ∞,

4.13) a) $\frac{7}{9}$, **б)** 1,

4.14) a) $\frac{1}{8}$, **б)** 6,

4.15) a) $-\infty$, **б)** 0,

4.16) a) 0, **б)** 1,

4.17) a) 3, **б)** 1,

4.18) a) 0, **б)** ∞,

4.19) a) $-\frac{3}{7}$, **б)** $\frac{1}{4}$,

4.20) a) $\frac{1}{2}$, **б)** 1,

4.21) a) $\frac{1}{2}$, **б)** 1,

4.22) a) $\frac{1}{2}$, **б)** 3,

4.23) a) $\frac{1}{3}$, **б)** $-\frac{1}{2}$

Mavzu:Yaqinlashish prinsipi.

Reja

1. Monoton ketma-ketlikning limiti, *e* soni.
2. Ichma-ich joylashgan segmentlar prinsipi. Qismiy ketma ketlik. Bolsano-Veyyershtrass teoremasi.
3. Ketma-ketlik yaqinlashishining Koshi kriteriyasi.

Monoton ketma-ketliklarning limiti

Teorema: Agar $\{x_n\}$ ketma-ketlik o'suvchi va yuqoridan chegaralangan bo'lsa, u holda $\{x_n\}$ ketma-ketlik yaqinlashuvchi va limitga ega bo'ladi.

Teorema: Agar $\{x_n\}$ ketma-ketlik kamayuvchi va quyidan chegaralangan bo'lsa, u holda $\{x_n\}$ ketma-ketlik yaqinlashuvchi va limitga ega bo'ladi.

Keyingi teoremaning isbotini ko'rib chiqamiz.

Isbot: Aytaylik $\{x_n\}$ ketma-ketlik teoremaning ikkala shartini bajarsin. Bu ketma-ketlikning barcha hadlaridan iborat to'plamni E bilan belgilaymiz. $E=\{x_1,x_2,x_3,....,x_n,...\}$.

Ravshanki E quyidan chegaralangan bo'lib, $E\neq\varnothing$. Unda to'plamning aniq quyi chegarasi mavjudligi haqidagi teoremaga muvofiq, $\inf E$ mavjud bo'ladi. Uni a bilan belgilaylik. Ya'ni $\inf E=a$.

To'plamning aniq quyi chegarasi ta'rifiga ko'ra, $\forall\varepsilon>0$ sonni olaylik.

1) $\forall n\in N$ uchun $x_n\geq a$
2) $\exists x_{n_0}\in E,\ x_{n_0}<a+\varepsilon$ bo'ladi.

Berilgan ketma-ketlikning kamayuvchiligidan, n_0 dan katta ixtiyoriy $\forall n>n_0$ lar uchun $x_n\leq x_{n_0}$ tengsizlik bajarilib, $x_n<a+\varepsilon$ bo'ladi. Bundan kelib chiqadiki, $\forall n>n_0$ lar uchun $a-\varepsilon<x_n<a+\varepsilon$ qo'sh tengsizlik bajariladi va ketma-ketlik limiti mavjud va $\lim\limits_{n\to\infty}x_n=a=\inf E$ ekanligi kelib chiqadi.

Masalan: Ushbu $x_n=\left(1+\frac{1}{n}\right)^n$ ketma-ketlik limiti mavjud ekanligini ko'rsating.

Yechish: avvalambor ketma-ketlikni monotonlikka tekshiramiz. Buning uchun $\frac{x_{n+1}}{x_n}$ nisbatni qaraymiz.

$$\frac{\left(1+\frac{1}{n+1}\right)^{n+1}}{\left(1+\frac{1}{n}\right)^n}=\frac{(n+2)^{n+1}\cdot n^n}{(n+1)^{2n+1}}=\frac{(n(n+2))^{n+1}\cdot(n+1)}{((n+1)^2)^{n+1}\cdot n}=\left(\frac{n^2+2n}{n^2+2n+1}\right)^{n+1}\cdot\frac{n+1}{n}=$$

$\left(1-\frac{1}{(n+1)^2}\right)^{n+1}\cdot\frac{n+1}{n}$ tenglikning ohirgi ifodasiga Bernulli tengsizligini qo'llab, quyidagiga ega bo'lamiz.

$$\left(1-\frac{1}{(n+1)^2}\right)^{n+1}\cdot\frac{n+1}{n}\geq\left(1-\frac{1}{(n+1)^2}\cdot(n+1)\right)\cdot\frac{n+1}{n}=\frac{n}{n+1}\,\frac{n+1}{n}=1$$ bo'lib $\frac{x_{n+1}}{x_n}\geq1$

ekanligi kelib chiqadi va bu esa ketma-ketlikning o'suvchi ekanligini bildiradi.

Keyingi bosqich ketma-ketlikning chegaralanganligini ko'rsatishdan iborat. Buning uchun $(1+\frac{1}{n})^n$ ko'phadnig darajasini Nyuton binomidan foydalanib ochib chiqamiz.

$$(1+\frac{1}{n})^n=1^n+C_n^{n-1}1^{n-1}\cdot\frac{1}{n}+C_n^{n-2}\cdot1^{n-2}\cdot\frac{1}{n^2}+C_n^{n-3}\cdot1^{n-3}\cdot\frac{1}{n^3}+C_n^{n-4}\cdot1^{n-4}\cdot\frac{1}{n^4}+$$
$$C_n^{n-5}\cdot1^{n-5}\cdot\frac{1}{n^5}+C_n^{n-6}\cdot1^{n-6}\cdot\frac{1}{n^6}+C_n^{n-7}\cdot1^{n-7}\cdot\frac{1}{n^7}+C_n^{n-8}\cdot1^{n-8}C_n^{n-9}\cdot1^{n-9}\cdot\frac{1}{n^9}+\ldots$$
$$+C_n^1\cdot1^1\cdot\frac{1}{n^{n-1}}+\frac{1}{n^n}=1+\frac{n!}{(n-1)!}\cdot\frac{1}{n}+\frac{n!}{(n-2)!\cdot2!}\cdot\frac{1}{n^2}+\frac{n!}{(n-3)!\cdot3!}\cdot\frac{1}{n^3}+$$
$$\frac{n!}{(n-4)!\cdot4!}\cdot\frac{1}{n^4}+\frac{n!}{(n-5)!\cdot5!}\cdot\frac{1}{n^5}+\frac{n!}{(n-6)!\cdot6!}\cdot\frac{1}{n^6}+\frac{n!}{(n-7)!\cdot7!}\cdot\frac{1}{n^7}+$$
$$\frac{n!}{(n-8)!\cdot8!}\cdot\frac{1}{n^8}+\frac{n!}{(n-9)!\cdot9!}\cdot\frac{1}{n^9}+\ldots+\frac{n!}{(n-1)!}\cdot\frac{1}{n^{n-1}}+\frac{1}{n^n}=1+n\cdot\frac{1}{n}+$$

$$\frac{n(n-1)}{2}\cdot\frac{1}{n^2}+\frac{n(n-1)(n-2)}{6}\cdot\frac{1}{n^3}+\frac{n(n-1)(n-2)(n-3)}{24}\cdot\frac{1}{n^4}+$$
$$\frac{n(n-1)(n-2)(n-3)(n-4)}{120}\cdot\frac{1}{n^5}+\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{720}\cdot\frac{1}{n^6}+$$
$$\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)}{5040}\cdot\frac{1}{n^7}+$$
$$\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)(n-7)}{40320}\cdot\frac{1}{n^8}+$$
$$\frac{(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)(n-7)(n-9)}{362880}\cdot\frac{1}{n^9}+\ldots+\frac{1}{n^n}$$

ushbu ifodaning ikkinchi hadidan boshlab barcha hadlarini taqqoslab baholasak u holda, ifodaning ikkinchi hadidan keyingi barcha hadlari mos ravishta $\frac{1}{2},\frac{1}{6},\frac{1}{24},\frac{1}{120},\frac{1}{720},\frac{1}{5040},\frac{1}{40320},\frac{1}{362880}$..... lardan kichik ekanligi kelib chiqadi va umumiy yig'indini topib

$$1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\frac{1}{720}+\frac{1}{5040}+\frac{1}{40320}+\frac{1}{362880}=$$

$$2+\frac{181440+60480+15120+3024+504+72+9+1}{362880}=2+\frac{260650}{362880}\approx$$

$2+0{,}7182815=2{,}7182815$ bo'lib, ketma ketlik chegaralanganligini ko'rishimiz mumkin va $0<x_n<3$ deb olib, o'suvchi va yuqoridan chegaralangan ketma-ketlikning limiti mavjud bo'lishi haqidagi teoremadan berilgan ketma-ketlikning limiti mavjudligini ko'rishimiz mumkin.

Ta'rif: $x_n=\left(1+\frac{1}{n}\right)^n$ ketma-ketlikning limiti e soni deyiladi. YA'ni

$$\lim_{n\to\infty}x_n=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e.$$

Ichma-ich joylashgan segmentlar prinsipi. Qismiy ketma-ketlik. Bolsano-Veyyershtrass teoremasi

Aytaylik $\{x_n\}$ ketma-ketlik berilgan bo'lsin, ushbu ketma-ketlikning biror n_1 hadini tanlab, x_{n_1} bilan, n_1 dan katta n_2 hadini tanlab x_{n_2} bilan va hokazo huddi shunday belgilashlarni amalga oshirib, $x_{n_1},x_{n_2},x_{n_3},....,x_{n_k},...$ ketma-ketlikni hosil qilamiz. Hosil qilingan ketma-ketlikka $\{x_n\}$ ketma-ketlikning qismiy ketma-ketligi deyiladi.

Qismiy ketma-ketliklarning limitlari ketma-ketlikning qismiy limitlari deyiladi.

Teorema: Agar $\{x_n\}$ ketma-ketlik limitga ega bo'lsa, uning har qanday qismiy ketma-ketligi ham shu limitga ega bo'ladi.

Isbot: Teorema shartiga ko'ra $\{x_n\}$ ketma-ketlik limitga ega bo'lsin, ya'ni $\lim x_n=a$ bo'lsin. Ketma-ketlik limiti ta'rifiga ko'ra $\forall\varepsilon>0,\ \exists n_0(\varepsilon),\ n>n_0\ |x_n-a|<\varepsilon$ tengsizlik bajariladi. Bu esa $\{x_n\}$ ketma-ketlikning n_0 nomerdan keyingi barcha hadlari $(a-\varepsilon,a+\varepsilon)$ oraliqda joylashishini bildiradi. Bundan esa $\{x_n\}$ ketma-ketlikning hadlaridan tuzilgan harqanday $\{x_{n_k}\}$ qismiy ketma-ketlik uchun shunday $n_k>n_0$ shartni qanoatlantiruvchi hadlari topiladiki va bu hadlarning barchasi $(a-\varepsilon,a+\varepsilon)$ oraliqqa tegishli bo'ladi. YA'ni $\forall\varepsilon>0,\exists n_0(\varepsilon),\ n_k>n_0\ |x_{n_k}-a|<\varepsilon$ tengsizlik bajariladi. Teorema isbot bo'ldi.

Izlatma: Ketma-ketlikning qismiy ketma-ketliklarining limitlari mavjud bo'lishidan, har doim ham berilgan ketma-ketlikning limiti mavjudligi kelib chiqavermaydi.

Masalan: $x_n=(-1)^n$ ketma-ketlikning limiti mavjud emas. Ammo bu ketma-ketlikning qismiy limitlari mavjud bo'ladi. Ya'ni ketma-ketlikning darajasi toq va juft sonlardan iborat bo'lgan ikkita ketma-ketlikka ajratsak, u holda $x_{2n-1}=-1$ va $x_{2n}=1$ bo'lgan ikkita qismiy ketma-ketliklar hosil bo'ladi va bu har ikkala ketma-ketlikning limiti mos ravishta -1 va 1 ga tengdir.

Shu o'rinda tabiiy savol tug'iladi. Qachon qismiy ketma-ketliklarning limitlari mavjud bo'ladi? Ushbu savolga quyidagi teorema javob beradi.

Teorema (Balsano-Veyrshtrass teoremasi): Har qanday chegaralangan ketma-ketlikdan chekli songa intiluvchi qismiy ketma-ketlik ajratish mumkin.

Ushbu teoremaning isbotini talabaning o'ziga mustaqil ishi sifatida qoldiramiz.

Ketma-ketlik yaqinlashishining Koshi kriteriyasi.

$\{x_n\}$ ketma-ketlik berilgan bo'lsin.

Ta'rif: agar ixtiyoriy $\forall\varepsilon>0$ musbat son olinganida ham shunday natural $n_0(\varepsilon)$ nomer topilsaki, istalgan $n>n_0$ va $m>n_0$ lar uchun $|x_n-x_m|<\varepsilon$ tengsizlik bajarilsa, $\{x_n\}$ ketma-ketlik fundamental ketma-ketlik deyiladi.

Ushbu ta'rifni qisqacha matematik tilda quyidagicha yozamiz. $\forall\varepsilon>0, \exists n_0(\varepsilon), \forall n>n_0, \forall m>n_0, |x_n-x_m|<\varepsilon$.

Masalan: $x_n=\dfrac{1}{2^n}$ ketma-ketlikni fundamental ketma-ketlik ekanligini isbotlang.

Yechish: berilgan ketma-ketlik uchun $\forall\varepsilon>0, \exists n_0(\varepsilon), \forall n>n_0, \forall m>n_0, |x_n-x_m|<\varepsilon$ shartning bajarilishini tekshiramiz. m va n larning ixtiyoriyligidan $m>n$ deb olib, $\left|\dfrac{1}{2^n}-\dfrac{1}{2^m}\right|=\left|\dfrac{2^{m-n}-1}{2^m}\right|<\dfrac{2^{m-n}}{2^m}=\dfrac{1}{2^n}$ bo'lib, $\forall\varepsilon>0$ ga ko'ra $n_0=\log_{\frac{1}{2}}\varepsilon$ deyilsa, u holda $\forall n>n_0, \forall m>n_0$ bo'lganda $|x_n-x_m|<\left(\dfrac{1}{2}\right)^{\log_{\frac{1}{2}}\varepsilon}=\varepsilon$ bo'ladi.

Teorema (Koshi teoremasi): Ketma-ketlik yaqinlashuvchi bo'lishi uchun uning fundamental bo'lishi zarur va yetarli.

Ta'rif: $\{x_n\}$ ketma-ketlikning qismiy limitlarining eng kattasi berilgan ketma-ketlikning yuqori limiti deyiladi va $\overline{\lim\limits_{n\to\infty}}x_n$ kabi belgilanadi. $\{x_n\}$ ketma-ketlikning qismiy limitlarining eng kichigi esa berilgan ketma-ketlikning quyi limiti deyiladi va $\underline{\lim\limits_{n\to\infty}}x_n$ kabi belgilanadi.

Mustaqil yechish uchun misollar

1-misol: Monoton ketma-ketlik limitining mavjudligi haqidagi tasdiqlardan va Koshi teoremasidan foydalanib, quyidagi $\{x_n\}$ ketma-ketliklarning yaqinlashuvchiligi isbotlansin.

1.1. $x_n=\dfrac{\sin\alpha}{2}+\dfrac{\sin 2\alpha}{2^2}+\ldots+\dfrac{\sin n\alpha}{2^n}$

1.2. $x_n=\dfrac{|\cos 1|}{3}+\dfrac{|\cos 2|}{3^2}+\ldots+\dfrac{|\cos n|}{3^n}$

1.3. $x_n=\dfrac{1}{1\cdot 2}-\dfrac{1}{2\cdot 3}+\ldots+\dfrac{(-1)^{n-1}}{n(n+1)}$

1.4. $x_n=\dfrac{1}{1\cdot 3}-\dfrac{1}{2\cdot 4}+\ldots+\dfrac{(-1)^{n-1}}{n(n+2)}$

1.5. $x_n=1+\dfrac{1}{2!}+\ldots+\dfrac{1}{n!}$

1.6. $x_n=\sin 1+\dfrac{\sin 2}{2^2}+\ldots+\dfrac{\sin n}{n^2}$

1.7. $x_n=1+\dfrac{1}{2^2}+\ldots+\dfrac{1}{n^2}$

1.8. $x_n=a_0+a_1q+\ldots+a_nq^n$,

bu yerda $|a_k|<M$ $(k=0,1,2,\ldots)$ ва $|q|<1$.

1.9. $x_n=\dfrac{\cos 1!}{1\cdot 2}+\dfrac{\cos 2!}{2\cdot 3}+\ldots+\dfrac{\cos n!}{n\cdot(n+1)}$

1.10. $x_n=\left(1-\dfrac{1}{2}\right)\cdot\left(1-\dfrac{1}{2^2}\right)\cdot\ldots\cdot\left(1-\dfrac{1}{2^n}\right)$

1.11. $x_n = 0,\underbrace{33...3}_{n \text{ та}}$

1.12. $x_n = 0,\underbrace{77...7}_{n \text{ та}}$

1.13. $x_n = \left(1+\frac{1}{n}\right)^n$

1.14. $x_n = \frac{2^n}{n!}$

1.15. $x_n = \frac{1}{2^2}+\frac{2}{3^2}+...+\frac{n}{(n+1)^2}$

1.16. $x_n = \left(1+\frac{1}{2}\right)\left(1+\frac{1}{4}\right)...\left(1+\frac{1}{2^n}\right)$

1.17. $x_1 = \sqrt{2}$, $x_2 = \sqrt{2+\sqrt{2}}, ...,$ $x_n = \underbrace{\sqrt{2+\sqrt{2+...+\sqrt{2}}}}_{n \text{ та илдиз}}, ...$

2-misol: Quyidagi ketma-ketliklarning yuqori va quyi limitlari topilsin $\left(\overline{\lim_{n\to\infty}} x_n - ?; \underline{\lim_{n\to\infty}} x_n - ?\right)$

2.1. $x_n = \frac{1+(-1)^n}{2}$

2.2. $x_n = \frac{(-1)^n}{n}+\frac{1+(-1)^n}{2}$

2.3. $x_n = 1+2\cdot(-1)^{n+1}+3\cdot(-1)^{\frac{n(n-1)}{2}}$

2.6. $x_n = n^{(-1)^n}$

2.7. $x_n = 1+\frac{n}{n+1}\cdot\cos^2\frac{n\pi}{2}$

2.8. $x_n = \frac{n-1}{n+1}\cdot\cos\frac{2n\pi}{3}$

2.4. $x_n = -n\cdot\left[2+(-1)^n\right]$

2.5. $x_n = \sqrt[n]{1+2^{n\cdot(-1)^n}}$

2.9. $x_n = \left(1+\frac{1}{n}\right)^n(-1)^n + \sin\frac{n\pi}{4}$

2.10. $x_n = \cos^n\frac{2n\pi}{3}$

II BOB FUNKSIYA VA UNING LIMITI

Mavzu: Bir o'zgaruvchili funksiya va uning xossalari

Reja:

1. Funksiyaning ta'rifi va berilish usullari.
2. Funksiyaning grafigi.

Funksiya va uning berilish usullari

Tabiatda ikki xil miqdorlar uchraydi, o'zgaruvchi va o'zgarmas miqdorlar. Bizga bir nechta to'rtburchak berilgan bo'lsin. Ularda quyidagi miqdorlar qatnashadi. Tomonlarning uzunliklari, burchaklarning kattaliklari, yuzalari va perimetrlari. Bu miqdorlardan ba'zilari o'zgarmaydi, ba'zilari o'zgarib turadi. Masalan, qaralayotgan hamma to'rtburchaklarda burchaklarining to'g'riligi, ularning soni to'rtta bo'lishligi va yig'indisi 360^0 ga tengligi o'zgarmaydi. Tomonlarining uzunliklari, perimetrlari, yuzlari esa o'zgarib turadi. Xuddi shuningdek, bir nechta doira chizsak, ularda aylana uzunliklarining o'z diametrlariga nisbati hammasida bir xil bo'lib, π ga teng, lekin ularning radiuslari, aylana uzunliklari, doira yuzlari o'zgarib turadi.

Ma'lum sharoitda faqat bir xil son qiymatlariga ega bo'lgan miqdorlar o'zgarmas miqdorlar deyiladi. Ma'lum sharoitda har xil son qiymatlariga ega bo'lgan miqdorlar o'zgaruvchi miqdorlar deyiladi. Odatda o'zgarmas miqdorlarni $a,b,c,d,...$ o'zgaruvchi miqdorlarni $x,y,z,u,v,...$ harflari bilan belgilanadi.

Matematikada ko'pincha o'zaro bir-biriga bog'liq ravishda o'zgaradigan miqdorlar bilan ish ko'riladi. Yuqoridagi misollarimizda doiraning yuzi uning radiusining o'zgarishiga qarab o'zgaradi, ya'ni doiraning radiusi ortsa, yuzi ham ortadi, va aksincha. Xuddi shuningdek, kvadratning tomoni bilan yuzi orasida ham shunday bog'lanish mavjud. Kvadratning yuzi uning tomoniga bog'liq ravishda o'zgaradi.

Ta'rif. Agar x miqdorning X sohadagi har bir qiymatiga biror f qonuniyatga ko'ra y miqdorning Y-sohadan aniq bir qiymati mos keltirilsa, y miqdor x miqdorning X-sohadagi *funksiyasi* deyiladi va $y=f(x)$ kabi yoziladi.

Bu holda x - argument yoki erkli o'zgaruvchi, y - esa funksiya yoki erksiz o'zgaruvchi deyiladi. Agar y, x ning funksiyasi bo'lsa, u holda x va y lar orasidagi bog'lanish funksiyali bog'lanish deyiladi va quyidagicha yoziladi: $y=f(x)$, $y=g(x)$, $y=\varphi(x),...$. Agar yuqoridagi misollarga e'tibor qaratsak, doiraning yuzi radiusning funksiyasi, kvadratning yuzi tomonining funksiyasi ekanligi ma'lum bo'ladi.

Argument qabul qilishi mumkin bo'lgan qiymatlari to'plami funksiyaning aniqlanish sohasi, funksiyaning o'zi qabul qilishi mumkin bo'lgan qiymatlari to'plami funksiyaning o'zgarish sohasi yoki qiymatlari to'plami deyiladi.

Funksiyaning berilish usullari. Funksiya sharoitiga qarab jadval, analitik va grafik usullar bilan berilishi mumkin.

Funksiya jadval usulida berilganda, argumentning ma'lum tartibdagi $x_1,x_2,x_3,...,x_n,...$ qiymatlari va funksiyaning ularga mos keluvchi $y_1,y_2,y_3,...,y_n,...$ qiymatlari jadval holida beriladi:

X	x_1	x_2	x_3	...	x_n	...
Y	y_1	y_2	y_3	...	y_n	...

Funksiyalarning jadval usulida berilishiga misol qilib kvadratlar, kublar, kvadrat ildizlar jadvallarni ko'rsatish mumkin. Bu usuldan ko'pincha miqdorlar orasida tajribalar o'tkazishda foydalaniladi.

Funksiyaning grafik usulda berilishi. $y=f(x)$ funksiyaning grafigi deb koordinatalari $y=f(x)$ ni to'g'ri tenglikka aylantiruvchi tekislikdagi barcha nuqtalar to'plamiga aytiladi.

Agar funksiyaning grafigi tasvirlangan bo'lsa, funksiya grafik usulda berilgan deyiladi.

Funksiyaning analitik usulda berilishi.

Formula yordamida berilgan funksiyalarga analitik usulda berilgan deyiladi. Masalan, $y=x^2$, $y=kx+b$, $y=a^x$, $y=\lg x$, $y=\sin x$, $y=\cos x$,... funksiyalar analitik usulda berilgan. Agar analitik usulda berilgan funksiyaning aniqlanish sohasi to'g'risida alohida shart qo'yilmagan bo'lsa, u holda $y=f(x)$ da o'ng tomonda turuvchi ifoda ma'noga ega bo'ladigan ning qiymatlari olinadi. Masalan, agar $y=x^2$ ni kvadratning tomoni bilan yuzi ifodalovchi bog'lanish sifatida olsak, u holda aniqlanish sohasi barcha musbat sonlardan iborat bo'ladi.

Funksiyaning aniqlanish sohasini topishga doir misollar keltiramiz. Quyidagi funksiyalarning aniqlanish sohasini toping:

1-misol. $y=\frac{3}{x}$ funksiyaning aniqlanish sohasini toping.

Yechish: Ma'lumki, kasr ma'noga ega bo'lishi uchun uning maxraji noldan farqli bo'lishi kerak. Demak, $x\neq 0$ ёki $x\in(-\infty;0)\cup(0;+\infty)$.

2-misol. $y=\frac{1}{2}(x-1)^{-1}$ funksiyaning aniqlanish sohasini toping.

Yechish: Xuddi yuqoridagidek muhokama yuritsak, $2x-1\neq 0$ yoki $2x\neq 1$. Demak, aniqlanish sohasi $(-\infty;\frac{1}{2})\cup(\frac{1}{2};+\infty)$ dan iborat.

3-misol. $y=\sqrt{3x+2}$ funksiyaning aniqlanish sohasini toping.

Yechish: Kvadrat ildiz ma'noga ega bo'lishi uchun ildiz ostidagi ifoda manfiy bo'lmasligi kerak, ya'ni $3x+2\geq 0$ bunda $x\geq -\frac{2}{3}$. Demak, aniqlanish sohasi $[-\frac{2}{3},+\infty)$ dan iborat.

4-misol. $y=\frac{1}{\sqrt{4x-5}}$ funksiyaning aniqlanish sohasini toping.

Yechish: Agar yuqoridagilarni hisobga olinsa, u holda $4x-5>0$ bo'ladi. Bundan $x>\frac{5}{4}$. Demak, aniqlanish sohasi $(\frac{5}{4},+\infty)$ dan iborat.

5-misol. $y=\lg(2x-1)$ funksiyaning aniqlanish sohasini toping.

Yechish: Logarifmik funksiya faqat musbat sonlar uchun aniqlangan. Demak, $2x-1>0$ bo'lishi kerak. Bundan, $x>\frac{1}{2}$. Demak, aniqlanish sohasi $(\frac{1}{2},+\infty)$ dan iborat.

6-misol. $y=\frac{1}{\lg(2x-1)}$ funksiyaning aniqlanish sohasini toping.

Yechish: Agar yuqoridagilarni inobatga olsak, $2x-1>0$, $2x-1\neq 1$ bo'ladi. Bundan $x>\frac{1}{2}$, $x\neq 1$ kelib chiqadi. Demak, aniqlanish sohasi $(\frac{1}{2};+1)\cup(1;+\infty)$ dan iborat.

A) analitik usul funksiyaning o'rganish jarayonida juda ko'p uchraydigan usuldir, lekin ba'zi xollarda funksiyaning qiymatini topish murakkab hisoblashlarga olib keladi,

Б) $y=f(x)$ yozuv hali funksiyaning analitik usulda berilishi bo'lmasligi mumkin. Masalan, ushbu Dirixle funksiyasini olaylik,

$$y=\begin{cases}1, & агар \quad x-рационал \quad сон \quad бўлса\\ 0, & агар \quad x-иррационал \quad сон \quad бўлса.\end{cases}$$

Demak $y=f(x)$ funksiya berilgan, uning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iborat, ammo funksiyaning analitik ifodasi berilgan emas,

В) funksiyaning jadval usulida berilishi qulaydir, chunki bir necha qiymatlar topilgan bo'ladi, lekin funksiyaning sohasi cheksiz to'plam bo'lganda, uning barcha qiymatlarini ko'rsatib bo'lmaydi,

Г) funksiyaning grafik usulda berilishi uning o'zgartirishlarini ko'rgazmali qilish imkonini beradi.

Funksiyaning grafigi – egri chiziq (hususiy holda to'g'ri chiziq), ba'zi hollarda biror nuqtalar to'plami bo'ladi.

Funksiya grafigi.

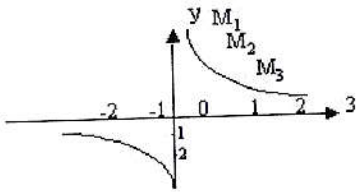
$y=f(x)$ funksiyaning grafigini hosil qilish uchun $M(x,f(x))$ nuqtalarni hosil qilib, ular bir-biriga juda yaqin bo'lganda, silliq chiziq bilan tutashtiriladi.

Masalan. 1) $y=\frac{1}{x}$ funksiyaning grafigi chizilsin. Bu funksiyaning aniqlanish sohasi $x\neq 0$ haqiqiy sonlar to'plami, ya'ni $(-\infty;0)\cup(0;+\infty)$ dan iborat.

Endi, aniqlanish sohasidan x ning bir necha qiymatlarini olib, uning ularga mos keladigan qiymatlarini topamiz.

x	1	2	3	-1	-2	-3	$\frac{1}{2}$	$-\frac{1}{2}$	...
y	1	$\frac{1}{2}$	$\frac{1}{3}$	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	2	-2	...

Koordinata tekisligida $M_1(1;1)$, $M_2(2;\frac{1}{2})$, $M_3(3;\frac{1}{3})$,... nuqtalarni hosil qilamiz. Bir biriga yaqin turga nuqtalarni uzluksiz chiziq yordamida tutashtirsak, funksiyaning grafigini ifoda qiladigan egri chiziq giperbola hosil bo'ladi. (2-chizma)



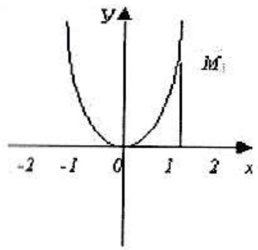
2-чизма

2) $y=x^2$ ning grafigi chizilsin.

Jadval tuzamiz

x	0	1	2	3	-1	-2	-3	...
$y=x^2$	0	1	4	9	1	4	9	...

$M_1(0;0)$, $M_2(1;1)$, $M_3(2;4)$ nuqtalarni hosil qilamiz. Ularni silliq chiziq bilan tutashtirsak, parabola egri yaizig'i hosil bo'ladi. (3-chizma)

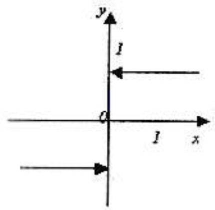


3-чизма.

3) 4-chizmada

$$y=\begin{cases}1, & агар \quad x>0 \quad бўлса,\\ 0, & агар \quad x=0 \quad бўлса,\\ -1, & агар \quad x<0 \quad бўлса.\end{cases}$$

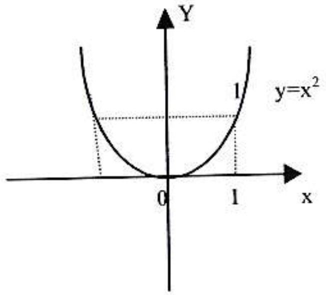
funksiyaning grafigi ko'rsatilgan. (4-chizma)



4-чизма.

Aksincha, agar tekislikda biror egri chiziq berilgan bo'lib, abssissalar o'qiga tik bo'lgan har qanday to'g'ri chiziq, bu egri chiziq bilan bittadan ko'p bo'lmagan nuqtada kesishsa, u holda bu egri chiziq funksiyani ifodalaydi.

Juft va toq funksiyalar. $y=f(x)$ funksiyaning aniqlanish sohasiga tegishli x o'zgaruvchining har bir qiymati bilan $-x$ qiymat ham shu funksiyaning aniqlanish sohasiga tegishli bo'lsa va bunda $f(-x)=f(x)$ tenglik bajarilsa, $y=f(x)$ funksiya juft funksiya deyiladi. Masalan, $y=x^2$ funksiya juft funksiyadir. Haqiqatdan, bu funksiya R to'plamda aniqlangan va aniqlanish sohasi har qanday x bilan $-x$ ni o'z ichiga oladi. Bundan tashqari, $f(-x)=(-x)^2=x^2=f(x)$ tenglik bajariladi. Juft funksiya grafigi ordinata o'qiga nisbatan simmetrik bo'ladi (5-chizma).



5-чизма

$y=\cos x$ juft funksiyadir. Haqiqatdan ham, har qanday x va $-x$ uchun P_α va $P_{-\alpha}$ nuqtalar absissalar o'qiga nisbatan simmetrik joylashgan (9-chizma). Bundan shu nuqtalarning absissalari bir xil, ordinatalari esa qarama-qarshi ekani kelib chiqadi. Bu kosinus ta'rifiga ko'ra, har qanday α da quyidagi tenglik to'g'ri ekanini bildiradi: $\cos\alpha=\cos(-\alpha)$. Umuman, har qanday juft funksiyaning grafigi ordinata o'qiga nisbatan simmetrikdir. $y=f(x)$ funksiyaning aniqlanish sohasiga tegishli x ning har bir qiymati bilan $-x$ qiymat ham shu funksiyaning aniqlanish sohasiga tegishli bo'lsa va bunda $f(-x)=-f(x)$ tenglik bajarilsa, $y=f(x)$ funksiya toq funksiya deyiladi. Toq funksiyaning grafigi koordinata boshiga nisbatan simmetrik joylashadi. Masalan, $f(x)=x^3$ funksiya toq funksiyadir. Haqiqatdan ham, $f(-x)=(-x)^3=-x^3=-f(x)$, ya'ni $f(-x)=-f(x)$ tenglik bajariladi. Bu

funksiyaning grafigi koordinata boshiga nisbatan simmetrik bo'lib, kubik paraboladan iboratdir (9-chizma). $y=\sin x$ toq funksiyadir. Haqiqatdan ham, chizmada P_α va $P_{-\alpha}$ nuqtalarning ordinatalari bir xil, lekin ishoralari qarama-qarshiligidan $sin(\alpha)=y_\alpha$, $sin(-\alpha)=-y_\alpha$ bo'ladi. Bundan esa $sin(-\alpha)=-\sin\alpha$ bo'ladi. Har qanday funksiya ham juft yoki toq bo'lishi shart emas.

Masalan, $y=2x+5$, $y=x^2+5x^3$, $y=\sin x+\cos x$ juft ham, toq ham emas. Demak funksiyalar har doim juft yoki toq bo'lishi shart emas ekan.

1-misol. $y=\frac{x(x^2+1)(2x-3)}{3x^2-6x+8}$ funksiyaning juft yoki toq ekanligini aniqlang.

Yechish: yuqoridagi ta'rifga ko'ra argument o'rniga qarama-qarshi ishoradagi argumentni qo'yib soddalashtirilganda, funksiyaning o'zi kelib chiqsa funksiya juft bo'ladi. Agar soddalashtirilgach funksya ham teskari ishora bilan kelib chiqsa, funksiya tok funksiya bo'ladi. Shuni tekshiramiz.

$$y(-x)=\frac{-x((-x)^2+1)(-2x-3)}{3(-x^2)+6x+8}=\frac{x(x^2+1)(2x+3)}{3x^2+6x+8}\neq y(x)\neq -y(x)$$

Bundan ko'rinadiki bu funksiya juft ham emas toq ham emas.

Endi funksiyaning juft yoki toq ekanligini tekshirishning yana bir usulini ko'rib chiqamiz. Sababi har doim ham maktab o'quvchilari funksiyani tekshirishda o'zgaruvchidan to'la foydalana olishmaydi. Shuning uchun, bu funksiyaning o'rniga har xil ishorali absolyut qiymati o'zaro teng o'zgarmas sonlarni qo'yib tekshiramiz. Agar xar hil ishorali sonlarni qo'yganimizda ham natija bir hil bo'lsa, funksiya juft funksiya bo'ladi. Agar har hil ishorali absolyut qiymatlari teng sonlar kelib chiqsa, u holda funksiya toq funksiya bo'ladi. Agar ikki hil qiymat kelib chiqsa, u holda funksiya juft ham emas toq ham emas bo'ladi. Buni tekshirish uchun yuqoridagi misoldan foydalanamiz.

$y=\frac{x(x^2+1)(2x-3)}{3x^2-6x+8}$ bu funksiyaning o'rniga 1 va -1 ni qo'yib tekshiramiz.

Odatda 1 va -1 dan foydalanamiz, lekin trigonametrik yoki logarifmik funksiyalar uchraganda hisoblashga va soddalashtirishga osonroq bo'ladigan qiymatlardan foydalanish mumkin.

$$y=\frac{x(x^2+1)(2x-3)}{3x^2-6x+8}\,,\qquad y(1)=\frac{1(1+1)(2-3)}{3-6+8}=-\frac{2}{5}\,,$$

$y(-1)=\dfrac{-1(1+1)(-2-3)}{3+6+8}=\dfrac{5}{17}$ bu yerda $y(1)\neq y(-1)$ bundan esa bu funksiya toq ham emas juft ham degan hulosaga kelamiz.

Davriy funksiyalar

Ta'rif. Agar $f(x)$ funksiya uchun shunday $t>0$ son mavjud va funksiyaning aniqlanish sohasidan olingan har bir x uchun $x+t$ va $x-t$ lar aniqlanish sohasiga joylashgan bo'lib, $f(x+t)=f(x)$ tenglik o'rinli bo'lsa, u holda $f(x)$ *davriy funksiya* deb ataladi. t sonlarni eng kichigi funksiyaning davri deyiladi.

1-misol. $y=sinx$, $y=cosx$, $y=tgx$, $y=x-[x]$ davriy funksiyalardir.

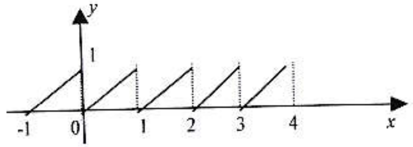
Davriy funksiyaning grafigini hosil qilish uchun uning bir davr ichidagi grafigini chizib, so'ngra uni chapga va o'ngga cheksiz ko'p marta ko'chirish kerak.

2-misol. $f(x)=x-[x]=x-E(x)$ funksiya berilgan. Bunda $E(x)=[x]$ ifoda x ning butun qismini bildiradi. (E – fransuzcha Entier -ante-butun so'zining birinchi harfi). Masalan, $[x]=m$ $(m\le x<m+1)$ m - butun son.

$f(x)=x-E(x)=\{x\}$ bu funksiya x ning kasr qismini bildiradi, ya'ni $f(1)=0$; $f(1{,}05)=0{,}05$;... $f(x)$ funksiya davriydir va uning davri t=1 dir. Haqiqatdan,

$$f(x+1)=x+1-E(x+1)=x+1-E(x)-1=x-E(x)=f(x).$$

Demak, har qanday butun son ham davr bo'ladi. Funksiyaning grafig 8-chizmada ko'rsatilgan.



8-chizma

3-misol. $y=\sin 2x+\cos\frac{1}{2}x+tg3x$ funksiyaning eng kichik davrini toping.

Yechish. Ushbu funksiyaning eng kichik davrini topish uchun berilgan funksiyani tashkil qilgan har bir funksiyaning eng kichik davrlarini topamiz va topilgan davrlarning eng kichik umumiy karralisini topamiz.

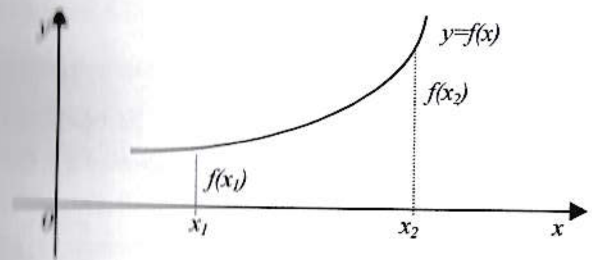
$y=\sin 2x,\ 2x=2\pi,\ x=\pi,\ y=\cos\frac{1}{2}x,\ \frac{1}{2}x=2\pi,\ x=4\pi$ va

$y=tg3x,\ 3x=\pi,\ x=\frac{\pi}{3}$ bo'lib, $\pi, 4\pi, \frac{\pi}{3}$ larning eng kichik umumiy karralilarini topamiz va 4π ga teng ekanligini ko'ramiz. Demak ushbu berilgan funksiyaning eng kichik davri 4π ga teng ekanligi kelib chiqadi.

Monoton va teskari funksiyalar

Ta'rif. $y=f(x)$ funksiyaning X sohadagi ihtiyoriy ikkita (x_1, x_2) qiymatlari uchun $x_1<x_2$ bo'lganda, $f(x_1)<f(x_2)$ tengsizlik o'rinli bo'lsa, u holda $y=f(x)$ funksiyasi X sohada o'suvchi funksiya deyiladi.

Ta'rifni geometrik nuqtai nazardan quyidagicha ko'rsatishimiz mumkin.

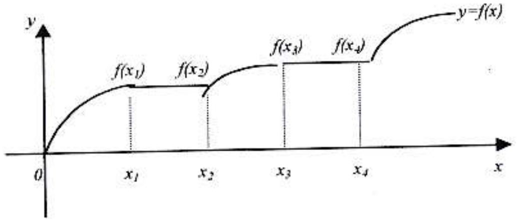


Yuqoridagi ta'rifdan ko'rinadiki, funksiya biror oraliqda o'suvchi bo'lishi uchun shu oraliqdagi argumentning kichik qiymatiga funksiyaning kichik qiymati, argumentning katta qiymatiga funksiyaning katta qiymati mos keladi.

1) $y=2^x$ funksiyasi butun sonlar o'qida o'suvchi.

2) $y=tgx$ funksiya ham o'suvchi funksiyadir.

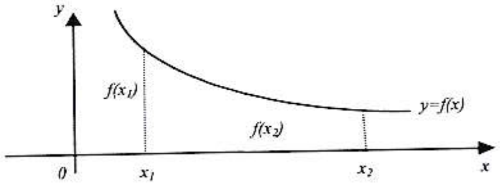
Ta'rif. $y=f(x)$ funksiyaning X sohadagi ixtiyoriy ikkita (x_1,x_2) qiymatlari uchun $x_1 \le x_2$ bo'lganda $f(x_1)\le f(x_2)$ tengsizlik o'rinli bo'lsa, u holda $y=f(x)$ funksiya (x_1,x_2) oraliqda kamaymaydigan funksiya deyiladi.



Ta'rif. $y=f(x)$ ning argumenti x ni $\forall(x_1,x_2)$ uchun $x_1 < x_2$, bo'lganda $f(x_1) > f(x_2)$ tengsizlik o'rinli bo'lsa, $y=f(x)$ ni (x_1,x_2) oralig'ida kamayuvchi funksiya deyiladi.

1-misol. $y=x^2$ funksiyaning olsak, bu funksiya $(-\infty,0)$ oralig'ida kamayuvchi, $(0,\infty)$ oralig'ida o'suvchi funksiyadir.

2-misol. $y=sinx$ funksiya $(0,\frac{\pi}{2})$ oraliqda monoton o'suvchi bo'lib, $(\frac{\pi}{2},\frac{3\pi}{2})$ oraliqda monoton kamayuvchidir.



Ta'rif. $y=f(x)$ ning argumentining ixtiyoriy (x_1,x_2) qiymatlari uchun $x_1 \le x_2$ bo'lganda $f(x_1)\ge f(x_2)$ bo'lsa, u holda $y=f(x)$ funksiyasi (x_1,x_2) oralig'ida *o'smaydigan* funksiya deyiladi.

Agar berilgan oraliqda argumentning katta qiymatiga funksiyaning katta qiymati mos kelsa, ya'ni shu oraliqdagi ixtiyoriy x_1 va x_2 uchun

$x_1 < x_2$ shartdan $f(x_2) > f(x_1)$ kelib chiqsa, $y=f(x)$ funksiya shu oraliqda o'suvchi deyiladi.

Ta'rif. Biror (x_1,x_2) oralig'ida o'suvchi va kamayuvchi funksiyalar *monoton funksiyalar* deyiladi.

Teskari funksiya tushunchasi.

Teskari trigonometrik funksiyalarga o'tishdan avval umuman teskari funksiyalarga izoh berib o'tamiz.

Faraz qilaylik, $y=f(x)$ funksiya biror X sohada berilgan bo'lsin va x argument X sohada o'zgarganda, bu funksiya qabul qilgan barcha qiymatlar to'plami Y bilan ifodalansin. Odatda, X va Y lar oraliqlardan iborat bo'ladi.

Biz Y sohadan biror $y=y_0$ qiymatni tanlaylik; bu vaqtda X sohadan bizning funksiyamiz xuddi shu y_0 ga teng bo'ladigan $x=x_0$ qiymat, albatta, topiladi, demak, $f(x_0)=y_0$ bo'ladi.

x_0 ning bunday qiymatlari bir qancha bo'lishi ham mumkin. Shunday qilib, Y sohadagi y ning har bir qiymatiga xning bitta yoki bir qancha qiymati mos keladi, shu bilan Y sohada bir qiymatli yoki ko'p qiymatli $x=g(y)$ funksiya aniqlanib, buni $y=f(x)$ funksiyaning teskari funksiyasi deyiladi.

Misollar qaraymiz:

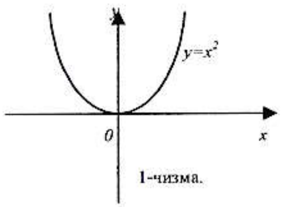
1) $y=a^x (a>1)$ funksiyani qaraylik, bu yerda x argument $X=(-\infty;+\infty)$ oraliqda o'zgaradi. Funksiya y ning qiymatlari $Y=(0;\ +\infty)$ oraliqni tashkil qiladi, shu bilan birga, bu oraliqdagi har bir y ga X dan bitta $x=\log_a y$ qiymat mos keladi. Bu holda teskari funksiya bir qiymatli bo'ladi.

2) Aksincha, $y=x^2$ funksiya uchun x argument $X=(-\infty;+\infty)$ oraliqda o'zgarsa, teskari funksiya ikki qiymatli bo'ladi, chunki $Y=[0;\ +\infty)$ oraliqdagi y ning har bir qiymati uchun X da ikkita $x=\pm\sqrt{y}$ qiymat mos keladi. Odatda, bu ikki qiymatli funksiya o'rniga $x=\sqrt{y}$ va $x=-\sqrt{y}$ funksiya (ikki qiymatli

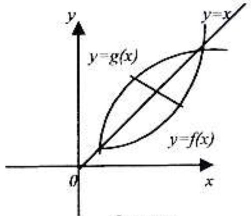
funksiyaning "shoxchalari") tekshiriladi. Bularning har birini alohida $y=x^2$ ga teskari funksiya deb qarash ham mumkin, faqat bu vaqtda x ning o'zgarish sohasi $[0;\ +\infty)$ yoki $(-\infty;\ 0]$ oraliq bilan chegaralangan, deb faraz qilish kerak.

Berilgan $y=f(x)$ funksiyaning grafigiga qarab, bunga teskari $x=g(y)$ funksiyaning bir qiymatli bo'lish yoki bo'lmasligini sezish oson. Agar x o'qqa parallel bo'lgan har bir to'g'ri chiziq bu grafikni faqat bitta nuqtada kessa, u holda teskari funksiya bir qiymatli bo'ladi. Aksincha, bunday to'g'ri chiziqlardan ba'zilari grafikni bir nechta nuqtada kesib o'tsa, teskari funksiya ko'p qiymatli deyiladi. Bu holda grafikka qarab, har bir bo'lakka bu funksiyaning bir qiymatli "shoxchasi" mos keladigan qilib, x ning o'zgarish oralig'ini bo'laklarga bo'lish mumkin. Masalan, 1-chizmadagi $y=x^2$ funksiyaning grafigi bo'lgan parabolaga birinchi qarashimizdayoq, uning teskari funksiyasi ikki qiymatli ekanini aniq ko'ramiz va teskari funksiyaning bir qiymatli "shoxchalarini" olish uchun parabolaning o'ng va chap bo'laklarini, ya'ni x ning musbat va manfiy qiymatlarini alohida qarash yetarli.

Agar $x=g(y)$ funksiyasi $y=f(x)$ funksiyaga teskari bo'lsa, u vaqtda bu ikki funksiyaning grafigi bir xil bo'lishi ravshan. Teskari funksiyaning argumentini ham x bilan belgilashni, ya'ni $x=g(y)$ funksiya o'rniga $y=g(x)$ deb yozishni talab etish mumkin. U vaqtda gorizontal o'qni y o'q deb va vertikal o'qni esa x o'q (yangi) gorizontal, y o'q (yangi) vertikal bo'lsin desak, u vaqtda bu o'qlarning o'rinlarini almashtirib, birining o'rniga ikkinchisini qo'yish kerak, bu esa grafikni ham o'zgartiradi. Buni amalga oshirish uchun xOy chizma tekisligini birinchi koordinata burchak bissektritsasi atrofida 180^0 ga aylantirish maqsadga muvofiq.



1-чизма.



2-чизма.

Shunday qilib, $y=g(x)$ ning grafigi $y=f(x)$ ning grafigini shu bissektrisaga nisbatan ko'zgudagi aksi deb olish mumkin.

Funksiyaning superpozitsiyasi.

Funksiyalarning superpozitsiyasi (yoki o'rniga qo'yish) tushunchasini kiritib chiqamiz. Bu tushuncha berilgan funksiyaning argumenti o'rniga boshqa argumentga bog'liq bo'lgan funksiyani qo'yishdan iborat. Masalan, $y=\sin x$ va $z=lgy$ funksiyalarning superpozitsiyasi $z=lgsinx$ funksiyani beradi; shunga o'xshash $\sqrt{1-x^2}$, $arctg\frac{1}{x}$ va hokazo funksiyalar ham hosil bo'ladi.

Umuman, $y=f(x)$ funksiya x ning barcha qiymatlari uchun $X=\{x\}$ sohada aniqlangan va shu bilan birga bu funksiyaning hamma qiymatlari esa $Y=\{y\}$ sohaga kirgan deb faraz qilamiz. Endi $z=\varphi(y)$ funksiya xuddi $Y=\{y\}$ sohada aniqlangan bo'lsin. U vaqtda z o'zgaruvchining o'zgaruvchi x ning funksiyasi bo'ladi, ya'ni: $z=\varphi(f(x))$. x ning X sohadagi berilgan qiymati bo'yicha avval yning Y dagi unga mos qiymatini (f belgi bilan harakterlangan qonun bo'yicha) topamiz, so'ngra yning bu qiymatiga muvofiq z ning qiymatini (φ belgi bilan harakterlangan qonun bo'yicha) aniqlaymiz; z ning bu qiymatini x ning tanlangan qiymatiga mos deb hisoblanadi. Hosil qilingan funksiyaning funksiyasi yoki murakkab funksiya $f(x)$ va $\varphi(y)$ funksiyalarning superpozitsiyasi natijasida vujudga keldi.

Bunday $f(x)$ funksiyaning qiymatlari, $\varphi(y)$ ni aniqlovchi Y sohadan chetga chiqmaydi degan farazimiz g'oyat muhimdir; agar bu farazni tushirib qoldirilsa, ma'nosizlik yuz berishi mumkin. Masalan, $z=lgy$, $y=sinx$ deb olib, biz faqat x ning $sinx>0$ ni qanoatlantiruvchi qiymatlarinigina olamiz, boshqacha $lgsinx$ ifoda ma'noga ega bo'lmay qoladi.

Murakkab funksiyaning harakteristikasi xvazorasidagi funksional munosabatning tabiati bilan emas, balki bu munosabatning berilish usuli bilangina bog'langanligini ta'kidlab o'tish foydali deb hisoblaymiz. Masalan,

$[-1,1]$ dagi y uchun $z=\sqrt{1-y^2}$ funksiya va $[-\frac{\pi}{2};\frac{\pi}{2}]$ dagi x uchun $y=\sin x$ funksiya berilgan bo'lsin.

U vaqtda: $z=\sqrt{1-\sin^2 x}=\cos x$.

Bu yerda $\cos x$ funksiyasi murakkab ko'rinishida berilgan bo'lib qoldi. Endi, funksiyalarning superpozitsiyasi tushunchasi to'la anglashilgandan keyin, analizda tekshiriladigan eng oddiy funksiyalar sinflarini harakterlashimiz mumkin: bular, yuqorida ko'rsatilgan elementar funksiyalar, so'ngra bulardan to'rtta arifmetik amalni ishlatish va superpozitsiyalashni chekli son marta ketma-ket qo'llash natijasida kelib chiqqan funksiyalardir. Bu funksiyalarni elementar funksiyalar orqali chekli ko'rinishda ifodalanuvchi funksiyalar deb, ba'zan esa faqat elementar funksiyalar deb ham ataladi.

Elementar funksiyalar

Bu yerda elementar funksiyalar deb atalgan funksiyalarning ba'zi bir sinflarini ko'rsatib o'tamiz.

Butun va kasr ratsional funksiyalar.

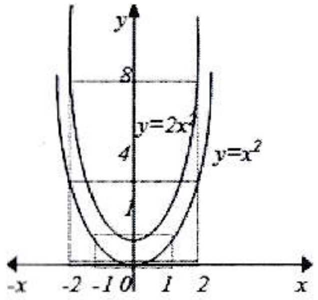
X ga nisbatan butun $y=a_0x^n+a_1x^{n-1}+...+a_{n-1}x+a_n$ ko'phad (bu yerda $a_0, a_1, a_2, ...$ o'zgarmas) bilan tasvirlanuvchi funksiya butun ratsional funksiya deyiladi.

Bunday ikki ko'phadning

$$y=\frac{a_0x^n+a_1x^{n-1}+...+a_{n-1}x+a_n}{b_0x^m+b_1x^{m-1}+...+b_{m-1}x+b_m}$$

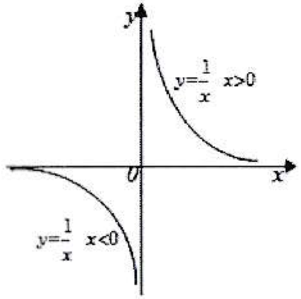
nisbati kasr ratsional funksiya deyiladi. Bu funksiya x ning maxrajini nolga aylantiruvchi qiymatlaridan boshqa barcha qiymatlari uchun aniqlangan bo'ladi.

Misol tariqasida 1-chizmada $y=ax^2$ funksiya (parabola) ning a koeffitsiyenti har xil qiymatlar qabul qilgandagi grafiklari berilgan.



1-чизма

2-chizmada esa $y=\frac{a}{x}$ funksiya (teng yonli giperbola) ning a har xil qiymatlarni qabul qilgandagi grafiklari berilgan.

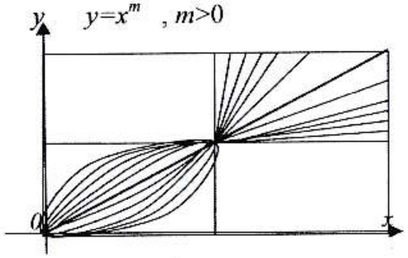


2-chizma

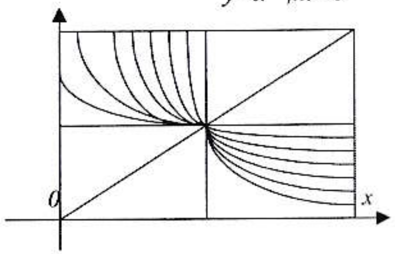
Darajali funksiya. Quyidagi $y=x^{\mu}$ ko'rinishdagi funksiyani darajali funksiya deyiladi, bu yerda μ ixtiyoriy o'zgarmas haqiqiy son. Agar μ kasr bo'lsa, biz ildizga ega bo'lamiz. Masalan, m natural son bo'lsin va: $y=x^{\frac{1}{m}}=\sqrt[m]{x}$.

Bu funksiya m toq bo'lganda, x ning barcha qiymatlari uchun va m juft bo'lganda, x ning faqat musbat qiymatlari uchun aniqlanadi. Bu holda biz ildizning faqat arifmetik qiymatini hisobga olamiz. Nihoyat, μ irratsional son bo'lsa, $x>0$ deb faraz etamiz ($x=0$ qiymat $\mu>0$ bo'lgandagina olinadi).

Quyida 3 va 4 - chizmalarda μ ning har xil qiymatlari uchun darajali funksiyaning grafiklari berilgan.

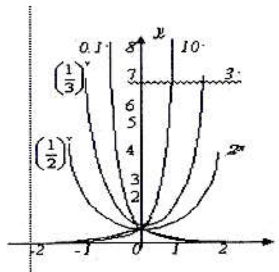


3-чизма.

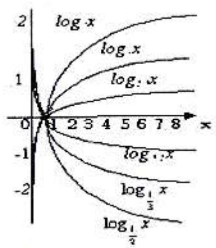


4-чизма.

Ko'rsatkichli funksiya, ya'ni $y=a^x$ ko'rinishdagi funksiyadir, bu yerda 1 dan farqli musbat son; x istalgan haqiqiy qiymat qabul qila oladi. 5-Chizmada a ning har xil qiymatlari uchun ko'rsatkichli funksiyaning grafiklari berilgan.



5-чизма



6-чизма

Logarifmik funksiya, ya'ni $y = \log_a x$ ko'rinishdagi funksiya, bu yerda a soni 1 dan farqli musbat sondir; x faqat musbat qiymatlar qabul qiladi.

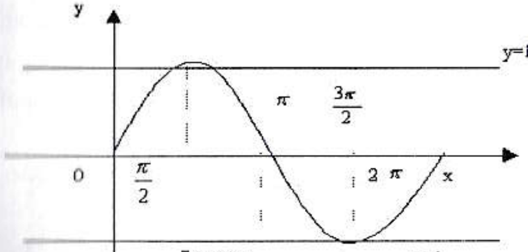
6-Chizmada bu funksiyaning a ning turli qiymatlaridagi grafiklari berilgan.

Trigonometrik funksiyalar:

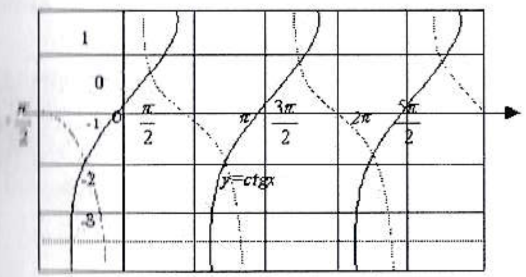
$y=\sin x$, $y=\cos x$, $y=tgx$, $y=ctgx$, $y=\sec x$, $y=\operatorname{cosec} x$

Agar trigonometrik funksiyalarning argumentlari burchaklarning o'lchovi sifatida qaralsa, ular bu burchaklarni har vaqt radianlarda ifodalaydi (agarda aksi aytilmagan bo'lsa). Buni har vaqt esda tutish kerak. Bunda $y=tgx$ va $y=\sec x$ lar uchun $(2k+1)\cdot\frac{\pi}{2}$ ko'rinishdagi qiymatlar, $y=ctgx$ va $y=\operatorname{cosec} x$ lar uchun $k\pi$ (bu yerda k-butun son) ko'rinishdagi qiymatlar mustasno.

$y=\sin x(\cos x)$ va $y=tgx(ctgx)$ funksiyalarning grafiklari 7-8 chizmalarda berilgan. Sinusning grafigi, odatda, sinusoida deyiladi.



7-чизма



8-chizma

Elementar funksiyalar va uning xossalariga oid misollar yechish.

1-misol. $y = \dfrac{\sin x + \cos x}{\sin x - \cos x}$ funksiyaning aniqlanish sohasini toping.

Yechish: ushbu misolni yechish uchun, kasr funksiya bo'lganligi sababli bu funksiyaning maxraji noldan farqli bo'lishi lozim. Shuning uchun $\sin x \neq \cos x$ bo'lishi lozim. Ya'ni ikki funksiya qaysi burchaklarda teng bo'ladi

2-misol. $y = \log_5(x^2 + 4x + 5)$ funksiyaning aniqlanish sohasini toping.

Yechish: funksiyaning aniqlanish sohasini topish uchun, logarifm xossasini esga oladigan bo'lsak, logorifm ostidagi ifoda noldan katta bo'lishi kerak. Shuning uchun $x^2+4x+5\geq 0$. Bu tengsizlikni yechish uchun ikki usuldan foydalanish mumkin. YA'ni bu kavadrat uchhadni kavs kvadratga keltirib va oraliqlar usulida yechish.

1) $x^2 + 4x + 5 = (x+2)^2 + 1$ ifodani kavs kvadratga keltirdik. Bundan ko'rinadiki ikki musbat qiymatlari yig'indisi yana musbat bo'ladi. Shuning uchun funksiyaning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iborat.

2) Oraliqlar usulida ishlaydigan bo'lsak, kvadrat uchhadning diskriminantini hisoblaymiz.

$x^2+4x+5\geq 0$, $D=16-20=-4<0$ diskriminant manfiy va kvadrat had oldidagi koiffitsiyent musbat shu sababdan kvadrat uchhadning grafigi obsissa o'qidan yuqorida yotadi. YA'ni funksiya argumentning barcha qiymatlarida musbat bo'ladi. Demak aniqlanish sohasi barcha haqiqiy sonlar to'plami.

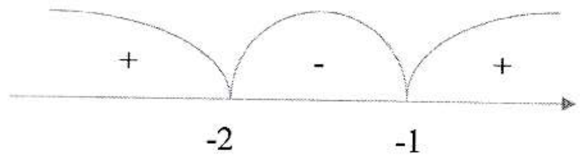
3-misol. $y = \lg(x^2 + 3x + 2) + \dfrac{1}{\sqrt{6+5x-x^2}}$ funksiyaning aniqlanish sohasining toping.

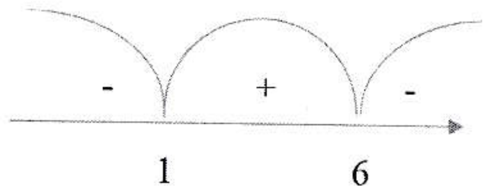
Yechish: ushbu funksiya uchta logorifmik, irrotsional va ratsional funksiyalardan tashkil topgan. Bu funksiyaning aniqlanish sohasini topadigan bo'lsak.

$y = \lg(x^2 + 3x + 2) + \dfrac{1}{\sqrt{6+5x-x^2}}$ tengsizlikda nolga teng qismi yo'qligiga sabab $\begin{cases} x^2 + 3x + 2 > 0 \\ 6 + 5x - x^2 > 0 \end{cases}$ logorifm ostadagi ifoda noldan farqli bo'lishi shart va irrotsional ifodakasr maxrajida bo'lganligi uchun ham noldan farqli bo'lishi kerak.

$\begin{cases} x^2 + 3x + 2 > 0 \\ 6 + 5x - x^2 > 0 \end{cases}$, $D=9-8=1$, $x_1 = \dfrac{-3+1}{2} = -1$, $x_2 = -2$

$D = 25 + 24 = 49$, $x_1 = \dfrac{-5+7}{-2} = 1$, $x_2 = 6$ har ikkala kvadrat uchhadning nollari topildi. Endi ushbu nollarni son o'qiga joylab oraliqlarda ifodalaymiz.





Endi har ikkala tengsizlikni yechimlar to'plamini yozamiz. $(-\infty;-2)\cup(-1;+\infty)$, bu oraliq sistemadagi birinchi tengsizlikning yechimlar to'plami. $(1;6)$ bu sistemadagi ikkinchi tengsizlikning yechimlar to'plami. Sistemaning umumiy yechimlari to'plamini topish uchun bu ikkita oraliqlarni kesishtirish lozim va $(1;6)$ oraliq sistemaning umumiy yechimi bo'ladi.

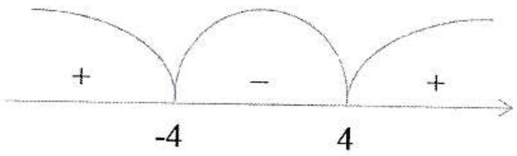
4-misol. $y = \sqrt{9 - 3\sqrt{x^2 - 16}}$ funksiyaning aniklanish sohasini toping.

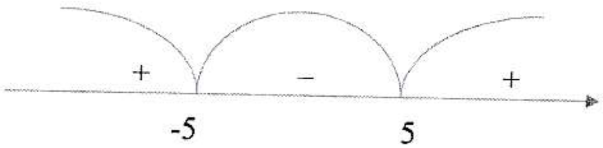
Yechish: Bu misolda berilgan funksiya murakkab va irratsional funksiya bo'lib, bu funksiyaning aniklanish sohasi ildiz ostidagi ifoda noldan katta yoki teng bo'lishi kerak. Bundan tashqari ildiz ostida yana bir ildiz joylashgani uchun, bu ifoda ham noldan katta yoki teng bo'lishi zarur. Shularni inobatga olib quyidagi tengsizlikni tuzamiz.

$$\begin{cases} 9-3\sqrt{x^2-16}\ge 0 \\ x^2-16\ge 0 \end{cases}, \quad \begin{cases} 3\sqrt{x^2-16}\le 9 \\ (x-4)(x+4)\ge 0 \end{cases}, \quad \begin{cases} x^2-16\le 9 \\ (x-4)(x+4)\ge 0 \end{cases}$$

$$\begin{cases} (x-5)(x+5)\le 0 \\ (x-4)(x+4)\ge 0 \end{cases}$$

endi oxirgi natijani oraliqlarda ifodalaydigan bo'lsak u holda,





Yuqoridagilardan funksiyaning aniqlanish sohasi $[-5;-4]\cup[4;5]$ ekanligi kelib chiqadi. Endi murakkab funksiya tuzish masalasi bilan shug'ullanamiz.

5-misol. $f(x)=x^3+1,\ g(x)=x^2+2$ funksiyalar berilgan bo'lsa, u holda $f(g(x))$ va $g(f(x))$ murakkab funksiyalarni tuzing.

Yechish: $f(g(x))$ ni topish uchun birinchi funksiyaning o'zgaruvchilari o'rniga ikkinchi funksiyani olib borib qo'yamiz.

$$f(g(x))=(x^2+2)^3+1=x^6+6x^4+12x^2+8+1=x^6+6x^4+12x^2+9$$

$$g(f(x))=(x^3+1)^2+2=x^6+2x^3+1+2=x^6+2x^3+3$$

Мустақил ечиш учун мисоллар.

1-misol: Quyida berilgan funksiyalarni juft yoki toqlikka ikki hil usulda tekshiring.

1.1. $y=\dfrac{6x^3}{(x+3)^2}$

1.2. $y=\dfrac{x(x-2)(x+2)}{x^2+4x-8}$

1.3. $y=\dfrac{3x^2}{x^4+4x^2}$

1.4. $y=7x^6+5x^2+2|x|+7$

1.5. $y=\dfrac{x^4-2x^2}{x}$

1.6. $y=\dfrac{(x-8)^2+5}{4x}$

1.7. $y=|x+13|+x^2$

1.8. $y=\dfrac{x^3+x-\sin x}{x^5}$

1.9. $y=2x|x|+x^3-4x$

1.10. $y=\dfrac{2x}{x^2-7}$

1.11. $y=\sqrt{x^2-2x+2}+\sqrt{x^2+2x+2}$

1.12. $y=\log_5 4x^2+x^4$

1.13. $y=x+4\log_2 3^x$

1.14. $y=\arccos x$

1.15. $y=\dfrac{3^x-3^{-x}}{4}$

1.16. $y=\dfrac{2^x+5^x}{2^x-5^x}$

1.17. $y=x^2+\cos x$

1.18. $y=x^2+\cos x$

1.19. $y=\sin 2x+tg4x$

1.20. $y=\sqrt[3]{(1-x)^2}+\sqrt[3]{(1+x)^2}$

1.21. $y=\ln\dfrac{1-x}{1+x}$

1.22. $y=\ln(x+\sqrt{1+x^2})$

1.23. $y=2^x+2^{-x}$

2-misol: Berilgan funksiyalarning eng kichik davrini toping.

2.1. $y=\sin 5x$

2.2. $y=\lg\cos 2x$

2.3. $y=tg3x+\cos 4x$

2.4. $y=tgx+3\sin\dfrac{x}{2}-3\cos\dfrac{x}{3}$

2.5. $y=5x^2+2\sin x-7\cos kx,\quad k\in Z$

2.6. $y = 4ctg\frac{x}{2} + 3tg\frac{x}{3}$

2.7. $y = \cos(8x-7)$, $y = \sin(4x+13)$

2.8. $y = ctg(8x+7)$, $y = tg(4x-7)$

2.9. $y = tg\frac{5\pi}{6}x$, $y = ctg\frac{5\pi}{3}x$

2.10. $y = tg3x$, $y = \sin(6x+5)$

2.11. $ctg6x$, $y = \cos(3x-1)$

2.12. $y = 2\sin\frac{\pi x}{3} + 3\cos\frac{\pi x}{4} + tg\frac{\pi x}{2}$

2.13.

$$y = (2+\sin\frac{x}{2})\cdot(1+\cos\frac{x}{6})\cdot tg(\frac{x}{3}-7)$$

2.14. $y = x^4 \sin 3x$ funksiyaning davriyligini aniqlang.

2.15. $y = x^4 - x^2 + x$ funksiyaning davriyligini aniqlang.

2.16. $y = \lg\cos x$ funksiyaning davriyligini aniqlang.

3-misol: Quyidagi 3.1-3.7 misollarda a) berilgan funksiyani berilgan oraliqlarda o'suvchi yoki kamayuvchiligini aniqlang, b) berilgan funksiyaga teskari funksiyani toping.

3.1. a) $y = x^2 + x$ $[1,2]$ б) $y = \frac{1}{x-2} + 1$.

3.2. a) $y = x^2$ $[0,2]$ б) $y = x^2 + 4x - 7$.

3.3. a) $y = -x^3$ $[1,2]$ б) $y = \frac{2x-1}{4-3x}$.

3.4. a) $y = \frac{1}{x+1}$ $[0,2]$ б) $y = 3x^2 + 4$.

3.5. a) $y = \frac{1}{x}$ $[1,2]$ б) $y = 3x^2 - 9$.

3.6. a) $y = \sin 2x$ $[0,1]$ б) $y = x^2 - 4x + 1$.

3.7. a) $y = \sin x$ $[0,1]$ б) $y = 2^{4x+5}$

3.8. a) $y = x^2 + x - 1$ funksiyaning o'sish oraliqlarini toping.

б) $y = 3^x + 7$ funksiyaga teskari funksiyani toping.

3.9. a) $y = x^2 + 2x - 4$ funksiyaning kamayuvchi oraliqlarini toping.

б) $y = 15\cdot 5^{6x-7} - 13$ funksiyaga teskari funksiyani toping.

3.10. a) $y = x^2 + x - 1$ funksiyaning kamayish oraliqlarini toping

б) $y = 3\cdot 4^{x+2} + 1$ funksiyaga teskari funksiyani toping.

3.11. a) $y = x^3 + 3$ funksiyaning grafigi yordamida o'sish oraliqlarini toping

б) $y = 4\cdot 8^{3x-5} + 11$ funksiyaga teskari funksiyani toping.

3.12. a) $y = -2x^3 + 3$ funksiyaning grafigi yordamida kamayish oraliqlarini toping

б) $y = 2\cdot 3^{x-12} - 25$ funksiyaga teskari funksiyani toping.

3.13. a) $y = 4x^3 + 3$ funksiyaning grafigi yordamida o'sish oraliqlarini toping

б) $y = \log_2 x + 4$ funksiyaga teskari funksiyani toping.

3.14. a) $y = ctgx + 3$ funksiyaning grafigi yordamida o'sish oraliqlarini toping

б) $y = \log_3(2x+3) - 4$ funksiyaga teskari funksiyani toping.

3.15. a) $y = x^4 - 1$ funksiyaning grafigi yordamida o'sish oraliqlarini toping

б) $y = \log_3(6x+12) - 15$ funksiyaga teskari funksiyani toping.

3.16. a) $y = x^2 + 3$ funksiyaning grafigi yordamida o'sish oraliqlarini toping

б) $y = \log_5 7x$ funksiyaga teskari funksiyani toping.

3.17. a) $y = x^2 + 3x - 7$ funksiyaning grafigi yordamida o'sish oraliqlarini toping

б) $y = \lg 10x + 2$ funksiyaga teskari funksiyani toping.

3.18. a) $y = x^3 + 3$ funksiyaning grafigi yordamida o'sish oraliqlarini toping

б) $y = \lg 100x^2 - 4$ funksiyaga teskari funksiyani toping.

3.19. a) $y = x + 3$ funksiyaning grafigi yordamida o'sish oraliqlarini toping

б) $y = \ln x^3 + 4$ funksiyaga teskari funksiyani toping.

3.20. a) $y = -x + 5$ funksiyaning grafigi yordamida o'sish oraliqlarini toping

б) $y = \ln 10x^2 + 11$ funksiyaga teskari funksiyani toping.

4-misol: Quyida berilgan 4.1-4.19 misollarda berilgan funksiyaning aniqlanish sohasini toping.

4.1. $y=\dfrac{1}{\sqrt{x^2-3x+2}}$

4.2. $y=\arcsin\dfrac{x-2}{2}$

4.3. $y=\dfrac{1}{\lg(4-x^2)}$

4.4. $y=\sqrt{25-x^2}+\lg\sin x$

4.5. $y=\sqrt{16-x^2}$

4.6. $y=3\cos x-1$

4.7. $y=3^{-x^2}$

4.8. $y=\sqrt{\dfrac{x(x+1)}{(x-1)(x-4)}}$

4.9. $y=\sqrt{\dfrac{2x^2-x-6}{x-3}}$

4.10. $y=\sqrt{8-2\sqrt{x^2+15}}$

4.11. $y=\dfrac{\sqrt{x-2}-\sqrt{x+1}}{\sqrt{x-3}-\sqrt{x+5}}$

4.12. $y=\dfrac{\sqrt{x+3}-\sqrt[4]{x+2}}{\sqrt{x-1}-\sqrt[4]{3-x}}$

4.13. $y=\ln\lg x$

4.14. $y=\lg\ln(3x-7)$

4.15. $y=\log_2\log_3\log_{\frac{1}{2}}(x^2-2x-3)$

4.16. $y=\log_{2x+4}(x^2-4)$

4.17. $y=\log_x(4x+5)$

4.18. $y=\dfrac{\lg(x^2+4x-5)}{\sqrt{x+3}}$

4.19. $y=\dfrac{\log_{x^2-4}(x+3)}{\sqrt{x^2-3x+2}}$

4.20. $f(x)=\dfrac{x^2+2}{x^2}$, $g(x)=\dfrac{1}{x^2}$ berilgan bo'lsa u holda $f(g(x))$, $g(f(x))$ va $f(g(1))$, $g(f(2))$ larni toping.

4.21. $f(\dfrac{3x-2}{2})=x^2-x-1$ ekanligi ma'lum bo'lsa, $f(x)$ va $f(2)$ ni toping.

4.22. $f(x+1)=3-2x$ va $f(g(x))=6x-3$ bo'lsa $g(x)$ ni va $g(5)$ ni toping.

4.23. $f(x+2)=x^3+6x^2+12x+3$ ekanligi ma'lum bo'lsa $f(x)$ funksiyani va $f(5)$ larni hisoblang.

4.24. $f(x)=x^4-2x^2+1$ bo'lsa $f(a+1)-f(1-a)$ nimaga teng?

4.25. $f(x)=(2x+3)(\dfrac{3}{x}-3)$ bo'lsa $f(x-1)$ ni toping.

4.26. $f(\dfrac{3x+2}{2})=x^2-x-1$ ekanligi ma'lum bo'lsa $f(3x-4)$ ni va $f(4)$ larni hisoblang.

4.27. $f(x)=\begin{cases}x^2-1, & x<0\\ 2x-1, & x\geq 0\end{cases}$ ekanligi ma'lum bo'lsa $f(f(-3))$ ni toping.

4.28. $f(x)=\dfrac{1}{x}-x$ funksiya berilgan bo'lsa $f(\dfrac{1}{a})+f(-\dfrac{1}{a})+a^2$ ni hisoblang.

4.29. $f(x)=2x^2-3x+7$ funksiya berilgan bo'lsa u holda $f(a)-f(a-1)+f(a+1)$ ni hisoblang.

4.30. $f(x+1)=x^2-2x-3$ bo'lsa $f(x-1)$ ni toping.

4.31. $f(x)=\begin{cases}|x+1|, & x>-2\\ 3-4|x|, & x\leq -2\end{cases}$ funksiya berilgan bo'lsa $f(5)+f(-\dfrac{9}{2})$ ni hisoblang.

4.32. $f(x)=6x-5$, $g(x)=10-2x$, $q(x)=3x+4$ funksiyalar berilgan bo'lsa, $f(g(q(x)))$ ni toping.

4.33. $f(x)=\dfrac{1}{x^2-\frac{2}{3}x+\frac{1}{9}}$ funksiya berilgan bo'lsa, $f(\dfrac{1}{3}-a)-f(\dfrac{1}{3}+a)+12$ ni hisoblang.

Javoblar:

1.1 juft ham emas toq ham emas; **1.3 juft;**

1.2 juft ham emas toq ham emas; **1.4 juft;**

1.5 toq;

1.6 juft ham emas toq ham emas;

1.7 juft ham emas toq ham emas;

1.8 juft;

1.9 toq;

1.10 toq;

1.11 juft;

1.12 juft;

1.13toq;

1.14 juft ham emas toq ham emas;

1.15 toq;

1.16 toq;

1.17 juft;

1.18 toq;

1.19 toq;

1.20 juft;

1.21toq;

1.22 toq;

1.23 juft:

2.1 $\frac{2\pi}{5}$;

2.2 π;

2.3 π;

2.4 12π;

2.5 2π;

2.6 6π;

2.7 $\frac{\pi}{2}$;

2.8 $\frac{\pi}{4}$;

2.9 $\frac{6}{5}$;

2.10 $\frac{\pi}{3}$;

2.11 $\frac{2\pi}{3}$;

2.12 24;

2.13 12π;

2.14 davriy emas;

2.15 davriy emas;

2.16 davriy.

3.1 a) o'suvchi, **б)** $y=\frac{1}{x-1}+2$;

3.2 a) o'suvchi, **б)** $y=\pm\sqrt{x+11}-2$;

3.3 a) kamayuvchi, **б)** $y=\frac{4x+1}{3x+2}$;

3.4 a) kamayuvchi, **б)** $y=\pm\sqrt{\frac{x-4}{3}}$;

3.5 a) kamayuvchi, **б)** $y=\pm\sqrt{\frac{x+9}{3}}$;

3.6 a) $[0;\frac{\pi}{6}]$ oraliqda o'suvchi,

$[\frac{\pi}{6};1]$ oraliqda kamayuvchi,

б) $y=\pm\sqrt{x+3}+2$;

3.7 a) o'suvchi, **б)** $y=\frac{\log_2 x-5}{4}$;

3.8 a) $[-\frac{1}{2};\infty)$ oraliqda o'suvchi, **б)** $y=\log_3(x-7)$;

3.9 a) $(-\infty;-1]$,

б) $y=\frac{1}{6}(\log_5\frac{x+13}{15}+7)$;

3.10 a) $(-\infty;-\frac{1}{2}]$,

б) $y=\log_4\frac{x-1}{3}-2$;

3.11 a) $(-\infty;\infty)$,

б) $y=\frac{1}{3}(\log_8\frac{x-11}{4}+5)$;

3.12 a) $(-\infty;\infty)$,

б) $y=\log_3\frac{x+25}{2}+12$;

3.13. a) $(-\infty;\infty)$ **б)** $y=2^{x-4}$;

3.14 a) $\varnothing$ **б)** $y=\frac{1}{2}(3^{x+4}-3)$;

3.15 a) $[0;\infty)$ **б)** $y=\frac{1}{6}(3^{x+15}-12)$;

3.16 a) $[0;\infty)$ **б)** $y=\frac{1}{2}\cdot 5^x$;

3.17 a) $[-\frac{3}{2};\infty)$ **б)** $y=10^{x-3}$;

3.18 a) $(-\infty;\infty)$ **б)** $y=10^{\frac{x+2}{2}}$;

3.19 a) $(-\infty;\infty)$**б)** $y=e^{\frac{x-4}{3}}$;

3.20 a) $\varnothing$ **б)** $y=\sqrt{\frac{e^{x-11}}{10}}$.

4.1 $(-\infty;1)$ $(2;\infty)$;

4.2 $[0;4]$;

4.3 $(-2;-\sqrt{3})$ $(-\sqrt{3};\sqrt{3})$ $(\sqrt{3};2)$;

4.4 $(-5;-\pi)$ $(0;\pi)$;

4.5 $[-4;4]$;

4.6 $(-\infty;\infty)$;

4.7 $(-\infty;\infty)$;

4.8 $(-\infty;-1]$ $[0;1)$ $(4;\infty)$;

4.9 $[-1,5;2]$ $(3;\infty)$;

4.10 $[-1;1]$;

4.11 $[3;\infty)$;

4.12 $[1;2)$ $(2;3]$;

4.13 $(1;\infty)$;

4.14 $(\frac{8}{3};\infty)$;

4.15 $(\frac{2-3\sqrt{2}}{2};-1)$ $(3;\frac{2+3\sqrt{2}}{2})$;

4.16 $(2;\infty)$;

4.17 $(0;1)$ $(1;\infty)$;

4.18 $(1;\infty)$;

4.19
$(-3;-\sqrt{5})$ $(-\sqrt{5};2)$ $(2;\sqrt{5})$
$(\sqrt{5};\infty)$;

4.20 $f(g(x))=1+2x^4$, $g(f(x))=\frac{x^4}{x^4+4x^2+4}$, $f(g(1))=3$, $g(f(2))=\frac{4}{9}$;

4.21 $f(x)=\frac{4x^2+2x-11}{9}$, $f(2)=1$;

4.22 $g(x)=4-3x$, $g(5)=-11$;

4.23 $f(x)=x^3-5$, $f(5)=120$;

4.24 $f(a+1)-f(1-a)=8a^3$;

4.25 $f(x-1)=\frac{(2x+1)(6-3x)}{x-1}$;

4.26 $f(3x-4)=\frac{36x^2-138x+121}{9}$, $f(4)=1$;

4.27 $f(f(-3))=17$;

4.28 $f(\frac{1}{a})+f(-\frac{1}{a})+a^2=\frac{1}{a^2}-a^2$;

4.29
$f(a)-f(a-1)+f(a+1)=2a^2-a+21$;

4.30 $f(x-1)=x^2-6x+5$;

4.31 -9;

4.32 $f(g(q(x)))=7-36x$.

Mavzu:Funksiya limiti

Reja:

1) Funksiyaning nuqtadagi limitining Geyne va Koshi ta'riflari.
2) Limitga ega bo'lgan funksiyalarning sodda xossalari.
3) Bir tomonli limitlar. Bir tomonli limitlar asosida funksiyaning chekli limitga ega bo'lish sharti.

To'plamning limit nuqtasi.

Bizga biror $X \subset R$ to'plam va $x_0 \in R$ nuqta berilgan bo'lsin.

Ta'rif. Agar $x_0 \in R$ nuqtaning ixtiyoriy $U_\varepsilon(x_0)=(x_0-\varepsilon, x_0+\varepsilon), (\forall\varepsilon>0)$ atrofida X to'plamning x_0 nuqtadan farqli kamida bitta nuqtasi, ya'ni $\forall\varepsilon>0, \exists x\in X, x\neq x_0: |x-x_0|<\varepsilon$ tengsizlik bajarilsa x_0 nuqta X to'plamning limit nuqtasi deyiladi.

Ta'rif. Agar $x_0 \in R$ nuqtaning ixtiyoriy $U_\varepsilon^+(x_0)=(x_0, x_0+\varepsilon)$ $(U_\varepsilon^-(x_0)=(x_0-\varepsilon, x_0)), (\forall\varepsilon>0)$ o'ng atrofida (chap atrofida) X to'plamning x_0 nuqtadan farqli kamida bitta nuqtasi bo'lsa x_0 nuqta X to'plamning o'ng (chap) limit nuqtasi deyiladi.

Ta'rif. Agar ixtiyoriy $c \in R$ uchun $U_c(+\infty)=\{x\in R | x>c\}$ to'plamda X to'plamning kamida bitta nuqtasi bo'lsa, "$+\infty$" X to'plamning limit nuqtasi deyiladi.

Agar ixtiyoriy $c \in R$ uchun $U_c(-\infty)=\{x\in R | x<c\}$ to'plamda X to'plamning kamida bitta nuqtasi bo'lsa, "$-\infty$" X to'plamning limit nuqtasi deyiladi.

Misollar:

1) $X=[0,1]$ to'plamning har bir nuqtasi to'plamning limit nuqtasi bo'ladi.

2) $X=(0,1)$ to'plamning ham har bir nuqtasi limit nuqtasi bo'ladi. Hatto 0, 1 ham to'plam uchun limit nuqta bo'ladi. Sababi 0 va 1 ning ixtiyoriy

atrofini olsak ham, ushbu atrofda 0 va 1 dan farqli kamida bitta to'plamga tegishli nuqta topiladi.

3) $N = \{1,2,3,......,n,...\}$ natural sonlar to'plami limit nuqtaga ega emas. Sababi to'plamning har bir nuqtasi yakkalangan nuqta bo'lib, bu nuqtalarning ixtiyoriy atrofi mavjud emas.

Teorema: agar $x_0 \in R$ nuqta $X \subset R$ to'plamning limit nuqtasi bo'lsa, u holda x_0 nuqtaning har qanday $U_\varepsilon(x_0) = (x_0 - \varepsilon, x_0 + \varepsilon), (\forall\varepsilon > 0)$ atrofida X to'plamning cheksiz ko'p nuqtalari bo'ladi.

Teorema: Agar $x_0 \in R$ nuqta $X \subset R$ to'plamning limit nuqtasi bo'lsa, u holda shunday sonlar ketma-ketligi topiladiki $\{x_n\}$,

1) $\forall n \in N,\ x_n \in X,\ x_n \neq x_0$
2) $\lim\limits_{n\to\infty} x_n = x_0$ bo'ladi.

Funksiya limiti ta'riflari

Faraz qilaylik $f(x)$ funksiya $X \subset R$ to'plamda berilgan bo'lib, $a \in R$ nuqta X to'plamning limit nuqtasi bo'lsin. a nuqtaga intiluvchi ixtiyoriy $\{x_n\}: x_1, x_2, x_3, ..., x_n, ..., (x_n \in X, x_n \neq a)$ ketma-ketlikni olib, ushbu ketma-ketlikning har bir hadiga mos keluvchi funksiyaning qiymatlaridan iborat $f(x_1), f(x_2), f(x_3), ..., f(x_n), ...,$ ketma-ketlikni hosil qilamiz.

Ta'rif (Geyne ta'rifi): Agar $n \to \infty$ da $x_n \to a\ (x_n \in X, x_n \neq a)$ bo'ladigan axtiyoriy $\{x_n\}$ ketma-ketlik uchun $n \to \infty$ da $f(x_n) \to A$ bo'lsa, A ga $f(x)$ funksiyaning a nuqtadagi limiti deyiladi va $x \to a$ da $f(x) \to A$ yoki $\lim\limits_{x\to a} f(x) = A$ kabi belgilanadi.

Ushbu ta'rifning ma'nosi shundaki a nuqtaga intiluvchi har kanday sonlar ketma-ketligini olganimizda ham, ushbu sonlar ketma-ketligining har bir qiymatiga mos keluvchi funksiyaning qiymatlaridan tashkil topgan $f(x_n)$ sonlar ketma-ketligi yagona A soniga intilsa, u holda A ga $f(x)$ funksiyaning a nuqtadagi limiti deyiladi.

Eslatma: Agar bir hil songa intiluvchi bir nechta ketma-ketlik olganimizda ham ketma-ketliklarga mos bo'lgan funksiyaning qiymatlaridan

tashkil topgan ketma-ketliklar turli songa intilsa u holda berilgan funksiyaning berilgan nuqtadagi limiti mavjud emas deyiladi. Ya'ni $n \to \infty$ da $x_n \to a_1\ (x_n \in X, x_n \neq a_1)$ ва $y_n \to a_2\ (y_n \in X, y_n \neq a_2)$ ketma-ketliklar uchun, $n \to \infty$ da $f(x_n) \to A_1$, $f(y_n) \to A_2$ bo'lib $A_1 \neq A_2$ bo'lsa, $f(x)$ funksiya $x_n \to x_0$ da limitga ega emas deyiladi.

Misol: $\lim\limits_{x\to 2}\dfrac{x^2+x-6}{x^2-4}$ limitni hisoblang.

Yechish: eng avvalo berilgan limit ostidagi ifodani soddalashtiramiz. $\lim\limits_{x\to 2}\dfrac{x^2+x-6}{x^2-4} = \lim\limits_{x\to 2}\dfrac{(x-2)(x+3)}{(x-2)(x+2)} = \lim\limits_{x\to 2}\dfrac{x+3}{x+2}$ bo'lib, $a = 2$ bo'lganligi uchun $x_n \to 2$ ketma-ketlikni qaraymiz va $\lim\limits_{x\to 2}\dfrac{x+3}{x+2} = \lim\limits_{n\to\infty}\dfrac{x_n+3}{x_n+2} = \dfrac{5}{4}$ ekanligi kelib chiqadi.

Ta'rif (Koshi ta'rifi). Ihtiyoriy $\forall\varepsilon > 0$ son uchun shunday $\delta(\varepsilon)$ son topilib, argument x ning $|x-a| < \delta(\varepsilon)$ tengsizlikni qanoatlantiruvchi a dan farqli barcha qiymatlarida $f(x)$ funksiya $|f(x) - A| < \varepsilon$ tengsizlikni qanoatlantirsa, u holda x argument a ga intilganda $f(x)$ *funksiya A ga teng limitga ega deyiladi.*

Koshi va Geyini ta'riflarining ma'nosi bir hil bo'lib, ya'ni ushbu ikkita ta'rif o'zaro ekvivalent ta'riflar hisoblanadi va bu ikki ta'rifning ekvivalent ta'riflar ekanligining isbotini talabalarga mustaqil ish sifatida qoldiramiz.

Misollar:

1. $\lim\limits_{x\to 3}(x^3 + x - 5) = 25$ ekanligini limit ta'rifi yordamida isbotlang.

$\forall\varepsilon > 0$ son uchun $\exists\delta(\varepsilon)$ son topilib, $|x-3| < \delta$ tengsizlik bajarilganda, $|x^3 + x - 30| < \varepsilon$ tengsizlik bajarilishi kerak.

$$|x^3 - 27 + x - 3| = |(x-3)(x^2+3x+9) + (x-3)| = |(x-3)(x^2+3x+10)| =$$
$$|(x-3)(x^2 - 6x + 9 + 9x + 1)| = |(x-3)((x-3)^2 + 9(x-3) + 28)| < |\delta^3 + 9\delta^2 + 28\delta| <$$
$$|\delta^3 + 9\delta^2 + 27\delta + 27| = |(\delta+3)^3| = \varepsilon \Rightarrow \delta + 3 = \sqrt[3]{\varepsilon} \Rightarrow \delta = \sqrt[3]{\varepsilon} - 3$$

bu esa tenglik to'g'ri ekanligini isbotlaydi. Chunki funksiya limiti ta'rifida $\forall\varepsilon > 0$ son uchun $\exists\delta(\varepsilon)$ topilib ya'ni δ son ε soniga bog'liq bo'lishi lozim.

Ushbu misolda ham δ son ε songa bog'liq ravishta kelib chiqadi. Demak, haqiqatda ham yuqoridagi limit o'rinli.

Quyida yana ikkita ta'rifni ko'rib chiqamiz, ya'ni $x\to\infty$ dagi limit va $x\to a$ dagi $f(x)\to\infty$ intiluvchi limitlarning ta'riflarini keltiramiz.

Ta'rif: Agar ihtiyoriy $\forall\varepsilon>0$ son uchun shunday $\delta(\varepsilon)$ son topilib, argument x ning $|x-a|<\delta(\varepsilon)$ tengsizlikni qanoatlantiruvchi a dan farqli barcha qiymatlarida $f(x)$ funksiya $f(x)>\varepsilon$ tengsizlikni qanoatlantirsa, u holda x argument a ga intilganda $f(x)$ *funksiya ∞ ga teng limitga ega* deyiladi.

Ta'rif: Agar ihtiyoriy $\forall\varepsilon>0$ son uchun shunday $\delta(\varepsilon)$ son topilib, argument $\forall x\in X$ ning $x>\delta$ tengsizlikni qanoatlantiruvchi barcha qiymatlarida $f(x)$ funksiya $|f(x)-A|<\varepsilon$ tengsizlikni qanoatlantirsa, u holda x argument ∞ ga intilganda $f(x)$ *funksiya A ga teng limitga ega deyiladi* va $\lim\limits_{x\to\infty} f(x)=A$ kabi belgilanadi.

Limitga ega bo'lgan funksiyalarning sodda xossalari.

Faraz qilaylik $f(x)$ funksiya $X\subset R$ to'plamda berilgan bo'lib, $a\in R$ nuqta X to'plamning limit nuqtasi bo'lsin.

1-xossa: Agar $x\to a$ da $f(x)$ funksiya limitga ega bo'lsa, u yagona bo'ladi.

2-xossa: Agar $\lim\limits_{x\to a} f(x)=A$ (A-chekli son) bo'lsa, u holda a nuqtaning shunday $U_\delta(a)$ atrofi topiladiki, bu atrofda $f(x)$ funksiya chegaralangan bo'ladi.

3-xossa: Agar $\lim\limits_{x\to a} f(x)=A$ bo'lib, $A<p$ bo'lsa, u holda a nuqtaning shunday $U_\delta(a)$ atrofi topiladiki, bu atrofda $f(x)<p$ bo'ladi.

4-xossa: Agar $\lim\limits_{x\to a} f(x)=A, \lim\limits_{x\to a} g(x)=B$ bo'lib, $\forall x\in X$ da $f(x)\le g(x)$ tengsizlik bajarilsa, u holda $A\le B$ ya'ni $\lim\limits_{x\to a} f(x)\le\lim\limits_{x\to a} g(x)$ bo'ladi.

5-xossa: Faraz qilaylik $\lim\limits_{x\to a} f(x)=A, \lim\limits_{x\to a} g(x)=B$ limitlar mavjud bo'lsin. U holda

а) $\forall c\in R$ да $\lim\limits_{x\to a}(c\cdot f(x))=c\cdot\lim\limits_{x\to a} f(x)$

б) $\lim\limits_{x\to a}(f(x)+g(x))=\lim\limits_{x\to a} f(x)+\lim\limits_{x\to a} g(x)$

в) $\lim\limits_{x\to a}(f(x)\cdot g(x))=\lim\limits_{x\to a} f(x)\cdot\lim\limits_{x\to a} g(x)$

г) агар $B\neq 0$ бўлса, $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$ бўлади.

Xossalardan kelib chiqadigan natijalar:

1. O'zgarmas ko'paytuvchini limit belgisi oldiga chiqarish mumkin.

$$\lim_{x\to a} k\cdot f(x)=k\cdot\lim_{x\to a} f(x)$$

2. Agar n natural son bo'lsa, u holda

$$\lim_{x\to a} x^n=a^n,\ \lim_{x\to a}\sqrt[n]{x}=\sqrt[n]{a}$$

bo'ladi.

3. Ushbu $P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_n$ ko'phadning $x\to a$ dagi limiti bu ko'phadning $x=a$ dagi qiymatiga teng, ya'ni $\lim\limits_{x\to a} P(x)=P(a)$ ga teng.

4. Ushbu $R(x)=\dfrac{P(x)}{Q(x)}=\dfrac{a_0x^n+a_1x^{n-1}+\ldots a_n}{b_0x^m+b_1x^{m-1}+\ldots b_m}$ kasr ratsional funksiyaning $x\to a$ dagi limiti,agar a bu funksiyaning aniqlanish sohasiga tegishli bo'lsa, bu funksiyaning $x=a$ dagi qiymatiga teng, ya'ni $\lim\limits_{x\to a} R(x)=R(a)$ ga teng.

Bir tomonli limitlar. Bir tomonli limitlar asosida funksiyaning chekli limitga ega bo'lish sharti

Ta'rif: Agar $\forall\varepsilon>0, \exists\delta(\varepsilon), \forall x\in(a-\delta,a):|f(x)-b|<\varepsilon$ bo'lsa, b son $f(x)$ funksiyaning a nuqtadagi chap limiti deyiladi va $\lim\limits_{x\to a-0} f(x)=b$ kabi belgilanadi.

Ta'rif: Agar $\forall\varepsilon>0, \exists\delta(\varepsilon), \forall x\in(a,a+\delta):|f(x)-b|<\varepsilon$ bo'lsa, b son $f(x)$ funksiyaning a nuqtadagi o'ng limiti deyiladi va $\lim\limits_{x\to a+0} f(x)=b$ kabi belgilanadi.

Quyida bir qator funksiyaning limitlarini hisoblashga oid misol va masalalarni tahlilini ko‘rib chiqamiz.

Misol: $\lim_{x\to 6}\frac{x-6}{\sqrt{x+3}-3}$ limitni hisoblang.

Yuqoridagi misolda funksiya mahrajini irratsionallikdan qutqaramiz.

$$\lim_{x\to 6}\frac{x-6}{\sqrt{x+3}-3}=\lim_{x\to 6}\frac{(x-6)(\sqrt{x+3}+3)}{(\sqrt{x+3}-3)(\sqrt{x+3}+3)}=\lim_{x\to 6}\frac{(x-6)(\sqrt{x+3}+3)}{x+3-9}=$$

$$\lim_{x\to 6}\frac{(x-6)(\sqrt{x+3}+3)}{x-6}=\lim_{x\to 6}(\sqrt{x+3}+3)=6$$

ekanligi kelib chiqadi.

Keyingi misollarda esa $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty-\infty$, $\infty+\infty$ ko‘rinishidagi aniqmasliklarni ko‘rib chiqamiz.

Misol. $\lim_{x\to\infty}x(\sqrt{4x^2-1}-2x)$ limitni hisoblang.

Yechish: ushbu misol ham $\infty-\infty$ ko‘rinishidagi aniqmaslik hosil bo‘ladi. Bu aniqmaslikni yechish uchun ham irratsionallikdan qutqarish ishini qilimiz, ya’ni ifodani qo‘shmasiga ko‘paytirib bo‘lamiz.

$$\lim_{x\to\infty}x(\sqrt{4x^2-1}-2x)=\lim_{x\to\infty}\frac{x(\sqrt{4x^2-1}-2x)(\sqrt{4x^2-1}+2x)}{(\sqrt{4x^2-1}+2x)}=\lim_{x\to\infty}\frac{x(4x^2-1-4x^2)}{(\sqrt{4x^2-1}+2x)}=$$

$$\lim_{x\to\infty}\frac{-x}{(\sqrt{4x^2-1}+2x)}$$

bundan ohirgi tenglikdan argumentni cheksizlikka intiltirib, limit hisoblanganda $\frac{\infty}{\infty}$ ko‘rinishidagi aniqmaslik hosil bo‘ladi. Bu aniqmaslikdan qutilish uchun quyidagicha amal bajariladi:

$$\lim_{x\to\infty}\frac{-x}{(\sqrt{4x^2-1}+2x)}=-\lim_{x\to\infty}\frac{x}{x(\sqrt{4-\frac{1}{x^2}}+2)}=-\lim_{x\to\infty}\frac{1}{\sqrt{4-\frac{1}{x^2}}+2}=-\frac{1}{4}.$$

Misol. $\lim_{x\to\infty}(\sqrt{x^2-4}-x)$ limitni hisoblang.

Yechish: ushbu limitni hisoblaganda ham $\infty-\infty$ ko‘rinishidagi aniqmaslik hosil bo‘ladi. Bu aniqmaslikni ochish uchun ham yuqoridagi kabi irratsionallikdan qutqarish ishini bajaramiz.

$$\lim_{x\to\infty}\frac{(\sqrt{x^2-4}-x)(\sqrt{x^2-4}+x)}{(\sqrt{x^2-4}+x)}=\lim_{x\to\infty}\frac{x^2-4-x^2}{(\sqrt{x^2-4}+x)}=\lim_{x\to\infty}\frac{-4}{(\sqrt{x^2-4}+x)}$$ ohirgi tenglikdan kasr maxrajiga cheksizlikni qo‘yib hisoblanganda bo‘ladi va limit nolga teng bo‘lar ekan.

Endi esa $\frac{\infty}{\infty}$ ko‘rinishidagi aniqmasliklarni ochishga to‘htalamiz.

Misol. $\lim_{x\to\infty}\frac{3x^5-2x^2}{x^3+3x-x^5}$ limitni hisoblang.

Yechish: ushbu misolda haqiqatdan ham $\frac{\infty}{\infty}$ ko‘rinishdagi aniqmaslik hosil bo‘ladi. Bunday misollarni ishlashda quyidagicha yo‘l tutiladi, ya’ni kasrning surat va maxrajidan argumentning eng katta darajasini chiqaramiz.

$$\lim_{x\to\infty}\frac{3x^5-2x^2}{x^3+3x-x^5}=\lim_{x\to\infty}\frac{x^5(3-\frac{2}{x^3})}{x^5(-1+\frac{1}{x^2}+\frac{3}{x^4})}=\lim_{x\to\infty}\frac{(3-\frac{2}{x^3})}{(-1+\frac{1}{x^2}+\frac{3}{x^4})}=\frac{3}{-1}=-3$$

ga teng bo‘ladi.

1-misol. $\lim_{x\to -2}\frac{\sqrt{2x+9}-\sqrt{x+7}}{x^2+2x+6}$ limitni hisoblang.

Yechish: ushbu limitni hisoblash uchun argument intilayotgan sonni funksiya o‘rniga qo‘yib tekshiramiz.

$$\lim_{x\to -2}\frac{\sqrt{2x+9}-\sqrt{x+7}}{x^2+2x+6}=\frac{\sqrt{2\cdot(-2)+9}-\sqrt{-2+7}}{(-2)^2+2\cdot(-2)+6}=\frac{\sqrt{5}-\sqrt{5}}{6}=\frac{0}{6}=0$$ ekanligi kelib chiqadi va limit nolga teng ekanligi kelib chiqadi.

2-misol. $\lim_{x\to 3}\frac{\sqrt{2x+3}-3}{\sqrt{x-2}-1}$ limitni hisoblang.

Yechish: Ushbu misolni ishlashda ham huddi yuqoridagi kabi argument intilayotgan sonni funksiya o‘rniga qo‘yib tekshiramiz. $\frac{\sqrt{2\cdot 3+3}-3}{\sqrt{3-2}-1}=\frac{\sqrt{6+3}-3}{\sqrt{3-2}-1}=\frac{0}{0}$ ekanligi kelib chiqadi va bu esa aniqmaslikka keladi. Yuqorida aniqmasliklarni ochishning ber nechtasini ko‘rib chiqqan edik. Endi bu aniqmaslikni ochish uchun kasr surati va maxrajlarini qo‘shmasiga ko‘paytirib bo‘lamiz, ya’ni

$$\lim_{x\to3}\frac{\sqrt{2x+3}-3}{\sqrt{x-2}-1}=\lim_{x\to3}\frac{(\sqrt{2x+3}-3)(\sqrt{2x+3}+3)(\sqrt{x-2}+1)}{(\sqrt{x-2}-1)(\sqrt{x-2}+1)(\sqrt{2x+3}+3)}=$$

$$\lim_{x\to3}\frac{(2x+3-9)(\sqrt{x-2}+1)}{(x-2-1)(\sqrt{2x+3}+3)}=\lim_{x\to3}\frac{(2x-6)(\sqrt{x-2}+1)}{(x-3)(\sqrt{2x+3}+3)}=\lim_{x\to3}\frac{2(x-3)(\sqrt{x-2}+1)}{(x-3)(\sqrt{2x+3}+3)}=$$

$\lim\limits_{x\to3}\dfrac{2(\sqrt{x-2}+1)}{\sqrt{2x+3}+3}$ ko'rinishga keladi. Endi argumentning intilayotgan nuqtasini funksiya o'rniga qo'yib hisoblansa, $\lim\limits_{x\to3}\dfrac{2(\sqrt{x-2}+1)}{\sqrt{2x+3}+3}=\dfrac{2\cdot2}{6}=\dfrac{2}{3}$ ga teng bo'ladi.

Demak limit $\dfrac{2}{3}$ ga teng.

3-misol. $\lim\limits_{x\to\infty}\dfrac{x+\sqrt{9x^2+1}}{2x+\sqrt{x^2-1}}$ limitni hisoblang.

Yechish: Ushbu limitni hisoblash uchun limit ostidagi funksiyaning surat va maxrajidan argumentni qavsdan tashqariga chiqaramiz. Sababi bu funksiya $\dfrac{\infty}{\infty}$ aniqmaslikka teng bo'lganligi uchun.

$$\lim_{x\to\infty}\frac{x+\sqrt{9x^2+1}}{2x+\sqrt{x^2-1}}=\lim_{x\to\infty}\frac{x(1+\frac{\sqrt{9x^2+1}}{x})}{x(2+\frac{\sqrt{x^2-1}}{x})}=\lim_{x\to\infty}\frac{1+\sqrt{\frac{9x^2+1}{x^2}}}{2+\sqrt{\frac{x^2-1}{x^2}}}=\lim_{x\to\infty}\frac{1+\sqrt{9+\frac{1}{x^2}}}{2+\sqrt{1-\frac{1}{x^2}}}=$$

$\dfrac{1+3}{2+1}=\dfrac{4}{3}$. Bundan limitning qiymati $\dfrac{4}{3}$ ga teng bo'ladi.

Mustaqil yechish uchun misollar:

1-misol: a soni berilgan $f(x)$ funksiyaning limiti ekanligini Koshi hamda Geyne ta'riflari yordamida isbotlang.

1.1 $\lim\limits_{x\to6}\dfrac{x-3}{x+2},\quad a=\dfrac{3}{8}$

1.2 $\lim\limits_{x\to1}\dfrac{x}{\sqrt{x+3}},\quad a=\dfrac{1}{2}$

1.3 $\lim\limits_{x\to1}\dfrac{x^2+1}{x+1},\quad a=1$

1.5. $\lim\limits_{x\to-1}\dfrac{x^2-1}{x+1},\quad a=-2$

1.4 $\lim\limits_{x\to1}\dfrac{x+1}{x^2+x+1},\quad a=\dfrac{2}{3}$

1.6 $\lim\limits_{x\to-2}\dfrac{x^2-4}{x^2+5x+6},\quad a=-4$

2-misol: Quyidagi limitlarni hisoblang.

2.1 a) $\lim\limits_{x\to+\infty}(\dfrac{x^3}{x^2-3}-x)$; b) $\lim\limits_{x\to\infty}\dfrac{1+x-x^2}{2x^2+3x}$;

2.2 a) $\lim\limits_{x\to3}\dfrac{\sqrt{-3x+11}-\sqrt{x-1}}{x^2-5x+6}$; b) $\lim\limits_{x\to\infty}\dfrac{2x^3-x+3}{x^3-8x+5}$;

2.3 a) $\lim\limits_{x\to+\infty}(\sqrt{x^2+2}-x)$; b) $\lim\limits_{x\to\infty}\dfrac{x+5x^2-x^3}{2x^3-x^2+7x}$;

2.4 a) $\lim\limits_{x\to1}\dfrac{\sqrt{2-x}-1}{\sqrt{5-x}-2}$; b) $\lim\limits_{x\to\infty}\dfrac{1-3x^2}{x^2+7x-2}$;

2.5 a) $\lim\limits_{x\to4}\dfrac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$; b) $\lim\limits_{x\to\infty}\dfrac{x^3+x}{x^4-3x^2+1}$;

2.6 a) $\lim\limits_{x\to-8}\dfrac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}$; b) $\lim\limits_{x\to\infty}\dfrac{x^5-2x}{2x^3+x^2+1}$;

2.7 a) $\lim\limits_{x\to0}\dfrac{\sqrt[4]{x+1}-1}{x(1+\sqrt{x})}$; b) $\lim\limits_{x\to\infty}\dfrac{(x+1)^2(3-4x)^2}{(2x-1)^4}$;

2.8 a) $\lim\limits_{x\to-1}\dfrac{\sqrt[3]{1+2x}+1}{\sqrt{2+x}+x}$; b) $\lim\limits_{x\to\infty}\dfrac{10+x\sqrt{x}}{x^2}$;

2.9 a) $\lim\limits_{x\to1}\dfrac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$; b) $\lim\limits_{x\to\infty}\dfrac{1+10x}{2x+\sqrt[3]{x^2}}$;

2.10 a) $\lim\limits_{x\to 0}\frac{\sqrt[3]{8-x}-2}{x}$; b) $\lim\limits_{x\to\infty}\frac{2x^2-3x-4}{\sqrt{x^4+1}}$;

2.11 a) $\lim\limits_{x\to 4}\frac{x^2-16}{\sqrt[3]{5-x}-\sqrt[3]{x-3}}$; b) $\lim\limits_{x\to\infty}\frac{10+x\sqrt{2}}{x^2}$;

2.12 a) $\lim\limits_{x\to -2}(\frac{1}{x+2}-\frac{12}{x^3+8})$; b) $\lim\limits_{x\to\infty}\frac{1+10x}{2x+\sqrt[3]{x^2}}$;

2.13 a) $\lim\limits_{x\to -7}(\frac{1}{x+7}+\frac{14}{x^2-49})$; b) $\lim\limits_{x\to\infty}\frac{2x^2-3x-4}{\sqrt{x^4+1}}$;

2.14 a) $\lim\limits_{x\to -1}(\frac{1}{x+1}+\frac{2}{x^2-1})$; b) $\lim\limits_{x\to\infty}(\frac{x^3}{x^2+1}-\frac{3x^2}{3x+1})$;

2.15 a) $\lim\limits_{x\to 3}(\frac{1}{x-3}-\frac{27}{x^3-27})$; b) $\lim\limits_{x\to+\infty}\frac{2x-3}{2+|x|}$;

2.16 a) $\lim\limits_{x\to\infty}\frac{x+\sqrt{9x^2+1}}{2x+\sqrt{x^2-1}}$; b) $\lim\limits_{x\to\infty}(\sqrt{x^2+2x+2}-\sqrt{x^2-2x-3})$;

2.17 a) $\lim\limits_{x\to\infty}\frac{2x-3}{2+|x|}$; b) $\lim\limits_{x\to-\infty}\frac{x+\sqrt{9x^2+1}}{2x+\sqrt{x^2-1}}$;

2.18 a) $\lim\limits_{x\to+\infty}x(\sqrt{x^2+1}-x)$; b) $\lim\limits_{x\to+\infty}(\sqrt{x^2+4}-x)$;

3-misol: Quyidagi funksiyalarning ko'rsatilgan nuqtadagi o'ng va chap limitlari topilsin.

3.1. $f(x)=\frac{x-|x|}{2x}$, $a=0$ **3.2.** $f(x)=2^{tgx}$, $a=\frac{\pi}{2}$

3.3. $f(x)=\begin{cases}\frac{\sin x}{x}, & \text{агар } x>0 \text{ бўлса},\\ \cos x, & \text{агар } x\le 0 \text{ бўлса}.\end{cases}$

3.4. $f(x)=\operatorname{sgn} x$, $a=0$

3.5. $f(x)=\operatorname{sgn}(\cos x)$, $a=\frac{\pi}{2}$

3.6. $f(x)=\frac{1}{x+3^{\frac{1}{3-x}}}$, $a=3$

3.7. $f(x)=\frac{1}{x-[x]}$, $a=-1$

3.8. $f(x)=x+[x^2]$, $a=10$

3.9. $f(x)=\frac{\sin x}{|x|}$, $a=0$

3.10. $f(x)=\frac{\sqrt{1-\cos 2x}}{x}$, $a=0$

Жавоблар:

2.1 a) 0 **b)** $-\frac{1}{2}$;

2.2 a) $-\frac{1}{\sqrt{2}}$ **b)** 2

2.3 a) 0 **b)** $-\frac{1}{2}$;

2.4 a) 2 **b)** -3;

2.5 a) $-\frac{1}{3}$ **b)** 0;

2.6 a) $-\frac{2}{3}$ **b)** ∞;

2.7 a) $\frac{1}{4}$ **b)** 1;

2.8 a) $\frac{2}{3}$ **b)** 0;

2.9 a) $\frac{4}{3}$ **b)** 5;

2.10 a) $-\frac{1}{12}$ **b)** 2;

2.11 a) -12 **b)** 0;

2.12 a) $-\frac{1}{2}$ **b)** 5;

2.13 a) $-\frac{1}{4}$ **b)** 2;

2.14 a) $-\frac{1}{2}$ **b)** $\frac{1}{3}$;

2.15 a) $\frac{1}{3}$ **b)** 2;

2.16 a) $\frac{4}{3}$ **b)** 2;

2.17 a) 2 **b)** $\frac{4}{3}$;

2.18 a) $\frac{1}{2}$ **b)** 0

3.1) $\lim\limits_{x\to 0+0} f(x)=+\infty,\ \lim\limits_{x\to 0-0} f(x)=1,$

3.2) $\lim\limits_{x\to \frac{\pi}{2}+0} f(x)=0,\ \lim\limits_{x\to \frac{\pi}{2}-0} f(x)=\infty,$

3.3) $\lim\limits_{x\to 0+0} f(x)=1,\ \lim\limits_{x\to 0-0} f(x)=1,$

3.4) $\lim\limits_{x\to 0+0} f(x)=1,\ \lim\limits_{x\to 0-0} f(x)=-1,$

3.5) $\lim\limits_{x\to \frac{\pi}{2}+0} f(x)=-1,\ \lim\limits_{x\to \frac{\pi}{2}-0} f(x)=1,$

3.6) $\lim\limits_{x\to 3+0} f(x)=\frac{1}{3},\ \lim\limits_{x\to 3-0} f(x)=0,$

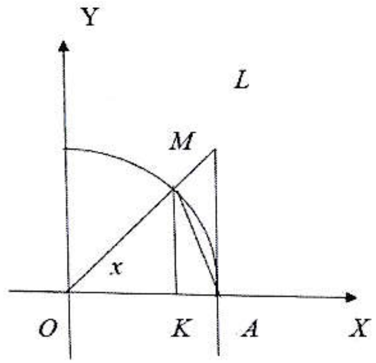
3.7) $\lim\limits_{x\to -1+0} f(x)=+\infty,\ \lim\limits_{x\to -1-0} f(x)=1,$

3.8) $\lim\limits_{x\to 0+0} f(x)=111,\ \lim\limits_{x\to 0-0} f(x)=109$

Mavzu: Ajoyib limitlar haqida qisqacha tushunchalar.

Hozirga qadar $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty-\infty$ ko'rinishidagi aniqmasliklarni ko'rib, ularni ochish o'rganildi. Endilikda esa $\frac{0}{0}$ aniqmaslikni va shu bilan birga 1^{∞} ko'rinishidagi aniqmasliklarni ham bartaraf qilishni o'rganamiz.

Birinchi ajoyib limit $\lim\limits_{x\to\infty}\frac{sinx}{x}=1$ ushbu tenglikni isbotlaymiz. Buning uchun koordinatalar sistemasida markazi O nuqtada bo'lgan va radiusi birga teng bo'lgan birlik aylana chizamiz va markaziy burchagi x ga teng bo'lgan yoyni qaraymiz.



Shakldan quyidagilarga ega bo'lamiz. $S_1=\Delta MOA$, $S_2=MOA$ sektor, $S_3=\Delta LOA$ uchburchak va sektorlarning yuzalari bo'lsa, u holda quyidagi tengsizliklar hosil bo'ladi. $S_1<S_2<S_3$ endi yuzalarni hisoblaymiz $S_1=\frac{1}{2}OA\cdot MK=\frac{1}{2}\cdot 1\cdot sinx=\frac{1}{2}\cdot sinx$, $S_2=\frac{1}{2}\cdot OA\cdot MA=\frac{1}{2}\cdot 1\cdot x=\frac{1}{2}x$, $S_3=\frac{1}{2}OA\cdot LA=\frac{1}{2}\cdot 1\cdot tgx=\frac{1}{2}tgx$, $\frac{1}{2}sinx<\frac{1}{2}<\frac{1}{2}tgx$ ekanligi kelib chiqadi. Tengsizlikni $sinx>0$ ga bo'lamiz. Bundan esa $1<\frac{x}{sinx}<\frac{1}{cosx}$ yoki

$cosx < \frac{sinx}{x} < 1$. Endi $x < 0$ bo'lsin. $\frac{\sin(-x)}{-x} = \frac{sinx}{x}$, $\cos\cos(-x) = cosx$ ekanligidan $x < 0$ da ham $cosx < \frac{sinx}{x} < 1$ tengsizlik o'rinli bo'ladi va tengsizlikning ikki chetki hadlari 1 ga intilganligi uchun o'rtadagi had ham 1 ga intiladi. Bundan esa $\lim_{x\to\infty}\frac{sinx}{x} = 1$ ekanligi kelib chiqadi.

Isbotlangan ikkita ajoyib limitlarga oid misollarni ko'rib chiqamiz.

Ikkinchi ajoyib limit bu $\lim_{x\to0}(1+x)^{\frac{1}{x}} = e$ yoki $\lim_{x\to\infty}(1+\frac{1}{x})^x = e$ limitlar bo'lib, ushbu ajoyib limitning ketma-ketliklardagi talqinini yuqorida isbot qilgan edik, bu ajoyib limitning funksiyalardagi talqinini talabalarga mustaqil vazifa sifatida qoldirib o'tamiz.

1-misol. $\lim_{x\to0}\frac{x - sinx}{x + sinx}$ funksiya limitini hisoblang.

Yechish: funksiyaning birinchi ajoyib limiti $\lim_{x\to\infty}\frac{sinx}{x} = 1$ ko'rinishda bo'lganligi uchun, yuqoridagi misolni ham shu ko'rinishga keltiramiz. Ya'ni kasrning surat va maxrajini x argumentga bo'lamiz va:

$$\lim_{x\to0}\frac{x - sinx}{x + sinx} = \lim_{x\to0}\frac{1 - \frac{sinx}{x}}{1 + \frac{sinx}{x}}$$

bu tenglikdan esa limit hisoblansa quyidagi ko'rinishga keladi: $\lim_{x\to0}\frac{x - sinx}{x + sinx} = \lim_{x\to0}\frac{1 - \frac{sinx}{x}}{1 + \frac{sinx}{x}} = \frac{1-1}{1+1} = \frac{0}{2} = 0$.

2-misol. $\lim_{x\to\pi}\frac{\sin 3x}{\sin 2x}$ funksiya limitini hisoblang.

Yechish: Bu funksiyaning limitini topish uchun argumentni nolga intiltirib ajoyib limitga keltirib hisoblaymiz. Buning uchun $x - \pi = t$ belgilash kiritamiz. Bunda $x \to \pi$, $t \to 0$ bo'ladi va argument $x = t + \pi$ endi topilgan ifodalarni yuqoridagi tenglikka olib borib qo'yib quyidagiga ega bo'lamiz.

$\lim_{x\to\pi}\frac{\sin 3x}{\sin 2x} = \lim_{t\to0}\frac{\sin(3\pi + 3t)}{\sin(2\pi + 2t)}$ bu tenglikda keltirish formulasi yordamida quyidagi tenglikka kelamiz.

$$\lim_{t\to0}\frac{-\sin 3t}{\sin 2t} = -\lim_{t\to0}\frac{\frac{\sin 3t}{3t}\cdot 3t}{\frac{\sin 2t}{2t}\cdot 2t} = -\frac{3}{2}.$$

3-misol. $\lim_{x\to0}(\cos 2x)^{1+ctg^2x}$ funksiya limitini hisoblang.

Yechish: funksiya limitini hisoblashdan oldin trigonometrik funksiyalarning ba'zi xossalaridan foydalanib soddalashtiramiz. U holda ifoda quyidagi ko'rinishga keladi.

4-misol. $\lim_{x\to0}(cos2x)^{1+ctg^2x}$ limitni ajoyib limitlar formulasidan foydalanib yeching.

Yechish:

$$\lim_{x\to0}(\cos 2x)^{1+ctg^2x} = \lim_{x\to0}(1 - 2\sin^2 x)^{\frac{1}{\sin^2 x}} = \lim_{x\to0}(1 + (-2\sin^2 x))^{\frac{1}{-2\sin^2 x}(-2)} = e^{-2}.$$

5-misol. $\lim_{x\to\infty}(1+\frac{k}{x})^x$, $k \in R$ limitni hisoblang.

Yechish: ushbu limitni hisoblash uchun ikkinchi ajoyib limit formulasidan foydalanamiz, ya'ni qavs ichidagi ifodaning o'zgaruvchisi bilan ko'rsatkichdagi o'zgaruvchining birhil lekin teskari bo'lishini ta'minlaymiz.

$$\lim_{x\to\infty}(1+\frac{k}{x})^x = \lim_{x\to\infty}(1+\frac{k}{x})^{\frac{x}{k}k} = \lim_{x\to\infty}[(1+\frac{k}{x})^{\frac{x}{k}}]^k = e^k.$$

6-misol. $\lim_{x\to0}(1-3x)^{\frac{2}{x}}$ limitni hisoblang.

Yechish: Bunday limitni hisoblash uchun kavs ichidagi ifodani yig'indi ko'rinishiga keltirish va yuqoridagi misol kabi ko'rsatkichini mutanosib qilish kerak. $\lim_{x\to0}(1-3x)^{\frac{2}{x}} = \lim_{x\to0}(1+(-3x))^{\frac{1}{-3x}(-2)} = e^{-2} = \frac{1}{e^2}$.

7-misol. $\lim_{x\to\infty}(\frac{5-x}{6-x})^{x+2}$ limitni hisoblang.

Yechish: bu limitni hisoblashda ham ajoyib limitdan foydalanamiz.

$$\lim_{x\to\infty}\left(\frac{5-x}{6-x}\right)^{x+2}=\lim_{x\to\infty}\left(\frac{x-5}{x-6}\right)^{x+2}=\lim_{x\to\infty}\left(\frac{x-6+1}{x-6}\right)^{x+2}=\lim_{x\to\infty}\left(1+\frac{1}{x-6}\right)^{x+2}=$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x-6}\right)^{x-6+8}==\lim_{x\to\infty}\left(1+\frac{1}{x-6}\right)^{x-6}\cdot\left(1+\frac{1}{x-6}\right)^{8}=\lim_{x\to\infty}\left(1+\frac{1}{x-6}\right)^{x-6}\cdot$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x-6}\right)^{8}=e\cdot 1=e.$$

Mustaqil yechish uchun misollar:

1-misol: Ajoyib limitlarning birinchi formulasidan foydalanib hisoblang

1.1 $\lim_{x\to 0}(1+kx)^{\frac{1}{x}},\ k\in R.$

1.2 $\lim_{x\to 0}\sqrt[x]{1+5x}$

1.3 $\lim_{x\to 0}\sqrt[2x]{1+3x}$

1.4 $\lim_{x\to\infty}\left(\frac{x}{x+2}\right)^{x}$

1.5 $\lim_{x\to\infty}\left(\frac{x+5}{x-3}\right)^{x+2}$

1.6 $\lim_{x\to\infty}\left(\frac{3x+2}{3x-1}\right)^{2x+3}$

1.7 $\lim_{x\to\infty}\left(\frac{2x+1}{2x-2}\right)^{3x-1}$

1.8 $\lim_{x\to\infty}\left(\frac{x+2}{x+3}\right)^{x+4}$

1.9 $\lim_{x\to\infty}\left(\frac{-x+6}{-x+4}\right)^{3x-1}$

1.10 $\lim_{x\to\infty}\left(\frac{4x+10}{4x+5}\right)^{4x-2}$

1.11 $\lim_{x\to\infty}\left(\frac{5x-3}{5x-2}\right)^{-2x-1}$

1.12 $\lim_{x\to\infty}\left(\frac{4x-2}{4x-3}\right)^{3x-1}$

1.13 $\lim_{x\to 0}(1-\sin x)^{\frac{1}{\sin x}}$

1.14 $\lim_{x\to 0}(\cos 2x)^{\frac{1}{\sin^2 x}}$

1.15 $\lim_{x\to 0}(1+tgx)^{ctgx}$

1.16 $\lim_{x\to 0}(1+\sin 2x)^{\frac{1}{arctg3x}}$

Javoblar:

1.1) e^{k},

1.2) e^{5},

1.3) $e^{\frac{3}{2}}$,

1.4) $\frac{1}{e^{2}}$,

1.5) e^{8},

1.6) e^{2},

1.7) $e^{\frac{9}{2}}$,

1.8) $\frac{1}{e}$,

1.9) e^{-6},

1.10) e,

1.11) $e^{\frac{2}{5}}$,

1.12) $e^{\frac{3}{4}}$,

1.13) $\frac{1}{e}$,

1.14) $\frac{1}{e^{2}}$,

1.15) e,

1.16) $e^{\frac{2}{3}}$

III BOB. FUNKSIYA HOSILASI VA UNING TADBIQLARI.

Mavzu: Bir o'zgaruvchili funksiyaning hosilasi

Reja

1. Hosila tushunchasiga olib keladigan masalalar.
2. Hosilaning ta'rifi, uning geometrik va mexanik ma'nolari.
3. Egri chiziq urinmasi va normalining tenglamalari.
4. Hosila tushunchasiga olib keladigan masalalar bilan tanishib chiqamiz.

Bunday masalalarga moddiy nuqtaning bosib o'tgan yo'lining vaqtga bog'liqligini ifodalovchi funksiya ma'lum bo'lsa, u holda, ushbu moddiy nuqtaning ma'lum vaqtdagi oniy tezligini topish yoki biror funksiyaning grafigi egri chiziqdan iborat bo'lib, shu egri chiziqqa argumentning biror qiymatida o'tkazilgan urinmasining burchak koeffitsiyentini topishlarni keltirish mumkin.

Masalan, metro stansiyasida tormoz belgisidan birinchi vagonning to'xtashigacha bo'lgan masofa 80m ga teng. Agar metro poyezdi to'htash belgisidan keyin 1.6 $\frac{M}{c^2}$ tekis sekinlanuvchan tezlanish bilan harakat qilsa, u holda metro poyezdi bu belgiga qanday tezlik bilan kelishi kerak?

Masalani yechish uchun poyezdning to'htash belgisidan o'tish momentidagi tezligini, ya'ni shu vaqt momentidagi oniy tezligini topish kerak. Tormoz yo'li $S=\frac{at^2}{2}$ formula bilan hisoblanadi, bunda a -tezlanish, t - tormozlanish vaqti. Mazkur holda $s=80m$, $a=1.6$ $\frac{M}{c^2}$ shuning uchun $80=0,8t^2$, bundan $t=10c$. $v=at$ formuladan oniy tezlikni topamiz: $v=1,6\cdot 10=16$ ya'ni $v=16$ $\frac{\text{m}}{c}$.

Nuqta to'g'ri chiziq bo'ylab harakat qilayotgan va harakat boshlangandan t vaqt o'tganda $s(t)$ yo'lni bosib o'tgan bo'lsin, ya'ni $s(t)$ funksiya berilgan bo'lsin.

Biror t momentini tayinlaymiz va t dan $t+h$ gacha vaqt oralig'ini qaraymiz, bunda h – ixtiyoriy kichik son. Nuqta t dan $t+h$ gacha vaqt oralig'ida $S(t+h)-S(t)$ masofa o'tadi.

Nuqta xarakatining shu vaqt oralig'idagi o'rtacha tezligi quyidagi nisbatga teng

$$v_{ypm}=\frac{s(t+h)-s(t)}{h}.$$

Fizika kursidan ma'lumki, h kamayishi bilan bu nisbat t vaqt momentidagi oniy tezlik deb ataluvchi va $v(t)$ kabi belgilanuvchi biror miqdorga yaqinlashadi. $v(t)$ miqdori bu nisbatning h nolga intilgandagi limiti deb ataladi va quyidagicha yoziladi:

$$v(t)=\lim_{h\to 0}\frac{s(t+h)-s(t)}{h}$$

Ta'rif. Agar $\Delta x\to 0$ da $\frac{\Delta y}{\Delta x}$ nisbatning limiti

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

mavjud va chekli bo'lsa, bu limit $f(x)$ funksiyaning x_0 nuqtadagi hosilasi deyiladi. $f(x)$ funksiyaning x_0 nuqtadagi hosilasi $f'(x_0)$ yoki $y'_{x=x_0}$ kabi belgilanadi.

Masalan, birinchi elementar funksiyalardan biri bu $y=c$, $c=const$ ya'ni funksiya o'zgarmas miqdor bo'lgan xolda funksiyaning hosilasi nimaga teng?

Ayirmali nisbatni tuzib olamiz $\frac{f(x+\Delta x)-f(x)}{\Delta x}=\frac{c-c}{\Delta x}=0$ bu ayirmali nisbatimiz h ga bog'liq bo'lmaganligi uchun funksiya o'zgarmas son bo'lganda hosilasi nolga teng bo'lishini ko'rish qiyin emas.

Asosiy elementar funksiyalardan bittasi, bu chiziqli funksiya $f(x)=kx+b$. Shu funksiyaning hosilasini topamiz. Buning uchun ayirmali nisbat tuzamiz

$$\frac{f(x+\Delta x)-f(x)}{\Delta x}=\frac{k(x+\Delta x)+b-kx-b}{\Delta x}=\frac{kx+k\Delta x+b-kx-b}{\Delta x}=\frac{k\Delta x}{\Delta x}=k.$$

Bundan ko'rinadiki, $f'(x)=k$ tenglik o'rinli.

Keyingi asosiy elementar funksiyalardan biri $f(x)=x^n$ ko'rinishidagi funksiyadir bu funksiyaning hosilasini topamiz.

Avval ayirmali nisbat tuzamiz, $\dfrac{f(x+\Delta x)-f(x)}{\Delta x}=\dfrac{(x+\Delta x)^n-x^n}{\Delta x}$. Bu ayirmali nisbatning suratidagi darajali qavsni Nyuton binom formulasidan ochib chiqamiz

$$\frac{(x+\Delta x)^n-x^n}{\Delta x}=\frac{x^n+nx^{n-1}\Delta x+\frac{n(n-1)}{2}x^{n-1}\Delta x^2+\ldots+\Delta x^n+x^n}{\Delta x}=$$

$$\frac{nx^{n-1}\Delta x+\frac{n(n-1)}{2}x^{n-1}\Delta x^2+\ldots+\Delta x^n}{\Delta x}$$

va bu ifodaning suratidan Δx ni qavsdan tashqariga chiqarib soddalashtirgach, quyidagiga kelamiz.

$$\frac{nx^{n-1}\Delta x+\frac{n(n-1)}{2}x^{n-1}\Delta x^2+\ldots+\Delta x^n}{\Delta x}=\frac{\Delta x(nx^{n-1}+\frac{n(n-1)}{2}x^{n-1}\Delta x+\ldots+\Delta x^{n-1})}{\Delta x}=$$

$$nx^{n-1}+\frac{n(n-1)}{2}x^{n-1}\Delta x+\ldots+\Delta x^{n-1}$$

Yuqoridagi ifodada $\Delta x\to 0$ ifodaning qiymati quyidagiga teng bo'ladi, bundan ko'rinadiki $f'(x)=nx^{n-1}$ ekanligi

$$\lim_{\Delta x\to 0}(nx^{n-1}+\frac{n(n-1)}{2}x^{n-1}\Delta x+\ldots+\Delta x^{n-1})=nx^{n-1}$$

kelib chiqadi. Shu bilan birga $f(x)=(x+a)^n$ va $f(x)=(ax+b)^n$ ko'rinishdagi funksiya xosilalarini ko'rib chiqamiz va albatta ayirmali nisbat tuzamiz.

$$\frac{f(x+\Delta x)-f(x)}{\Delta x}=\frac{(x+\Delta x+a)^n-(x+a)^n}{\Delta x}=\frac{((x+a)+\Delta x)^n-(x+a)^n}{\Delta x}=$$

$$\frac{(x+a)^n+n(x+a)^{n-1}\Delta x+\frac{n(n-1)}{2}(x+a)^{n-2}\Delta x^2+\ldots+\Delta x^n-(x+a)^n}{\Delta x}=$$

$$\frac{n(x+a)^{n-1}\Delta x+\frac{n(n-1)}{2}(x+a)^{n-2}\Delta x^2+\ldots+\Delta x^n}{\Delta x}=n(x+a)^{n-1}+$$

$$\frac{n(n-1)}{2}(x+a)^{n-2}\Delta x+\ldots+\Delta x^{n-1}.$$

Bu ifodadan $\Delta x\to 0$ da quyidagi tenglikka kelamiz $f'(x)=n(x+a)^{n-1}$.

Funksiyaning darajasi natural son emas balki haqiqiy son bo'lgan holda ham yuqoridagi formula o'rinli ekanligini ko'rib chiqamiz. Buning uchun ajoyib limitdan foydalanamiz.

Ya'ni birinchi ajoyib limitdan kelib chiqadigan quyidagi natijadan foydalanamiz. $\lim\limits_{x\to 0}\dfrac{(1+x)^\alpha-1}{x}=\alpha$ formuladan quyidagini keltirib chiqaramiz.

$$\lim_{\Delta x\to 0}\frac{(x+\Delta x)^\alpha-x^\alpha}{\Delta x}=\lim_{\Delta x\to 0}\frac{x^\alpha(1+\frac{\Delta x}{x})^\alpha-x^\alpha}{\Delta x}=\lim_{\Delta x\to 0}\frac{x^\alpha((1+\frac{\Delta x}{x})^\alpha-1)}{\Delta x}=$$

$$x^\alpha\lim_{\Delta x\to 0}\frac{((1+\frac{\Delta x}{x})^\alpha-1)}{\frac{\Delta x}{x}\Delta x}=x^\alpha\Delta\alpha\Delta\frac{1}{x}=\alpha\Delta x^{\alpha-1}$$

bundan esa yuqoridagi formula darajali funksiyaning darajasi ixtiyoriy son bo'lgan holda ham o'rinli ekanligi kelib chiqadi.

Ushbu funksiya hosilasini boshqacha usul bilan ham hisoblash mumkin.

$$\lim_{\Delta x\to 0}\frac{(x+\Delta x)^\alpha-x^\alpha}{\Delta x}=\lim_{\Delta x\to 0}\frac{x^\alpha((1+\frac{\Delta x}{x})^\alpha-1)}{\Delta x}=\lim_{\Delta x\to 0}\frac{x^\alpha((1+\frac{\Delta x}{x})^{\frac{x}{\Delta x}\cdot\frac{\Delta x}{x}\alpha}-1)}{\Delta x}=$$

$$\lim_{\Delta x\to 0}\frac{x^\alpha(e^{\frac{\alpha\Delta x}{x}}-1)}{\Delta x}=\lim_{\Delta x\to 0}\frac{x^\alpha(e^{\frac{\alpha\Delta x}{x}}-1)}{\frac{\alpha\Delta x}{x}\frac{x}{\alpha}}=\frac{\alpha x^\alpha}{x}=\alpha x^{\alpha-1}.$$

Ajoyib limitlardan funksiya $y = x^{\alpha}$ bo'lgan holda, funksiyaning hosilasi $y' = \alpha x^{\alpha-1}$ ga teng bo'ladi.

Endi $f(x) = (ax+b)^n$ ko'rinishdagi funksiya xosilasini ko'rib chikamiz. Buning uchun yana

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{(a(x+\Delta x)+b)^n-(ax+b)^n}{\Delta x} = \frac{((ax+a\Delta x)+b)^n-(ax+b)^n}{\Delta x} =$$

$$\frac{((ax+b)+a\Delta x)^n-(ax+b)^n}{\Delta x} =$$

$$\frac{(ax+b)^n+n(ax+b)^{n-1}a\Delta x+\frac{n(n-1)}{2}(ax+b)^{n-2}\Delta x^2+\ldots+\Delta x^n-(ax+b)^n}{\Delta x} =$$

$$= \frac{n(ax+b)^{n-1}(a\Delta x)+\frac{n(n-1)}{2}(ax+b)^{n-2}(a\Delta)^2+\ldots+(a\Delta x)^n}{\Delta} = n(ax+b)^{n-1}a+$$

$$\frac{n(n-1)}{2}(ax+b)^{n-2}a\Delta x+\ldots+(a\Delta x)^{n-1}.$$

Bu ifodada $\Delta x \to 0$ da $f'(x) = na(ax+b)^{n-1}$ ekanligini ko'ramiz

Funksiya ko'rsatkichli funksiya bo'lgan holda funksiya hosilasini topamiz $y = a^x$.

Bu funksiyaning hosilasini topishda ham hosila ta'rifidan funksiya orttirmasini argument orttirmasiga nisbatini qaraymiz. $\lim\limits_{\Delta x\to 0}\frac{a^{x+\Delta x}-a^x}{\Delta x} =$

$$\lim_{\Delta x\to 0}\frac{a^x(a^{\Delta x}-1)}{\Delta x} = a^x\ln a \text{ ekanligini topamiz.}$$

Keyingi funksiyalardan biri bu logarifmik funksiya bo'lib, logarifmik funksiyaning hosilasini topish uchun ham ayirmali nisbat qaraymiz va bu nisbatning limitini topish uchun yana ajoyib limitlar va ajoyib limitlardan kelib chiqadigan natijalardan foydalanamiz. $\lim\limits_{\Delta x\to 0} y = \log_a x$ bu funksiyaning ayirmali nisbati quyidagicha bo'ladi.

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x\to 0}\frac{\log_a(x+\Delta x)-\log_a x}{\Delta x} = \lim_{\Delta x\to 0}\frac{\log_a(1+\frac{\Delta x}{x})}{\Delta x} =$$

$$\lim_{\Delta x\to 0}\frac{x}{\Delta x}\cdot\frac{1}{x}\cdot\log_a\left(1+\frac{\Delta x}{x}\right) = \lim_{\Delta x\to 0}\frac{1}{x}\cdot\log_a\left(1+\frac{\Delta x}{x}\right)^{\frac{x}{\Delta x}} = \frac{1}{x}\cdot\log_a x = \frac{1}{x\ln a}$$

Bulardan logarifmik funksiyaning hosilasi $y' = \frac{1}{x\ln a}$ ekanligi kelib chiqadi.

Endi trigonometrik funksiyalarning hosilasini topish bilan shug'ullanamiz. Masalan $y = \sin x$ bu funksiyaning hosilasini topish uchun yana funksiya orttirmasini tekshiramiz. $\lim\limits_{\Delta x\to 0}\frac{\sin(x+\Delta x)-\sin x}{\Delta x}$ bundan limitda kasr suratiga trigonometriyaning yana bir hossasini qo'llab ya'ni ayirmani ko'paytmaga aylantirish formulasidan quyidagiga ega bo'lamiz:

$$\lim_{\Delta x\to 0}\frac{2\sin\frac{\Delta x}{2}\cdot\cos(x+\frac{\Delta x}{2})}{\Delta x} = \lim_{\Delta x\to 0}\frac{\sin\frac{\Delta x}{2}\cdot\cos(x+\frac{\Delta x}{2})}{\frac{\Delta x}{2}} =$$

$$= \lim_{\Delta x\to 0}\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}\cdot\cos(x+\frac{\Delta x}{2}) = \cos x.$$

1. Bundan esa $y' = \cos x$ ekanligi kelib chiqadi. Huddi shu kabi $y = \cos x$ funksiyaning hosilasi $y' = -\sin x$ ekanligini ham huddi shu kabi ko'rsatishimiz mumkin.

2. Trigonometrik funksiyalardan yana biri ya'ni $y = tgx$ ning hosilasini topamiz. Bu funksiyaning hosilasini topishda ikki usuldan foydalanishimiz mumkin. Birinchi usul bu albatta o'zimiz biladigan funksiya hosilasi ta'rifidan foydalanib hisoblash. Ikkinchi usulidan foydalanish uchun hosila olish qoidalarni bilish kerak.

Funksiya hosilasini hisoblash qoidalari.

a) Ikki funksiyaning algebraik yig'indisining hosilasi, shu funksiyalar hosilalarining algebraik yig'indilariga teng. YA'ni $y = f(x) \pm g(x)$ ko'rinishdagi funksiya berilgan bo'lsa, u holda funksiya hosilasi quyidagicha bo'ladi. $y' = f'(x) \pm g'(x)$ ga teng bo'ladi. Buni isbotlash uchun ham hosila ta'rifidan foydalanamiz.

$$\lim_{\Delta x\to 0}\frac{\left(f(x+\Delta x)\pm g(x+\Delta x)\right)-(f(x)\pm g(x))}{\Delta x}=$$

$$\lim_{\Delta x\to 0}\frac{(f(x+\Delta x)-f(x))\pm(g(x+\Delta x)-g(x))}{\Delta x}==\lim_{\Delta x\to 0}\frac{(f(x+\Delta x)-f(x))}{\Delta x}\pm$$

$$\lim_{\Delta x\to 0}\frac{(g(x+\Delta x)-g(x))}{\Delta x}=f'(x)+g'(x).$$

b) Ikki funksiyaning ko'paytmasining hosilasini topamiz. YA'ni funksiya $y=f(x)\cdot g(x)$ ko'rinishda berilgan bo'lsa, bu funksiyaning hosilasini topamiz. $y'=f'(x)\cdot g(x)+f(x)\cdot g'(x)$ bu formula ikki funksiya ko'paytmasining hosilasini beradi. Shuni isbotlaymiz. Bu formulani isbotlash uchun ham funksiya orttirmasini argument orttirmasiga nisbatini qaraymiz.

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)g(x+\Delta x)-f(x)g(x)}{\Delta x}=$$

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)g(x+\Delta x)-f(x+\Delta x)g(x)+f(x+\Delta x)g(x)-f(x)g(x)}{\Delta x}=$$

$$\lim_{x\to 0}\frac{f(x+\Delta x)(g(x+\Delta x)-g(x))+g(x)(f(x+\Delta x)-f(x))}{\Delta x}=$$

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)(g(x+\Delta x)-g(x))}{\Delta x}+\lim_{\Delta x\to 0}\frac{g(x)(f(x+\Delta x)-f(x))}{\Delta x}=$$

$$f'(x)\cdot g(x)+f(x)\cdot g'(x)$$

c) ekanligi kelib chiqadi.

d) Endi ikki funksiya bo'linmasining hosilasini qaraylik,

$y'=\left(\frac{f(x)}{g(x)}\right)'=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)}$ bo'linmaning hosilasi ushbu formula bilan topiladi. Bu formulaning isboti ham juda oddiy va maktab o'quvchilariga tushuntirishda o'qituvchilar qiyinchilikka uchramaydi. Bu formulani isbotlashda ham hosila ta'rifidan foydalanamiz. Buning uchun ayirmali nisbatni ko'rib chiqamiz.

e) $$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\frac{f(x+\Delta x)}{g(x+\Delta x)}-\frac{f(x)}{g(x)}}{\Delta x}=$$

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)g(x)-g(x+\Delta x)f(x)}{\Delta x\cdot g(x+\Delta x)\cdot g(x)}=$$

$$\lim_{x\to 0}\frac{f(x+\Delta x)g(x)-f(x)g(x)+f(x)g(x)-g(x+\Delta x)f(x)}{\Delta x\cdot g(x+\Delta x)\cdot g(x)}=$$

$$\lim_{\Delta x\to 0}\frac{g(x)(f(x+\Delta x)-f(x))-f(x)(g(x+\Delta x)-g(x))}{\Delta x\cdot g(x+\Delta x)\cdot g(x)}=$$

$$\lim_{\Delta x\to 0}\frac{1}{g(x+\Delta x)\cdot g(x)}\cdot\frac{g(x)(f(x+\Delta x)-f(x))-f(x)(g(x+\Delta x)-g(x))}{\Delta x}=$$

$$\lim_{\Delta x\to 0}\frac{1}{g(x+\Delta x)\cdot g(x)}\cdot\left(\frac{g(x)(f(x+\Delta x)-f(x))}{\Delta x}-\frac{f(x)(g(x+\Delta x)-g(x))}{\Delta x}\right)=$$

$$\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)}$$

Endi yuqoridagi trigonometrik funksiyani hosilasini hisoblaymiz. Yuqorida ta'kidlab o'tganimizdek $y=tgx$ funksiyaning hosilasini ikki hil usulda topishimiz mumkin.

Funksiya orttirmasini argument orttirmasiga nisbatini qaraylik.

$$\lim_{\Delta x\to 0}\frac{tg(x+\Delta x)-tgx}{\Delta x}=\lim_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x\cdot\cos(x+\Delta x)\cos x}=\lim_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x}\cdot\frac{1}{\cos(x+\Delta x)\cos x}=$$

$$\frac{1}{(\cos x)^2}$$

ekanligi kelib chiqadi.

Endi yuqoridagi formula bo'yicha bo'linmaning hosilasi formulasidan foydalanib,

$$y'=\frac{\sin' x\cdot\cos x-\sin x\cos' x}{\cos^2 x}=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}$$

ekanligini keltirib chiqaramiz. Bundan ko'rinadiki har ikki usulda ham natija bir hil bo'ladi.

Elementar funksiyalarning hosilasi jadvali

1) $y=c,\ c=const \quad y'=0$

2) $f(x)=kx+b \quad y'=k$

3) $f(x)=x^{\alpha} \quad y'=\alpha x^{\alpha-1}$

4) $y'=a^{x} \quad y'=a^{x}\ln a$

5) $y=e^{x} \quad y'=e^{x}$

6) $y=\log_{a}x \quad y'=\dfrac{1}{x\ln a}$

7) $y=\ln x \quad y'=\dfrac{1}{x}$

8) $y=\sin x \quad y'=\cos x$

9) $y=\cos x \quad y'=-\sin x$

10) $y=tgx \quad y'=\dfrac{1}{\cos^{2}x}$

11) $y=ctgx \quad y'=-\dfrac{1}{\sin^{2}x}$

1-misol. $y=\dfrac{x^4}{4}-5\sqrt{x}+\dfrac{8}{x^2}-\dfrac{1}{3\sqrt[3]{x}}$ funksiyaning hosilasini toping.

Yechish: ushbu funksiyaning hosilasini topish uchun oldin funksiyaning har bir yig'indisini argumentni darajasi shaklida yozamiz. $y=\dfrac{x^4}{4}-5\sqrt{x}+\dfrac{8}{x^2}-\dfrac{1}{3\sqrt[3]{x}}=\dfrac{1}{4}x^4-5x^{\frac{1}{2}}+8x^{-2}-\dfrac{1}{3}x^{-\frac{1}{3}}$ ko'rinishga keltirilgandan keyin darajaning hosilasi formulasi va bir nechta funksiya yig'indisidan olingan hosila xossasidan foydalanib, ushbu funksiya hosilasini topamiz.

$$y'=(\frac{1}{4}x^4-5x^{\frac{1}{2}}+8x^{-2}-\frac{1}{3}x^{-\frac{1}{3}})'=(\frac{1}{4}x^4)'-(5x^{\frac{1}{2}})'+(8x^{-2})'-(\frac{1}{3}x^{-\frac{1}{3}})'=$$

$$\frac{1}{4}\cdot 4x^3-5\cdot\frac{1}{2}x^{-\frac{1}{2}}-8\cdot 2x^{-3}+\frac{1}{3}\cdot\frac{1}{3}x^{-\frac{4}{3}}=x^3-\frac{5}{2\sqrt{x}}-\frac{16}{x^3}+\frac{1}{9\sqrt[3]{x^4}}.$$

2-misol. $y=\sin x\cdot e^{x}$ funksiya hosilasini hisoblang.

Yechish: ushbu funksiya hosilasini hisoblashda ko'paytmaning hosilasi formulasidan $y'=(\sin x\cdot e^{x})'=\sin' x\cdot e^{x}+\sin x\cdot(e^{x})'=e^{x}(\cos x+\sin x)$ ekanligi kelib chiqadi.

Murakkab funksiyaning hosilalari

Teorema. $u=\varphi(x)$ funksiya $x=x_0$ nuqtada hosilaga ega bo'lsa, $y=f(u)$ funksiyasi ($x=x_0$ nuqtaga mos keluvchi) $z=z_0$ nuqtada hossilaga ega bo'lsa, u holda bulardan tuzilgan $y=f(\varphi(x))$ murakkab funksiyaning $x=x_0$ nuqtadagi hosilasi $y'=(f(\varphi(x)))'=f'(\varphi(x))\cdot\varphi'(x)$ ga teng bo'ladi.

Bu teoremaning isbotini talabalarga mustaqil ishi sifatida keltirib o'tamiz.

1-misol. $y=\cos(x^2+5x-7)$ ushbu funksiyaning hosilasini hisoblang.

Yechish: bu misolda ko'rinib turibdiki funksiya murakkab funksiya, ya'ni trigonometrik funksiyaning ichida kvadrat funksiya. Bu funksiyadan hosila olish uchun trigonometrik funksiyaning o'zidan hosila olinadi va ichidan hosila olinadi.

$$y'=(\cos(x^2+5x-7))'=\cos'(x^2+5x-7)\cdot(x^2+5x-7)'=$$
$$-\sin(x^2+5x-7)\cdot(2x+5)=-(2x+5)\sin(x^2+5x-7)$$

2-misol. $y=\sin^2 x$ funksiyaning hosilasini toping.

Yechish: bunday olib qaraganda bu funksiya ham murakkab funksiya sanaladi. Chunki kvadrat funksiyaning ichiga trigonometrik funksiya qo'yib hosil qilingan. Lekin bu misolni ikki hil usulda yechish mumkin. YA'ni birinchisi murakkab funksiyadan olingan hosila bo'yicha va trigonometrik funksiyaning xossalarini qo'llash bo'yicha.

A) $y'=(\sin^2 x)'=2\sin x\cdot(\sin x)'=2\sin x\cos x=\sin 2x$

B) $y=\sin^2 x$ funksiyaning darajasini pasaytiramiz. $y=\sin^2 x=\dfrac{1-\cos 2x}{2}$ endi bu funksiyadan hosila olamiz. Unitmaslik kerakki bu funksiyaning hosilasi ham murakkab funksiyaning hosilasidan kelib chiqadi. Chunki trigonometrik funksiyaning ichida birinchi darajali chiziqli funksiya turibdi. $y'=(\dfrac{1-\cos 2x}{2})'=\dfrac{1}{2}(\sin 2x\cdot 2)\sin 2x$ bu yerda ishlash usuliga bog'liq bo'lmagan holda natija bir hil chiqadi.

Mustaqil yechish uchun misollar:

1-misol:

1.1. $y=x^3+3x^2-4$ funksiyani hosila ta'rifi yordamida hosilasini hisoblang.

1.2. $y=2x^4-\dfrac{2}{3}x^3-4x+5$ функцияни ҳосила таърифи ёрдамида ҳосиласини ҳисобланг.

1.3. $y=3x^5-3\sqrt[3]{x}-\dfrac{1}{2x^3}+\dfrac{10}{\sqrt[5]{x^4}}$ funksiya hosilasini hosila olish qoidalari va xossalaridan foydalanib hisoblang.

1.4. $y=\sin x\cdot e^x$ funksiya hosilasini ko'paytmaning hosilasi formulasidan foydalanib hisoblang.

1.5. $y=x\,arctgx$ funksiya hosilasini ko'paytmaning hosilasi formulasidan foydalanib hisoblang.

1.6. $y=\sqrt[4]{x}\ln x$ funksiya hosilasini ko'paytmaning hosilasi formulasidan foydalanib hisoblang.

1.7. $y=x^2tgx$ funksiya hosilasini ko'paytmaning hosilasi formulasidan foydalanib hisoblang.

1.8. $y=\dfrac{\cos x}{\ln x}$ funksiya hosilasini bo'linmaning hosilasi formulasidan foydalanib yeching.

1.9. $y=\dfrac{3^x}{2x+1}$ funksiya hosilasini bo'linmaning hosilasi formulasidan foydalanib yeching.

1.10. $y=\dfrac{x^2+1}{x^2-1}$ funksiya hosilasini bo'linmaning hosilasi formulasidan foydalanib yeching.

1.11. $y=\dfrac{1+e^x}{1-e^x}$ funksiya hosilasini bo'linmaning hosilasi formulasidan foydalanib yeching.

1.12. $y=\dfrac{\ln x}{\sin x}+xctgx$ funksiyani yig'indi, ko'paytma va bo'linmaning hosilalari formulalaridan foydalanib yeching.

2-misol: Quyidagi funksiyalarning hosilalarini toping.

2.1 a) $y=\sin 2x$ b) $y=\arcsin x^3$

2.2 a) $y=\arcsin 4x$ b) $y=\sqrt{1-x^2}$

2.3 a) $y=\log_3(2x-5)$ b) $y=\sqrt{1+5\cos x}$

2.4 a) $y=tg(5x+1)$ b) $y=\dfrac{1}{2}e^x(\sin x+\cos x)$

2.5 a) $y=(3x-8)^7$ b) $y=\ln\ln x$

2.6 a) $y=\ln\sqrt{x}$ b) $y=arc\sin\sqrt{x}$

2.7 a) $y=\sin^2 x$ b) $y=\arcsin\frac{x}{5}$

2.8 a) $y=\cos^3 x$ b) $y=\ln 3x$

Javoblar:

1.1 $y'=3x^2+6x$;

1.2 $y'=8x^3-2x^2-4$;

1.3 $y'=15x^4-\frac{1}{\sqrt[3]{x^2}}+\frac{3}{2x^4}-\frac{8}{\sqrt[5]{x^9}}$;

1.4 $y'=e^x(\cos x+\sin x)$;

1.5 $y'=arctgx+\frac{x}{1+x^2}$;

1.6 $y'=\frac{\ln x+4}{4\sqrt[4]{x^3}}$;

1.7 $y'=\frac{x\sin 2x+x^2}{\cos^2 x}$;

1.8 $y'=\frac{x\sin x\cdot\ln x-\cos x}{x\ln^2 x}$;

1.9 $y'=\frac{3^x(2x\ln 3+\ln 3-2)}{(2x+1)^2}$;

1.10 $y'=-\frac{4x}{(x^2-1)^2}$;

1.11 $y'=\frac{2e^x}{(e^x-1)^2}$.

2.1 a) $y'=2\cos 2x$ **b)** $y'=\frac{3x^2}{\sqrt{1-x^6}}$;

2.2 a) $y'=\frac{4}{\sqrt{1-16x^2}}$

b) $y'=-\frac{x}{\sqrt{1-x^2}}$

2.3 a) $y'=\frac{2}{(2x-5)\ln 3}$

b) $y'=-\frac{5\sin x}{\sqrt{1+5\cos x}}$;

2.4 a) $y'=\frac{5}{\cos^2(5x+1)}$

b) $y'=e^x\cos x$;

2.5 a) $y'=21(3x-8)^7$

b) $y'=\frac{1}{x\ln x}$;

2.6 a) $y'=\frac{1}{2x}$ **b)** $y'=\frac{1}{2\sqrt{x-x^2}}$;

2.7 a) $y'=\sin 2x$ **b)** $y'=\frac{1}{\sqrt{25-x^2}}$;

2.8 a) $y'=-3\sin x\cos^2 x$ **b)** $y'=\frac{1}{x}$.

Mavzu: Funksiya hosilasi. Hosila olish qoidalari va murakkab funksiyalarning hosilalariga oid misollar yechish.

1-misol. $y=x^4+5x^3-7x^2+8x-15$ funksiyaning hosilasini toping.

***Yechish*:** bu funksiyaning hosilasini topish uchun hosila topish qoidalaridan funksiyalarning yig'indisining hosilasi funksiyalar hosilalari yig'indisiga tengdir. Shundan foydalanamiz.

$$y'=(x^4+5x^3-7x^2+8x-15)'=(x^4)'+(5x^3)'-(7x^2)'+(8x)'-15'=$$
$$4x^3+15x^2-14x+8$$

2-misol. $y=\cos x\cdot\ln x$ funksiyaning hosilasini toping.

***Yechish*:** Bu funksiya ikki funksiyaning ko'paytmasidan tashkil topgan. Shuning uchun ikki funksiya ko'paytmasi hosilasidan formulasidan foydalanamiz.

$$y'=(\cos x\cdot\ln x)'=\cos' x\cdot\ln x+\cos x\cdot\ln' x=-\sin x\cdot\ln x+\frac{\cos x}{x}=$$
$$\frac{\cos x-x\sin x\cdot\ln x}{x}.$$

3-misol. $y=\frac{\sin x}{e^x}$ funksiyaning hosilasini toping.

***Yechish*:** Funksiya ikki funksiyaning nisbatidan iborat. Shuning uchun bu funksiyaning hosilasini topishda bo'linmaning hosilasi formulasidan foydalanamiz.

$$y'=(\frac{\sin x}{e^x})'=\frac{\sin' x\cdot e^x-\sin x\cdot(e^x)'}{(e^x)^2}=\frac{(\cos x-\sin x)e^x}{e^{2x}}=\frac{(\cos x-\sin x)}{e^x}.$$

4-misol. $y=(3x+2)^4$ funksiyaning hosilasini toping.

***Yechish*:** bu funksiyaning hosilasini topishda ikki hil usuldan foydalanish mumkin.Birinchi usul hosila olishning oddiy qoidalaridan foydalanish maqsadida Nyuton binomidan yoki Paskal uchburchagidan foydalanib kavsni ochib chiqish. Hozir shu usuldan foydalanib funksiya hosilasini hisoblaymiz.

$$y=(3x+2)^4=81x^4+4\cdot 2\cdot 27x^3+6\cdot 9x^2\cdot 4+4\cdot 3x\cdot 8+16=$$
$$=81x^4+216x^3+216x^2+96x+16$$

funksiya 4 -darajali funksiya, ko'phad ko'rinishiga keldi. Endilikda bu funksiyaning hosilasini hisoblaymiz.

$$y'=(81x^4+216x^3+216x^2+96x+16)'=(81x^4)'+(216x^3)'+(216x^2)'+(96x)'+$$
$$(16)'==324x^3+648x^2+432x+96.$$

Yuqoridagi funksiya hosilasini topishning ikkinchi usuli bu murakkab funksiya hosilasidan foydalanish demakdir.

$$y'=((3x+2)^4)'=4(3x+2)^3\cdot(3x+2)'=12(3x+2)^3.$$

Yuqoridagi natijani soddalashtirsak ushbu javob bilan bir hil bo'ladi.

5-misol. $y=3^{4x^2-5x+1}$ funksiyani hosilasini toping.

Yechish: ko'rinib turibdiki bu funksiya murakkab funksiyadir,

$$y'=(3^{4x^2-5x+1})'=3^{4x^2-5x+1}(4x^2-5x+1)'\cdot\ln 3=(8x-5)3^{4x^2-5x+1}\cdot\ln 3.$$

6-misol. $y=\ln\dfrac{x^3-2x+1}{2x+4}$ funksiyaning hosilasini toping.

Yechish: $y'=(\ln\dfrac{x^3-2x+1}{2x+4})'=\dfrac{1}{\dfrac{x^3-2x+1}{2x+4}}\cdot(\dfrac{x^3-2x+1}{2x+4})'=\dfrac{2x+4}{x^3-2x+1}\cdot$

$$\frac{(3x^2-2x)(2x+4)-2(x^3-2x+1)}{(2x+4)^2}=\frac{2x+4}{x^3-2x+1}\cdot$$
$$\frac{6x^3+12x^2-4x-8-2x^3+4x-2}{(2x+4)^2}=\frac{4x^3+12x^2-10}{2(x+2)(x^3-2x+1)}=\frac{2x^3+12x^2-10}{(x+2)(x^3-2x+1)}$$

7-misol. $y=\cos(2^{x^3})$ funksiyaning hosilasini toping.

Yechish: bu funksiyaning hosilasini olish uchin ham murakkab funksiya hosilasidan foydalaniladi. Lekin bu funksiyada ikkita funksiya emas balki uchta funksiya ichma- ich joylashgan. Bunday holatda ham hosila murakkab

funksiyaniki kabi bajariladi. YA'ni $y=f(g(\varphi(x)))$ bo'lgan holatda ham hosila huddi shunday amalga oshiriladi.

$y'=(f(g(\varphi(x))))'=f'(g(\varphi(x)))\cdot g'(\varphi(x))\cdot\varphi'(x)$ formula orqali.

Demak

$$y'=(\cos(2^{x^3}))'=\sin(2^{x^3})\cdot(2^{x^3})'\cdot(x^3)'=2^{x^3}\cdot\ln 2\cdot\sin(2^{x^3})\cdot 3x^2=$$
$$3\ln 2\cdot x^2\cdot 2^{x^3}\cdot\sin(2^{x^3}).$$

Mustaqil yechish uchun misollar:

1-misol: Berilgan misollarda a) funksiyani berilgan nuqtadagi hosilasini toping, b) berilgan murakkab funksiya hosilasini hisoblang.

1.1 a) $y=\sqrt[5]{4-3x^5}+4^x\dfrac{2}{\ln 4}$ функцияни $x=1$ нуқтадаги ҳосиласини топинг. b) $y=\dfrac{1}{2}arctg\dfrac{x}{2}$

1.2 a) $y=\ln(\sin 5x)-\dfrac{4x^2}{\pi}+\dfrac{4}{5}$, $y'(x_0)=?$ $x_0=\dfrac{\pi}{10}$ b) $y=arctge^x$

1.3 a) $y=\ln\sqrt{(x-4)^3}+(x-4)^3$, $y'(x_0)$, $x_0=5$ b) $y=\dfrac{1}{2}\ln\dfrac{x-1}{x+1}$

1.4 a) $y=e^{x-1}(4x-5)$, $y'(x_0)$, $x=\ln 2$ b) $y=\ln(x+\sqrt{x^2+1})$

1.5 a) $y=(x+1)arctge^{-2x}$, $y'(x_0)$, $x=0$ b) $y=\ln(x+\sqrt{x^2+1})$

1.6 a) $y=\ln\dfrac{2+tgx}{2-tgx}$, $y'(x_0)$, $x_0=\dfrac{\pi}{3}$ b) $y=\ln\sin x$

1.7 a) $y=\arcsin\dfrac{x-1}{x}$, $y'(x_0)$, $x=5$ b) $y=\ln\cos x$

1.8 a) $y=(4x^2-3x+1)^3$, $y'(x_0)$, $x_0=0$ b) $y=\sqrt{1-x^2}$

1.9 a) $y=\sin(7x^2+x)$, $y'(x_0)$, $x_0=0$ b) $y=\sqrt{1+5\cos x}$

1.10 a) $y=e^{4x^2-5x}$, $y'(x_0)$, $x_0=0$ b) $y=\dfrac{1}{2}e^x(\sin x+\cos x)$

1.11 a) $y=\arcsin\dfrac{1}{x}$, $y'(x_0)$, $x=2$ b) $y=\ln\ln x$

Javoblar:

1.1 a) 5 **b)** $y'=\dfrac{1}{2\sqrt{4+x^2}}$;

1.2 a) -0.8 **b)** $y'=\dfrac{e^x}{1+e^{2x}}$;

1.3 a) 4.5 **b)** $y'=\dfrac{1}{x^2-1}$;

1.4 a) $\dfrac{2}{e}(4\ln 2-1)$; **b)** $y'=\dfrac{1}{\sqrt{x^2+1}}$;

1.5 a) $\dfrac{\pi}{4}-1$ **b)** $y'=\dfrac{1}{\sqrt{x^2+1}}$;

1.6 a) $\dfrac{16}{13}$ **b)** $y'=ctgx$;

1.7 a) $\dfrac{1}{15}$ **b)** $y'=-tgx$;

1.8 a) -9 **b)** $y'=-\dfrac{x}{\sqrt{1-x^2}}$;

1.9 a) 1 **b)** $y'=-\dfrac{5\sin x}{2\sqrt{1+5\cos x}}$;

1.10 a) -5 **b)** $y'=e^x\cos x$;

1.11 a) $-\dfrac{1}{2\sqrt{3}}$ **b)** $y'=\dfrac{1}{x\ln x}$

Mavzu: Funksiya differensiali

Reja

1) Differensiallanuvchanlik va hosilaning orasidagi bog'lanish.
2) Funksiya differensialining geometrik va fizik ma'nolari.
3) Differensialning taqribiy hisoblashlarga tatbiqlari.

Differensiallanuvchanlik va hosilaning orasidagi bog'lanish

Faraz qilaylik $f(x)$ funksiya (a,b) oraliqda berilgan bo'lib, $x_0\in(a,b), x_0+\Delta x\in(a,b)$ bo'lsin. Yuqorida aytib o'tganimizdek $\Delta f(x_0)=f(x_0+\Delta x)-f(x_0)$ ayirma $f(x)$ funksiyaning x_0 nuqtadagi orttirmasi deyiladi.

Ta'rif: Agar $\Delta f(x_0)$ ni ushbu $\Delta f(x_0)=A\cdot\Delta x+\alpha\Delta x$ ko'rinishida ifodalash mumkin bo'lsa, $f(x)$ funksiya x_0 nuqtada differensiallanuvchi deyiladi, bunda $A=const, \Delta x\to 0\Rightarrow\alpha\to 0$.

Teorema: $f(x)$ funksiya $x\in(a,b)$ nuqtada differensiallanuvchi bo'lishi uchun uning shu nuqtada chekli $f'(x)$ hosilaga ega bo'lishi zarur va yetarli.

Ta'rif: Funksiya orttirmasidagi $f'(x_0)\cdot\Delta x$ ifoda $f(x)$ funksiyaning x_0 nuqtadagi differensiali deyiladi va $df(x_0)$ kabi belgilanadi. Funksiya differensiali $df(x_0)=f'(x_0)\cdot\Delta x$ kabi yoziladi.

Endi $f(x)=x$ **funksiya deb faraz qilaylik va bu funksiyaning differensialini topamiz.** $df(x)=(x)'\cdot\Delta x=\Delta x$ bo'lib, $dx=\Delta x$ ekanligi kelib chiqadi va funksiyaning differensiali sifatida $df(x_0)=f'(x_0)\cdot dx$ kabi ko'rinishda ifodalashimiz mumkin.

Elementar funksiyalarning differensiallari.

1) $d(x^\alpha)=\alpha x^{\alpha-1}\cdot dx, (x>0)$
2) $d(a^x)=a^x\ln a\cdot dx, (a>0, a\neq)$

3) $d(\log_a x) = \frac{1}{x\ln a}\cdot dx, (x>0, a>0, a\neq 1)$

4) $d(\sin x) = \cos x\cdot dx,$

5) $d(\cos x) = -\sin x\cdot dx,$

6) $d(tgx) = \frac{dx}{\cos^2 x},$

7) $d(ctgx) = -\frac{dx}{\sin^2 x},$

8) $d(\arcsin x) = \frac{dx}{\sqrt{1-x^2}}, (-1\leq x\leq 1)$

9) $d(\arccos x) = -\frac{dx}{\sqrt{1-x^2}}, (-1\leq x\leq 1)$

10) $d(arctgx) = \frac{dx}{1+x^2},$

11) $d(arcctgx) = -\frac{dx}{1+x^2},$

12) $d(shx) = chxdx,$

13) $d(chx) = shxdx,$

14) $d(thx) = \frac{dx}{ch^2 x},$

15) $d(cthx) = -\frac{dx}{sh^2 x},$

Funksiyalarni differennsiallashdagi bir nechta sodda qoidalar.

$f(x)$ va $g(x)$ funksiyalar (a,b) oraliqda berilgan va $x\in(a,b)$ nuqtada differensiallanuvchi bo'lsin. U holda $x\in(a,b)$ nuqtada

1) $d(c\cdot f(x)) = c\cdot d(f(x)), c = const$

2) $d(f(x)+g(x)) = d(f(x)) + d(g(x))$

3) $d(f(x)g(x)) = g(x)d(f(x)) + f(x)d(g(x))$

4) $d\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)d(f(x)) - f(x)d(g(x))}{g^2(x)}, (g(x)\neq 0)$

Differensialning taqribiy hisoblashlarga tatbiqlari

Faraz qilaylik $f(x)$ funksiya (a,b) oraliqda berilgan bo'lib, $x_0\in(a,b)$ chekli hosilaga ega bo'lsin. U holda Faraz qilaylik $f(x)$ funksiya (a,b) oraliqda berilgan bo'lib, $\Delta x\to 0$ da Faraz qilaylik $f(x)$ funksiya (a,b) oraliqda berilgan bo'lib, $\Delta f(x_0) = f'(x_0)\cdot\Delta x + o(\Delta x)$ bo'ladi.

Yuqorida ko'rib o'tganimizdek berilgan funksiya $x_0\in(a,b)$ nuqtada differensiallanuvchi bo'lib, $df(x_0) = f'(x_0)\cdot\Delta x$ ga teng bo'ladi. Bundan esa $\Delta f(x_0) - df(x_0) = o(\Delta x)$ bo'lib, $\Delta x\to 0$ da $\Delta f(x_0) - df(x_0)\to 0$, bo'lib $\Delta f(x_0)\approx df(x_0)$ ga teng bo'lgan taqribiy formulaga kelamiz. Bundan esa $f(x_0+\Delta x) - f(x_0)\approx df(x_0)\Rightarrow f(x_0+\Delta x)\approx f(x_0) + f'(x_0)\cdot\Delta x$ ko'rinishidagi taxminiy formulaga kelamiz.

Misol: $\sin 29^0$ ni taqribiy qiymatini hisoblang.

Yechish: ushbu trigonometrik funksiyaning taqribiy qiymatini hisoblash uchun, $x_0 = 30^0$, $\Delta x = -1^0$ deb olamiz va yuqorida keltirib chiqargan taqribiy formulaga qo'yib, quyidagiga ega bo'lamiz.

$$\sin 29^0 = \sin 30^0 + \sin'(30^0)\cdot(-1^0) = \frac{1}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\pi}{360^0} = 0,4848 .$$

Shu o'rinda yuqoridagi misolning yechimida irrotsional ifoda qatnashganligi uchun, irrotsional ifodaning ham taqribiy qiymatini topish uchun ham taqribiy qiymat uchun keltrib chiargan formulamizni qo'llashimiz mumkin. Ya'ni $f(x) = \sqrt{x}, x_0 = 4, \Delta x = -1$ deb olib, yuqoridagi formulaga qo'yamiz va $\sqrt{3}\approx\sqrt{4} + \frac{1}{2\sqrt{4}}\cdot(-1) = 2 - \frac{1}{4} = 1,75$ bo'lib, 1,75 ning kvadratini hisoblasak 3,0625 ga teng bo'ladi va taqribiy formula sonning o'ndan birlar xonasigacha to'g'ri hisoblashini ko'rishimiz mumkin.

Endi quyida biz $x_0 = 0$ deb olib, $f(\Delta x)\approx f(0) + f'(0)\cdot\Delta x$ formulaga kelamiz va bir nechta ifodalarni $x_0 = 0$ nuqtadagi tahminiy formulalarini keltirib chiqaramiz.

Masalan, $f(x)=(1+x)^{\alpha}$, ushbu ifodaga yuqoridagi formulani qo'llasak, $(1+\Delta x)^{\alpha} \approx 1^{\alpha}+\alpha\cdot 1^{\alpha-1}\cdot\Delta x=1+\alpha\cdot\Delta x$ bo'lib, bundan esa $x_0=0$ nuqtaning atrofida $(1+x)^{\alpha}\approx 1+\alpha x$ tahminiy formula kelib chiqadi.

Bundan tashqari huddi shu kabi $x_0=0$ nuqtaning atrofida quyidagi bir qator formulalarni keltirib chiqarishimiz mumkin.

$$\sqrt{1+x}\approx 1+\frac{1}{2}x$$

$$e^{x}\approx 1+x$$

$$\ln(1+x)\approx x$$

$$\sin x\approx x$$

$$tgx\approx x$$

Mustaqil yechish uchun misollar

1-misol: quyida berilgan miqdorlarning taqribiy qiymatini hisoblang.

$\sin 31^0, \cos 29^0, tg59^0, ctg44^0, \sqrt{1,1}, \sqrt{1,01}, \sqrt{15}, \sqrt[3]{26}, \sqrt[3]{8,1}, \sqrt[3]{9}$,

$\ln 3,8$, $\ln 5$, $\ln 10$, $\lg 5$, $\lg 7$, $\lg 12$, $2^{2,2}$, $3^{0,4}$, $5^{0,1}$

2-misol: u, v funksiyalar differensiallanuvchi va ularning differensiallari du, dv ga teng deb faraz qilaylik. Quyidagi funksiyalarning differensiallarini toping.

1) $y=\sin(e^{u+v})$

2) $y=tg(u^2+uv+v^3)$

3) $y=e^{\sin u+\cos v}$

4) $y=(u+v)^3$

5) $y=u^2+3u^2v+v^4$

Mavzu: Funksiya hosilasi va uning tadbiqlari

Reja

1) Funksiyaning monotonligi. Funksiyaning ekstremumlari.

2) Birinchi tartibli hosila yordamida funksiyani ekstremumga tekshirish.

3) Funkiyaning eng katta va eng kichik qiymatlari.

Teorema. $f(x)$ funksiya $[a,b]$ intervalda chekli $f'(x)$ xosilaga ega bulsin. Bu funksiya shu intervalda o'suvchi (kamayuvchi) bulishi uchun $[a,b]$ интервалда $f'(x)>0$ $(f'(x)<0)$ tengsizlik o'rinli bo'lishi zarur va yetarli

Ta'rif. Agar $x_0\in(a,b)$ nuktaning shunday atrofi $U_\delta(x_0)\{x \mid x\in x_0-\delta<x<x_0+\delta;\delta>0\}\subset(a;b)$ mavjud bo'lsaki, $\forall x\in U_\delta(x_0)$ uchun $f(x)\le f(x_0)$ $(f(x)\ge f(x_0))$ tengsizlik o'rinli bulsa, u holda $f(x)$ funksiya x_0 nuqtada maksimumga (minimumga) ega deyiladi, $f(x_0)$ qiymat $f(x)$ funksiyaning $U_\delta(x_0)$ dagi maksimumi (minimumi) deyiladi.

Ekstremumning zaruriy sharti. $f(x)$ funksiya (a,b) intervalda aniqlangan bo'lib, $x_0\in(a,b)$ nuktada maksimum (minimum) ga erishsin. Demak, ta'rifga ko'ra x_0 nuqtaning shunday $U_\delta(x_0)\subset(a,b)$ atrofi topiladiki, $\forall x\in U_\delta(x_0)$ da $f(x)\ge f(x_0)$ $(f(x)\le f(x_0))$ tengsizlik o'rinli bo'ladi.

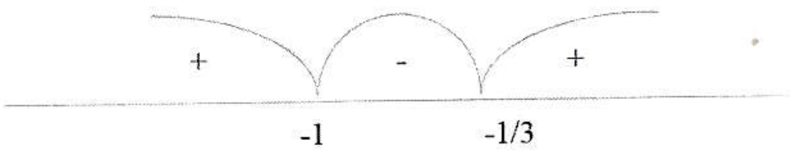
Teorema. Agar $f(x)$ **funksiya** $x_0\in(a,b)$ nuktada chekli $f'(x_0)$ hosilaga ega bo'lib, bu nuqtada $f(x)$ funksiya ekstremumga erishsa , u holda $f'(x_0)=0$ bo'ladi.

Ekstremumning yetarli shartlari. Endi funksiyaning ekstremumga ega bo'lishining yetarli shartlarini qaraymiz, ya'ni $f'(x_0)$ xosila x_0 nuktadan o'tishda o'z ishorasini « + » dan – ga o'zgartirsa, u holda $f(x)$ funksiya x_0 nuktada maksimumga ega bo'ladi.

Yuqoridagi ta'rif va teoremalardan ko'rinadiki funksiya hosilasining funksiya uchun tutgan yana bir o'rni mavjud bo'lib bu funksiyaning o'suvchi yoki kamayuvchiligini, oraliqda yoki nuqtada eng katta va eng kichik qiymatlarini aniqlashdan iborat. Bu esa funksiya grafigini chizishda qaysi oraliqlarda o'suvchi va qaysi oraliqlarda kamayuchi, oraliqlarning qaysi nuqtalarida eng katta va eng kichik qiymatlarni qabul qilishi mumkin ekanligini bilishda qo'l keladi. Buni misollar yordamida ko'rib chiqamiz.

Misol. $y=x^3+2x^2+x-7$ funksiyaning o'sish kamayish oraliqlari va ekstremum nuqtalarini toping.

Yechish: yuqoridagi ta'rif va teoremalarga asosan funksiyadan hosila olamiz va $y'=(x^3+2x^2+x-7)'=3x^2+4x+1$ ekanligi kelib chiqadi. Keyin chiqqan javobni nolga tenglashtiramiz. $D=16-12=4$, $x_1=\frac{-4+2}{6}=\frac{-1}{3}$, $x_2=-1$ tenglamaning javobini oraliqlar usulidan, sonli oraliqlarga joylashtiramiz. Keyin funksiyaning ham o'sish kamayish oraliqlarini, minimum va maksimum nuqtalarini aniqlaymiz ya'ni,

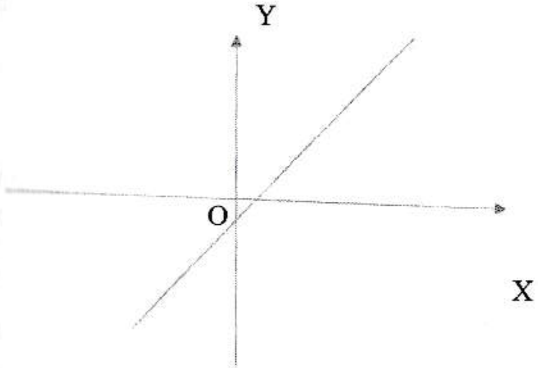


Bulardan funksiya $(-\infty;-1)$ $(-\frac{1}{3};\infty)$ oraliqda o'suvchi va $(-1;-\frac{1}{3})$ gacha oraliqda esa kamayuvchi bo'lib hisoblanadi. -1 nuqta maksimum nuqtasi va $-\frac{1}{3}$ nuqta esa minimum nuqtasidir.

Funksiya hosilasining yana ko'plab tadbiqlari mavjud bo'lib shulardan yana bittasi bu biror funksiya grafigiga qandaydir nuqtada o'tkazilgan urinma tenglamasini tuzish masalasidir. Bu masalaning yechimi quyidagi formulada berilgan.Biror funksiya grafigiga nuqtada o'tkazilgan urinma tenglamasi $y=f(x)$ funksiya berilgan bo'lsa u holda, $y_{ur}=f(x_0)+f'(x_0)(x-x_0)$ tenglama orqali topiladi.

1-misol. $y=x^3$ funksiyaga $x=1$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

Yechish: $y(1)=1$, $y'(1)=3x^2=3$ ekanligini topdik. Endi urinma tenglamasidan $y_{ur}=1+3(x-1)=1+3x-3=3x-2$, ya'ni $y_{ur}=3x-2$ ekanligi kelib chiqadi. Buni funksiya grafigida tasavvur qiladigan bo'lsak, quyidagiga kelamiz.



Chizmadagi funksiya grafigiga o'tkazilgan urinma tenglamasi yuqoridagi formula kabi bo'lar ekan.

Funksiya hosilasi mavzusining yana bir tadbiqlaridan biri bu funksiyani tekshirishdir, ya'ni funksiya grafigini chizishni hosila yordamida tekshirib chizishdir.

Funksiyalarni tekshirish va ularning grafiklarini yasashni kuyidagi sxema bo'yicha olib borish maqsadga muvofiqdir:

1 Funksiyaning aniqlanish soxasini topish;

2 Funksiyani uzluksizlikka tekshirish va uzilish nuqtalarini topish;

3 Funksiyaning juft, toq hamda davriyligini aniqlash;

4 Funksiyani monotonlikka tekshirish;

5 Funksiyani ekstremumga tekshirish;

6 Funksiya grafigining qavariq xamda botiqligini aniqlash, egilish nuktalarini topish;

7 Funksiya grafigining asimptotalarini topish;

8 Funksiyaning haqiqiy ildizlarini (agar ular mavjud bo'lsa), shuningdek argument xning bir nechta xaryakterli qiymatlarida funksiyaningkiymatlarini yasash

Ya'ni xar qanday grafigini chizish mumkin bo'lgan murakkab funksiyaning grafigini yasash bosqichlari shu tartibda olib boriladi.

2-misol. $y=\frac{x^3}{3}-\frac{5x^2}{2}+6x-7$ funksiyaning ekstremum nuqtalarini toping.

Yechish: Funksiyaning ekstremum nuqtalarini topish uchun yuqorida ta'kidlaganimizdek, funksiyaning hosilasini topib uni nolga tenglashtiramiz. $y'=(\frac{x^3}{3}-\frac{5x^2}{2}+6x-7)'=x^2-5x+6$ hosilani hisobladik, endi nolga tenglashtirib tenglama ishlaymiz va oraliqlar usuliga qo'yib $x=2$ maksimum nuqta va $x=3$ minimum nuqta ekanligiga guvoh bo'lamiz.

Mustaqil yechish uchun misollar:

1-misol: Quyida berilgan funksiyalarning ekstremum nuqtalarini toping.

1.1. $y=e^{x^2-2x}$

1.2. $y=\ln(3x^2+2x+1)$

1.3. $y=x^2c^{-x}$

1.4. $y=\sqrt{12x-3x^3}$

1.5. $y=x+arctgx$

1.6. $y=x^4-2x^2+5$

1.7. $y=\frac{x}{x^2-6x-16}$

1.8. $y=x^3-3x^2-9x+7$

1.9. $y=\sqrt[3]{(x^2-6x+5)^2}$

1.10. $y=x-\ln(1+x)$

1.11. $y=x\ln^2 x$

1.12. $y=\sqrt[3]{(x^2-1)^2}$

1.13. $y=x^4+4x^3-2x+5$

1.14. $y=e^{3-6x-x^2}$

1.15. $y=x^{\frac{2}{3}}-x$

1.16. $y=x^3-6x^2+12x$

1.17. $y=\sqrt{3x-7}$

1.18. $y=2x^3-6x^2-18x+7$

1.19. $y=\frac{4x+1}{4x^2+x+8}$

1.20. $y=\frac{1}{10x^2+x+2x}$

1.21. $y=\sqrt{3x^2+4x+63}$

Javoblar:

1.1 $x_{min}=1$;

1.2 $x_{min}=-\frac{1}{3}$;

1.3 $x_{min}=0,\ x_{max}=\frac{2}{\ln c}$;

1.4 $x_{min}=-\frac{2}{\sqrt{3}},\ x_{max}=\frac{2}{\sqrt{3}}$;

1.5 ekstremumga ega emas.

1.7 ekstremumga ega emas.

1.8 $x_{min}=3,\ x_{max}=-1$;

1.9 $x_{min}=3$;

1.10 $x_{min}=0$;

1.11 $x_{min}=0,\ x_{max}=\frac{1}{e^2}$;

1.12 $x_{min}=0$;

1.14 $x_{max}=-3$;

1.15 $x_{max}=\frac{8}{27}$;

1.16 ekstremum nuqtaga ega emas.

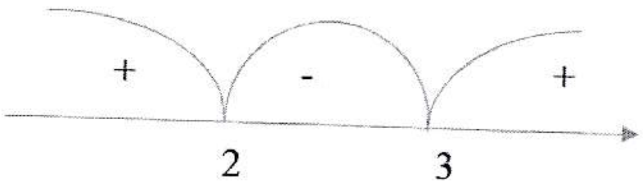
1.17 ekstremum nuqtaga ega emas.

1.18 $x_{min}=3,\ x_{max}=-1$.

Mavzu: Funksiya hosilasining tadbiqlariga oid misollar yechish

1-misol. $y=\frac{x^3}{3}-\frac{5x^2}{2}+6x-7$ funksiyaning ekstemumlarini va o'sish, kamayish oraliqlarini aniqlang.

Yechish: funksiyaning ekstremumlari va o'sish, kamayish oraliqlarini topish uchun, avvalambor funksiyadan hosila olamiz va nollarini topamiz. $y'=(\frac{x^3}{3}-\frac{5x^2}{2}+6x-7)'=x^2-5x+6=0$ kvadrat tenglamani ishlaymiz. $x^2-5x+6=0$, $D=25-24=1$, $x_1=\frac{5+1}{2}=3$, $x_2=2$ oraliqlar usuliga qo'yamiz va



Bundan ko'rinadiki $(-\infty;2)$ $(3;\infty)$ oraliqda funksiya o'suvchi, $(2;3)$ oraliqda esa funksiya kamayuvchi. $x_1=2$ nuqta maksimum nuqta, $x_2=3$ minimum nukta.

2-misol. $y=e^{2x^2-4x+7}$ funksiyaning ekstremum nuqtalari va o'sish kamayish oraliqlarini toping.

Yechish: bu misolni yechish uchun ham albatta hosila olamiz. $y'=(e^{2x^2-4x+7})'=e^{2x^2-4x+7}\cdot(2x^2-4x+7)'=(4x-4)\cdot e^{2x^2-4x+7}=0$ tenglamani ishlaymiz va argumentning qaysi qiymatlarida hosila musbat va qaysi qiymatlarida manfiy ekanligini aniqlaymiz, $(4x-4)\cdot e^{2x^2-4x+7}=0, x=1$. Har qanday musbat sonning ixtiyoriy darajasi musbat son bo'ladi. Shuning uchun tenglamani 1 дан farqli boshqa yechimlari mavjud emas. Shu bilan birga funksiya $(-\infty;1)$ oraliqda kamayuvchi, $(1;\infty)$ oraliqda esa o'suvchi bo'ladi. Shuningdek $x=1$ nuqta funksiyaning minimum nuqtasi hisoblanadi.

3-misol. $y=\ln(2x^2+2x+1)$ funksiyaning ekstremumlarini, o'sish va kamayish oraliqlarini aniqlang.

Yechish:

$$y'=(\ln(2x^2+2x+1))'=\frac{1}{2x^2+2x+1}\cdot(4x+2)=\frac{2(2x+1)}{2x^2+2x+1}=0$$

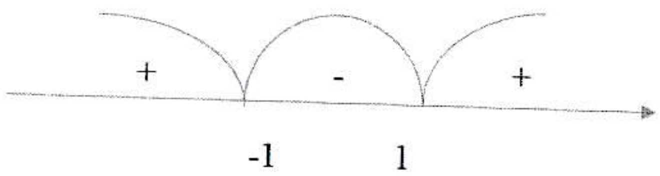
tenglamani ishlaydigan bo'lsak, $x=-\frac{1}{2}$ ekanligini topamiz. Ifodaning maxraji diskriminanti manfiy bo'lganligi uchun va kvadrat hadning koeffitsiyenti musbat bo'lganligi uchun ifodaning maxraji o'zgaruvchining barcha qiymatlarida musbat bo'ladi. Bundan esa $(-\infty;-\frac{1}{2})$ oraliqda funksiya kamayuvchi va $(-\frac{1}{2};\infty)$ oraliqda esa o'suvchi, $-\frac{1}{2}$ minimum nuqtasi bo'ladi.

4-misol. $y=x+arctgx$ funksiyaning ekstremumlarini, o'sish va kamayish oraliqlarini toping.

Yechish: $y'=(x+arctgx)'=1+\frac{1}{1+x^2}$ hosilani nolga tenglashtiramiz. $y'=1+\frac{1}{1+x^2}=0$, $\frac{1}{1+x^2}=-1$ bu tenglamani yechimi mavjud emas. Sababi o'zgaruvchining barcha qiymatlarida hosila musbat bo'ladi. Bu esa funksiya argumentning barcha qiymatlarida o'suvchi.

5-misol. $y=x^3-3x$ funksiyaning $[0,2]$ oraliqdagi eng katta va eng kichik qiymatlarini toping.

Yechish: ushbu misolni yechish uchun funksiyadan hosila olamiz va nollarini topamiz. Keyin sonli oraliqqa qo'yib minimum va maksimum nuqtalarini topamiz. Agar minimum yoki maksimum nuqtalar oraliqqa tegishli bo'lsa u holda ekstremum nuqtalardagi qiymatlari hisoblanadi va oraliqning chetki nuqtalaridagi qiymatlari ham hisoblanadi va qaysi nuqtadagi qiymati katta bo'lsa shu qiymat oraliqdagi eng katta qiymat hisoblanadi. Qaysi nuqtadagi qiymat eng kichik bo'lsa u holda bu qiymat eng kichik qiymat hisoblanadi. $y'=(x^3-3x)'=3x^2-3=0$ $x_1=1$, $x_2=-1$ tenglama nollarini son o'qiga joylab quyidagini topamiz.



$x_2=-1$ nuqta funksiyaning maksimum niqtasi va $x_1=1$ nuqta esa funksiyaning minimum nuqtasidir. Endi shu va oraliqning chetki nuqtalarida funksiyani tekshiramiz. $y(0)=x^3-3x=0$, $y(1)=x^3-3x=1^3-3=-2$ va $y(2)=x^3-3x=2^3-3\cdot 2=2$ bulardan ko'rinadiki, funksiyaning oraliqdagi eng katta qiymat 2 va eng kichik qiymati –2 ga teng.

6-misol. $y=\frac{x^3}{3}-\frac{5x^2}{2}+7x+4$ funksiya abchissalar o'qining qaysi nuqtasida o'tkazilgan urinmasi absissalar o'qi bilan tashkil qilgan burchagi $\frac{\pi}{4}$ ga teng bo'ladi?

Yechish: ma'lumki funksiyaning grafigiga biror nuqtada o'tkazilgan urinma absissalar o'qi bilan tashkil qilgan burchagining tangensi funksiyaning shu nuqtadagi hosilasining qiymatiga teng. Shuni inobatga olib funksiyadan hosila olamiz va burchak tangensiga tenglashtiramiz.

$$y'=(\frac{x^3}{3}-\frac{5x^2}{2}+7x+4)'=x^2-5x+7\,,\quad tg\frac{\pi}{4}=x^2-5x+7\,,\quad x^2-5x+7=1\,,$$

$x^2-5x+6=0$ tenglamani yechamiz. $D=25-24=1$ $x_1=\frac{5-1}{2}=2$, $x_2=3$. Bundan ko'rinadiki funksiya grafigiga $x_1=2$, $x_2=3$ nuqtalarda o'tkazilgan urinmalarning absissalar o'qi bilan tashkil qilgan burchagi $\frac{\pi}{4}$ ga teng bo'ladi.

7-misol. $y=3x^2-4x+6$ egri chiziqqa qaysi nuqtada o'tkazilgan urinma va $8x-y-5=0$ to'g'ri chiziqlar o'zaro parallel bo'ladi?

Yechish: funksiya grafigiga nuqtada o'tkazilgan urinmaning burchak koefitsienti bilan to'g'ri chiqning burchak koefitsiyentlari o'zaro teng bo'lsa u holda bu urinma va to'g'ri chiziqlar o'zaro parallel deyiladi. Shuning uchun egri chiziq tenglamasidan hosila olamiz va to'g'ri chiziqning burchak koefitsiyenti bilan tengglashtirib tenglama ishlaymiz va urinma o'tkazilgan nuqta kelib chiqadi. $y'=(3x^2-4x+6)'=6x-4$, $6x-4=8$, $6x=12$, $x=2$ bundan ko'rinadiki $x=2$ nuqtada funksiya grafigiga o'tkazilgan urinma va $8x-y-5=0$ to'g'ri chiziqlar o'zaro parallel bo'ladi.

8-misol. $y=\ln x$ funksiya grafigiga $x_0=1$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

Yechish: funksiya grafigiga o'tkazilgan urinma tenglamasi formulasini yozaylik, $f(x)=f(x_0)+f'(x_0)(x-x_0)$. Bundan ko'rinadiki urinma tenglamasini tuzish uchun $f(x_0)=\ln 1=0$ va $f'(x_0)=\frac{1}{x}=\frac{1}{1}=1$ endi urinma tenglamasini tuzamiz. $f(x)=0+1(x-1)=x-1$

Mustaqil yechish uchun misollar:

1.1. $y=\frac{x^3}{3}-\frac{5x^2}{2}+6x-7$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.2. $y=e^{x^2-2x}$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.3. $y=\ln(3x^2+2x+1)$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.4. $y=x^2e^{-x}$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.5. $y=\sqrt{12x-3x^3}$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.6. $y = x + arctgx$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.7. $y = x^4 - 2x^2 + 5$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.8. $y = x^3 - 3x^2 - 9x + 7$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.9. $y = x - \ln(1 + x)$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.10. $y = x\ln^2 x$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.11. $y = x^4 + 4x^3 - 2x + 5$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.12. $y = e^{3-6x-x^2}$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.13. $y = x^{\frac{2}{3}} - x$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.14. $y = x^3 - 6x^2 + 12x$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.15. $y = \sqrt{3x - 7}$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.16. $y = 2x^3 - 6x^2 - 18x + 7$ funksiyaning ekstremum nuqtalari va o'sish, kamayish orliqlarini toping.

1.17. $y = x^4 - 4x^3 + x - 1$ funksiyaga $x = 1$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.18. $y = 2x^3 + \ln x$ funksiyaga $x = 1$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.19. $y = e^{-x^2/2}$ funksiyaga $x = 2$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.20. $y = \ln(1 + x^2)$ funksiyaga $x = 0$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.21. $y = \dfrac{1}{4 - x^2}$ funksiyaga $x = 1$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.22. $y = \dfrac{2x^2}{1 + x^2}$ funksiyaga $x = 0$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.23. $y = xarctgx$ funksiyaga $x = 1$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.24. $y = arctgx - x$ funksiyaga $x = 0$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.25. $y = \dfrac{1 + x^2}{1 - x^2}$ funksiyaga $x = 2$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.26. $y = (3x + 6)e^{x/3}$ funksiyaga $x = 3$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.27. $y = \dfrac{x^2}{x - 2}$ funksiyaga $x = 3$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.28. $y = xe^x$ funksiyaga $x = 1$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.29. $y = \dfrac{x^3}{x^2 + 12}$ funksiyaga $x = 0$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.30. $y = \dfrac{e^x}{x}$ funksiyaga $x = 1$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.31. $y = x^5 - 10x^2 + 7x$ funksiyaga $x = 1$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

1.32. $y = x^3 - 3x$ funksiyaning $[0,2]$ oraliqdagi eng katta va eng kichik qiymatini toping.

1.33. $y = x^4 - 8x^2$ funksiyaning $[1,3]$ oraliqdagi eng katta va eng kichik qiymatini toping.

1.34. $y = x^5 - x^3 - 2x + 1$ funksiyaning $[-2,0]$ oraliqdagi eng katta va eng kichik qiymatini toping.

1.35. $y = x - \sqrt{x}$ funksiyaning $[0,1]$ oraliqdagi eng katta va eng kichik qiymatini toping.

1.36. $y = x - \ln x$ funksiyaning $[1/e, e]$ oraliqdagi eng katta va eng kichik qiymatini toping.

1.37. $y = (x^2 + 3x + 3)e^{-x}$ funksiyaning $[-4,0]$ oraliqdagi eng katta va eng kichik qiymatini toping.

1.38. $y = 2x^3 + 3x^2 - 12x + 1$ funksiyaning $[-1,5]$ oraliqdagi eng katta va eng kichik qiymatini toping.

1.39. $y = x + \sqrt[3]{x}$ funksiyaning $[-1,1]$ oraliqdagi eng katta va eng kichik qiymatini toping.

1.40. $y = 2x - \sqrt{x}$ funksiyaning $[0,4]$ oraliqdagi eng katta va eng kichik qiymatini toping.

1.41. $y = x^2 - 4x + 1$ funksiyaning $[-3,3]$ oraliqdagi eng katta va eng kichik qiymatini toping.

1.42. $y = tgx - x$ funksiyaning $[-\pi/4, \pi/4]$ oraliqdagi eng katta va eng kichik qiymatini toping.

1.43. $y = x^4 - 8x^2 + 3$ funksiyaning $[-2,2]$ oraliqdagi eng katta va eng kichik qiymatini toping.

1.44. $y = x^3 - 3x$ funksiyaning $[0,2]$ oraliqdagi eng katta va eng kichik qiymatini toping.

Javoblar:

1.1 $(-\infty,2]$ $[3;\infty)$ oraliqda o'suvchi, $[2,3]$ kamayuvchi.

1.2 $(-\infty,1]$ kamayuvchi, $[1,\infty)$ o'suvchi.

1.3 $(-\infty,-\frac{2}{6}]$ kamayuvchi, $[-\frac{1}{3},\infty)$ o'suvchi.

1.4 $(-\infty;-\frac{2}{\sqrt{3}}]$ $[\frac{2}{\sqrt{3}};\infty)$ kamayuvchi, $[-\frac{2}{\sqrt{3}};\frac{2}{\sqrt{3}}]$ o'suvchi.

1.6 $(-\infty;\infty)$ o'suvchi.

1.7 $(-\infty;-1]$ $[0,1]$ kamayuvchi, $[-1;0]$ $[1;\infty)$ o'suvchi.

1.8 $(-\infty;-1]$ $[3;\infty)$ o'suvchi, $[-1;3]$ kamayuvchi.

1.9 $(-\infty;-1)$ $[0;\infty)$ o'suvchi, $(-1;0]$ kamayuvchi.

1.10 $(-\infty;\frac{1}{e^2}]$ $[1;\infty)$ o'suvchi, $[\frac{1}{e^2};1]$ kamayuvchi.

1.12 $(-\infty;-3]$ o'suvchi, $[-3;\infty)$ kamayuvchi.

1.13 $(-\infty;0)$ $(\frac{8}{27};\infty)$ kamayuvchi, $(0;\frac{8}{27})$ o'suvchi.

1.14 $(-\infty;\infty)$ o'suvchi.

1.15 $(-\infty;\infty)$ o'suvchi.

1.16 $(-\infty;-1]$ $[3;\infty)$ o'suvchi, $[-1;3]$ kamayuvchi.

1.17 $y = -7x + 4$.

1.18 $y = 7x - 5$.

1.19 $y = -\frac{2}{e^2}x + \frac{5}{e^2}$.

1.20 $y = 0$.

1.21 $y = \frac{2}{9}x + \frac{1}{9}$.

1.22 $y = 0$.

1.23 $y = \frac{\pi + 2}{4}x - \frac{1}{2}$.

1.24 $y = 0$.

1.25 $y = \frac{8}{9}x - \frac{23}{9}$.

1.26 $y = 8ex - 9e$.

1.27 $y = 18 - 3x$.

1.28 $y = 2ex - e$.

1.29 $y=0$.

1.30 $y=e$.

1.31 $y=-8x+6$.

1.32 $y_{\min}(1)=-2,\ y_{\max}(2)=2$.

1.33 $y_{\min}(2)=-16,\ y_{\max}(3)=9$.

1.34 $y_{\min}(-2)=-19,\ y_{\max}(-1)=3$.

1.35

$y_{\min}(\frac{1}{4})=-\frac{1}{4},\ y_{\max}(0)=y_{\max}(1)=0$.

1.36 $y_{\min}(1)=1,\ y_{\max}(e)=e-1$.

1.37 $y_{\min}(-1)=e,\ y_{\max}(-4)=7e^{4}$.

1.38 $y_{\min}(1)=-6,\ y_{\max}(5)=266$.

1.39 $y_{\min}(-1)=-2,\ y_{\max}(1)=2$.

1.40

$y_{\min}(0)=y_{\min}(\frac{1}{4})=0,\ y_{\max}(4)=6$.

1.41. $y_{\min}(2)=-3,\ y_{\max}(-3)=22$.

Funksiya hosilasining fizika, mexanika va iqtisodiy masalalardagi tadbiqlari.

Funksiya hosilasining fizika fanida tutgan o'rnini ta'kidlash maqsadida ba'zi bir tadbiqlarini keltirib o'tamiz.

1. Agar $Q=Q(t)$ o'tkazgichning ko'ndalang kesimi orqali vaqtning t onida o'tuvchi elektr toki bo'lsa, u holda elektr tokining tondagi momenti

$$I(t)=\lim_{\Delta t\to 0}\frac{\Delta Q}{\Delta t}=\lim_{\Delta t\to 0}\frac{Q(t+\Delta t)-Q(t)}{\Delta t}.$$

2. Agar $m=m(x)$ bir jinsli bo'lmagan sterjenning $O(0,0)$ va $M(x,0)$ nuqtalar orasidagi massasi bo'lsa, u holda sterjenning x nuqtadagi zichligi

$$\lim_{\Delta t\to 0}\frac{m(t+\Delta t)-m(t)}{\Delta t}.$$

3. Induktiv g'altakda hosil bo'luvchi o'z induksiya elektr yurituvchi kuchi g'altakdan o'tuvchi tok kuchining tezlanishi ya'ni tok kuchidan vaqt bo'yicha birinchi tartibli hosilasiga to'g'ri proporsionaldir.

$$\varepsilon=-L\cdot\frac{dI}{dt}$$

4. Magnit maydonidan ayriluvchi simli ramkada hosil bo'luvchi elektr yurituvchi kuch,

$$\varepsilon=-N\frac{d\varphi}{dt}..$$

Magnit ramkasi kesib o'tuvchi magnit oqimining o'zgarish tezligiga proporsionaldir.

$$\Phi=BS\cos\cos(\omega t+\varphi_0)$$

$$\text{Ya'ni}\ \frac{d\Phi}{dt}=\left(BS\cos\cos\left(\omega t+\varphi_0\right)\right)'=-BS\omega\sin\sin(\omega t+\varphi_0).$$

Bulardan tashqari funksiya hosilasining nafaqat fizikaning yuqoridagi kabi misollarida balki hayotiy misollarini yechishda ham o'rni bor, ya'ni moddiy nuqtaning harakat troyektoriyasi (bosib o'tgan yo'li) biror formula bilan berilgan bo'lsa u holda hammamizga ma'lumki yo'ldan olingan hosila moddiy nuqtaning momentdagi tezligiga teng. Tezlik formulasidan olingan hosila esa moddiy nuqtaning momentdagi tezlanishiga teng.

1-misol. Moddiy nuqta $S(t)=lnt+\frac{1}{16}t$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Harakat boshlangandan keyin qancha vaqt o'tgach nuqtaning tezligi $\frac{1}{8}м/c$ ga teng bo'ladi.

Yechish: Yuqorida ko'rib chiqqanlarimizdan ma'lumki moddiy nuqtaning harakat troyektoriyasi formula bilan berilgan bo'lsa, u holda bu nuqtaning ma'lum vaqtdagi tezligini topish uchun yo'l formulasidan hosila olish kerak. Ushbu masalada ham tezlikni topish uchun yo'l formulasidan hosila olamiz. $v(t)=S(t)'=\frac{1}{t}+\frac{1}{16}$ tezlik formulasini keltirib chiqargach, uni $\frac{1}{8}$ ga tenglashtiramiz. $\frac{1}{t}+\frac{1}{16}=\frac{1}{8}\ \Rightarrow\ \frac{1}{t}=\frac{1}{8}-\frac{1}{16},\quad \frac{1}{t}=\frac{1}{16}\leftrightarrow t=16$ bundan ko'rinadiki 16 sekund vaqt o'tgach moddiy nuqtaning tezligi $\frac{1}{8}м/c$ ga teng bo'ladi.

2-misol. Ikki modniy nuqta $S_1(t)=2t^3-5t^2-3t$ va $S_2(t)=2t^3-3t^2-11t+7$ qonuniyatlar bo'yicha harakatlanyapti. Bu ikki nuqtaning tezliklari teng bo'lgan paytda birinchi nuqtaning tezlanishini toping.

Yechish: bu masalani yechish uchun har ikki moddiy nuqtaning harakat formulasidan hosila olamiz va tenglashtiramiz. $S_1'=6t^2-10t$, $S_2'=6t^2-6t-11$

$\Rightarrow\ 6t^2-10t=6t^2-6t-11$; ushbu tenglamani ishlaymiz. $4t=11\ \Rightarrow$; $t=\frac{11}{4}$

vaqt $\frac{11}{4}$ ga teng bo'lgan paytda bu ikkita moddiy nuqtaning tezliklari o'zaro teng bo'ladi. Yuqoridagi masalada vaqt $\frac{11}{4}$ ga teng bo'lgan holda birinchi moddiy nuqtaning tezlanishi so'ralgan. Buning uchun yana funksiya hosilasidan foydalanamiz, ya'ni birinchi funksiya hosilasidan kelib chiqqan funksiyadan yana hosila olamiz. Chunki moddiy nuqta harakat troyektoriyasi formulasidan ikki marta olingan hosila moddiy nuqta tezlanishini beradi:

$S''(t)=12t-10S''\left(\frac{11}{4}\right)=12\cdot\frac{11}{4}-10=23$

3-misol. Moddiy nuqta to'g'ri chiziq bo'ylab $S(t)=-\frac{1}{12}t^4+\frac{2}{3}t^3-\frac{3}{2}t^2$ qonuniyat bo'yicha harakatlanyapti. Harakat boshlangandan qancha sekund o'tgach, uning tezlanishi eng katta bo'ladi?

Yechish: tabiiyki bu masalani yechish uchun yo'l formulasidan ikki marta hosila olamiz. $S'=-\frac{1}{3}t^3+2t^2-3t$; $S''=-t^2+4t-3$ tezlanish formulasini topgach, buning eng katta qiymatini topish uchun yana hosila olish mumkin yoki kvadrat funksiyaning hossalaridan foydalanishimiz mumkin.

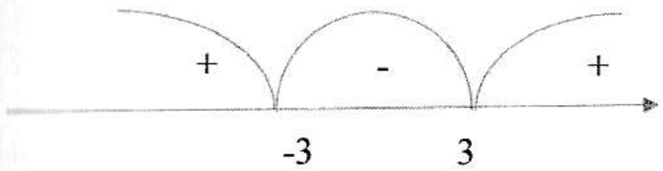
$(-t^2+4t-3)'=0$, $-2t+4=0\ \Rightarrow\ t=2$ demak $t=2$ bo'lganda moddiy nuqta eng katta tezlanishga ega bo'ladi.

Endi iqtisodiy masalalardagi tadbiqlarini ko'rib chiqamiz.

4-misol. AQSH dagi qaysidir supermarketning kunlik savdo natijalari taxminan $y=100x^3-2700x-700$ AҚШ dollarini tashkil qiladi (bu yerda x

o'zgaruvchi miqdor vaqt va u ertalabgi soat 8 dan kechqurungi soat 24 ga qadar o'zgaradi ya'ni o'zgarish oralig'i [1;16] gacha). Ushbu supermarketda kunning qaysi qismida savdo ko'rsatkichi eng katta bo'ladi va qaysi qismida savdoning o'sish tezligi eng katta bo'ladi?

Yechish: ushbu misolni yechish uchun yuqoridagi funksiyadan hosila olamiz. $y'=(100x^3-2700x-700)'=300x^2-2700$ va nolga tenglashtiramiz. $300x^2-2700=0\ \Rightarrow\ 300x^2=2700\ \Rightarrow\ x^2=9\ \Rightarrow\ x_{1,2}=\pm3$ tenglamini yechimini sonlar o'qiga joylashtirib, oraliqlar usulidan foydalanamiz.



Oraliqlardan ko'rinadiki $x=3$ минимум нуқтаси ва $x=-3$ nuqta esa maksimum nuqta bo'lib, vaqt manfiyda hisoblanmaydi. Shuning uchun oraliqning chekka nuqtalaridagi qiymatlari va minimum nuqtadagi qiymatini hisoblab qo'yamiz. Bundan tashqari $(3;\infty)$ oraliqda funksiya o'suvchi va shuning uchun [1;16] oraliqning yuqori nuqtasida funksiya eng katta qiymatga erishadi va 365700 AҚШ dollarini tashkil qiladi.

IV BOB. BOSHLANG'ICH FUNKSIY, ANIQMAS INTEGRAL VA ANIQ INTEGRALLAR.

Mavzu: Aniqmas integral va uni topishning sodda usullari.

Reja

1. Boshlang'ich funksiya va aniqmas integral tushunchalari. Aniqmas integralning xossalari. Aniqmas integrallar jadvali.
2. Integrallash usullari: o'zgaruvchilarni almashtirish va bo'laklab integrallash.
3. Ba'zi trigonometrik funksiyalarni integrallash.

Boshlang'ich funksiya va aniqmas integral tushunchalari. Aniqmas integralning xossalari. Aniqmas integrallar jadvali

Yuqorida harakatning tenglamasi, ya'ni vaqt o'tishi bilan yo'lning o'zgarish qonuni berilgan deb faraz etib, avval tezlikni so'ngra tezlanishni topdik. Biroq amalda ko'proq teskari masalani yechishga to'g'ri keladi: ya'ni / tezlanish t vaqtning funksiyasi $a=a(t)$ sifatida berilgan bo'lib, t vaqt o'tilgan s yo'lni va tezlik v ni topish talab etiladi. Shunday qilib bu yerda hosilasi $a(t)$ bo'lgan $v=v(t)$ funksiyani $a=a(t)$ funksiyadan topib, so'ngra hosilasi v bo'lgan $S=S(t)$ funksiyani topish kerak.

Shunga o'hshash, x o'qning $[0,x]$ to'g'ri chiziqli kesmasi bo'ylab uzliksiz taqsimlangan $m=m(x)$ q $\rho=\rho(x)$ "чизиқли" зичликни топиш мумкин. Лекин tabiiy shunday savol tug'iladi; zichlikning berilgan $\rho=\rho(x)$ o'zgarish qonuni bo'yicha taksimlangan massaning miqdorini topish, ya'ni berilgan $\rho(x)$ funksiya bo'yicha hosilasi $\rho(x)$ bo'lgan $m=m(x)$ funksiyani topish talab etiladi.

Agar berilgan $\rho=X$ oraliqning barcha nuqtalarida $f(x)$ funksiya $F(x)$ ning hosilasi, ya'ni $F'(x)=f(x)$ bo'lsa, u holda $F(x)$ funksiya berilgan oraliqda $f(x)$ funksiyaning boshlang'ich funksiyasi yoki aniqmas integrali deyiladi.

Teorema. Agar biror (chekli yoki cheksiz, ochiq yoki yopiq) X oraliqda $F(x)$ funksiya $f(x)$ funksiyaning boshlang'ich funksiyasi bo'lsa, u holda $F(x)+C$ ham (bu yerda C ihtiyoriy o'zgarmas son) boshlang'ich funksiya bo'ladi. Aksincha, X oraliqda $f(x)$ ning har bir boshlang'ich funksiyasini shu ko'rinishda yozish mumkin.

Isbot: $F(x)$ funksiya bilan birgalikda $F(x)+C$ funksiya ham $f(x)$ ning boshlang'ich funksiyasi ekanligini ko'rsatish uchun har ikkala funksiyadan hosila olishning o'zi yetarli va bulardan ko'rinadiki $(F(x)+C)'=F(x)'=f(x)$ bo'ladi bu esa teoremani isbotlaydi.

Ushbu teoremadan berilgan $f(x)$ funksiyaning hamma boshlang'ich funksiyalarini topish uchun faqat bitta boshlang'ich funksiyani topish yetarli ekanligi kelib chiqadi, chunki ular bir biridan o'zgarmas qo'shiluvchigagina farq qiladilar.

Bunga ko'ra $F(x)+C$ ifoda, bu yerda C ihtiyoriy o'zgarmas son, $f(x)$ hosilaga ega bo'lgan funksiyaning umumiy ko'rinishi bo'ladi. Bu ifoda $f(x)$ ning aniqmas integrali deyiladi va $\int f(x)dx$ kabi belgilanadi.

Yuqoridagi ta'rif va teoremalarni o'quvchilarga tushuntirgach boshlang'ich funksiya yoki aniqmas integral hisoblashning ba'zi hossa va qoidalar tushuntirilishi shart. Buning sababini har bir xossa va qoidalarni tushuntirib berayotganda yoritib o'tamiz.

1^0 $d\int f(x)dx=f(x)dx$ ya'ni d va $\int$ belgilar birinchisi ikkinchisidan oldin yozilgan bo'lsa, o'zaro qisqaradi. Ya'ni integrallashdan olingan hosila shu integral belgisi ostidagi funksiyaning o'ziga teng bo'ladi.

2^0 $F(x)$ funksiya $F'(x)$ ning boshlang'ich funksiyasi bo'lgani uchun $\int F(x)'dx=F(x)+C$ ga ega bo'lamiz, buni $\int dF(x)=F(x)+C$ ko'rinishda ham yozish mumkin. Bundan $F(x)$ oldidagi d va $\int$ belgilar d belgi $\int$ dan keyin kelsa ham qisqaradi, faqat bunda $F(x)$ ga ixtiyoriy o'zgarmas son qo'shish kerak.

Aniqmas integral hisoblashning ba'zi sodda qoidalari.

I. Agar a o'zgarmas son bo'lsabu holda $\int af(x)dx = a\int f(x)dx$. Ya'ni o'zgarmas ko'paytuvchini integral belgisi ostidan chiqarish mumkin.

II. $\int (f(x) \pm g(x))dx = \int f(x)dx + \int g(x)dx$ arifmetik yig'indining integrali integrallar arifmetik yig'indisiga teng.

Bu ikkala formula haqida quyidagini aytib o'taylik. Bularga ikkita har biri ihtiyoriy o'zgarmasni o'z ichiga oladigan aniqmas integral kiradi. Bu tipdagi tenglikni o'ng va chap tomoni orasidagi ayirma o'zgarmasga teng degan ma'noda tushuniladi. Bu tengliklarni asl ma'noda tushunish ham mumkin lekin u vaqtda bunda ishtirok etgan integrallardan bittasi ihtiyoriy boshlang'ich funksiya bo'lmay qoladi: tenglikdagi o'zgarmas son boshqa integrallardagi o'zgarmaslarni topilgandan keyin aniqlanadi.

III. Агар $\int f(t)dt = F(t) + C$ bo'lsa, u holda: $\int f(ax+b)dx = \frac{1}{a}F(ax+b) + C'$ bo'ladi.

Misol. $\int (6x^2 - 3x + 5)dx$ aniqmas integralni hisoblang.

Yechish: bu misolni yechish uchun yuqoridagi qoida va xossalardan foydalanamiz:

$$\int (6x^2 - 3x + 5)dx = \int 6x^2dx - \int 3xdx + \int 5dx = 6\int x^2dx - 3\int xdx + 5\int dx =$$

$$2x^3 - \frac{3}{2}x^2 + 5x + C.$$

Quyida biz asosiy elementar fuksiyalarning aniqmas integrallarini hisoblash formulalarining hosila yordamidagi isbotlari tahlilini ko'rib chiqamiz.

Masalan: 1) $\int 0dx = C$ bu formulani isbotlash uchun yuqorida ta'kidlaganimizdek funksiya uchun hosila formulasidan foydalanamiz. YA'ni o'zgarmas sonning hosilasi 0 ga teng bo'ladi.

2) keyingi elementar funksiyalarimizdan biri bu o'zgarmas sondir, o'zgarmas sonning boshlang'ich funksiyasi $\int kdx = kx + C$ ga teng bo'ladi. Shuni isbotlasak. $(kx + C)' = (kx)' + C' = k$ demak haqiqatdan ham formula o'rinli ekan.

3) navbatdagi elementar funksiyalardan biri bu darajali funksiya, ya'ni darajasi ixtiyoriy son bo'lgan elementar funksiyadir. Bu funksiyaning boshlang'ich funksiyasi $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C$ ga ya'ni, funksiya darajasiga birni qo'shib shu darajaga bo'linadi. Bu formulaning isboti ham yuqoridagi kabi hosila olish yordamida amalga oshiraladi.

4) keyingi elementar funksiyalardan yana biri bu darajasi -1 ga teng bo'lgan funksiyadir ya'ni, $\int \frac{1}{x}dx = \ln x + C$ formula o'rinli. Bu formulaning isboti ham yuqoridagi kabi hosila olish orqali isbotlanadi. Yuqoridagi va ushbu formulalarni talabalariga mustaqil ish sifatida tagdim qilish mumkin. Huddi shu kabi quyidagi

5) $\int \frac{1}{1+x^2}dx = arctgx + C$ 6) $\int \frac{1}{\sqrt{1-x^2}}dx = \int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$

7) $\int \sin xdx = -\cos x + C$ 8) $\int \cos xdx = \sin x + C$ 9) $\int \frac{1}{\cos^2 x}dx = tgx + C$

formulalarni ham mustaqil ish sifatida talabalarga berib o'tamiz va quyida bir qancha murakkabroq bo'lgan formulalarga to'xtalamiz.

10) $\int a^x dx = \frac{a^x}{\ln a} + C$ bu formulani isbotlashda $\frac{a^x}{\ln a} + C$ funksiyadan hosila olamiz. $(\frac{a^x}{\ln a} + C)'$ ushbu ifodada $\frac{1}{\ln a}$ o'zgarmas son bo'lganligi uchun, hosilada o'zgarmas sonni kavsdan tashqariga chiqarib hisoblash mumkin degan qoidadan kavsdan tashqariga chiqarib yuboramiz. $(\frac{a^x}{\ln a} + C)' = \frac{1}{\ln a}(a^x + c_1)'$ бу ерда $c_1 = C\ln a$ ga teng. Endi yana hosila olish qoidasidan yig'indining har bir hadidan hosila olamiz va

$\frac{1}{\ln a}(a^x + c_1)' = \frac{1}{\ln a} \cdot a^x \cdot \ln a + \frac{1}{\ln a} \cdot 0 = a^x$ bundan esa formulaning o'rinli ekanligi kelib chiqadi.

1-misol. $\int (3x-5)^2 dx$ aniqmas integralni hisoblang.

Yechish: Ushbu aniqmas integralni hisoblash uchun integral ostidagi ifodani kavsni ochamiz va soddalashtirib keyin integrallaymiz. $(3x-5)^2 = 9x^2 - 30x + 25$ ga teng. Endi bu funksiyaning boshlang'ich funksiyasini topamiz:

$$\int (3x-5)^2 dx = \int (9x^2 - 30x + 25)dx = \int 9x^2 dx - \int 30x dx +$$

$$\int 25 dx = \frac{9x^2}{3} - \frac{30x^2}{2} + 25x + C = 3x^3 - 15x^2 + 25x + C.$$

2-***мисол.*** $\int \frac{\sqrt{x^4 + x^{-4} + 2}}{x^3} dx$ аниқмас интегрални ҳисобланг.

Yechish: ushbu aniqmas integralni hisoblash uchun, integral ostidagi ifodani soddalashtiramiz: $\sqrt{x^4 + x^{-4} + 2} = \sqrt{\frac{x^8 + 2x^4 + 1}{x^4}} = \sqrt{\frac{(x^4+1)}{x^4}} = \frac{x^4+1}{x^2}$.

Soddalashtirgach integralga qo'yib aniqmas integralni topamiz:

$$\int \frac{\sqrt{x^4 + x^{-4} + 2}}{x^3} dx = \int \frac{\frac{x^4+1}{x^2}}{x^3} dx = \int \frac{x^4+1}{x^5} dx = \int \left(\frac{1}{x} + \frac{1}{x^5}\right)dx = \int \left(\frac{1}{x} + x^{-5}\right)dx =$$

$$\ln x - \frac{1}{4x^4} + C.$$

3-misol. $\int \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2 dx$ aniqmas integralni hisoblang.

Yechish: $\int \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2 dx = \int \left(\sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2} + \cos^2\frac{x}{2}\right)dx =$

$$\int (1 + \sin x)dx = x - \cos x + C.$$

4-misol. $\int ctg^2 x dx$ aniqmas integralni hisoblang.

Yechish:

$$\int ctg^2 x dx = \int \frac{\cos^2 x}{\sin^2 x} dx = \int \frac{1-\sin^2 x}{\sin^2 x} dx = \int \left(\frac{1}{\sin^2 x} - 1\right)dx = -ctgx - x + C.$$

5-misol. $\int \frac{dx}{1+\cos 2x}$ aniqmas integralni hisoblang.

Yechish:

$$\int \frac{dx}{1+\cos 2x} = \int \frac{dx}{1+\cos^2 x - \sin^2 x} = \int \frac{dx}{2\cos^2 x} = \frac{1}{2}\int \frac{dx}{\cos^2 x} = \frac{1}{2} tgx + C.$$

6-misol. $\int \sin^2\frac{x}{4}\cos^2\frac{x}{4} dx$ aniqmas integralni hisoblang.

Yechish: $\int \sin^2\frac{x}{4}\cos^2\frac{x}{4} dx$ integralni hisoblash uchun trigonometrik funksiyalarning xossalaridan foydalanamiz. Ya'ni

$$\sin^2\frac{x}{4}\cos^2\frac{x}{4} = \frac{1}{4} \cdot 4 \cdot \sin^2\frac{x}{4}\cos^2\frac{x}{4} = \frac{1}{4}\left(2\sin\frac{x}{4}\cos\frac{x}{4}\right)^2 = \frac{1}{4}\sin^2\frac{x}{2} = \frac{1-\cos x}{8}$$

olingan natijaga ko'ra integralni hisoblaymiz.

$$\int \sin^2\frac{x}{4}\cos^2\frac{x}{4} dx = \int \frac{1-\cos x}{8} dx = \frac{1}{8}\int (1-\cos x)dx = \frac{1}{8}(x - \sin x) + C$$

Integrallash usullari: o'zgaruvchilarni almashtirish va bo'laklab integrallash

O'zgaruvchilarni almashtirish yo'li bilan integrallash: funksiyalarni integrallashda kuchli usullardan biri bo'lgan o'zgaruvchilarni almashtirish yoki o'rniga qo'yish usulini byon qilamiz. Buning asosida quyidagi sodda izoh yotadi.

Agar $\int g(t)dt = G(t) + C$ ekani ma'lum bo'lsa, u holda: $\int g(\omega(x))\omega'(x)dt = G(\omega(x)) + C$.

Bu to'g'ridan- to'g'ri murakkab funksiyani differensiallash usulidan kelib chiqadi: $(G(\omega(x)))' = G'(\omega(x)) \cdot \omega'(x) = g(\omega(x)) \cdot \omega'(x)$

Buni, $dG(t) = g(t)dt$ munosabat t erkli o'zgaruvchining $\omega(x)$ funksiya bilan almashtirganda ham o'z kuchini saqlaydi deb, yana boshqacha ifodalash mumkin. $\int f(x)dx$ integralni hisoblash talab etilsin, deylik. Ko'p hollarda yangi o'zgaruvchi sifatida x ning funksiyasini tanlash mumkin bo'ladi: $t = \omega(x)$ va bunda integral ostidagi ifoda $f(x)dx = g(\omega(x)) \cdot \omega'(x)dx$ shaklda yoziladi, bu yerdagi $g(t)$ funksiyani integrallash $f(x)$ ni integrallashga qaraganda qulayroq bo'ladi. U vaqtda yuqorida aytilgandek, $\int g(t)dt = G(t) + C$ integralni topish yetarli, unda $t = \omega(x)$ almashtirishni bajarib izlangan integral topiladi.

Masalan. $\int \sin^3 x \cos x dx$ integralni hisoblang.

Ushbu misolni ikki hil usulda hisoblash mumkin. Bu ikkala usul ham o'zgaruvchilarni almashtirish usuliga tayanadi.

1) Birinchi usul shundan iboratki integral belgisi ostidagi ifodaning bir qismini differensiallash belgisi ostiga kiritamiz. $\int \sin^3 x \cos x dx = \int \sin^3 x d(\sin x)$ bu ifodadan $t = \sin x$ belgilash kiritamiz. Bundan integral quyidagi ko'rinishga keladi. $\int \sin^3 x d(\sin x) = \int t^3 dt$ bu esa oddiy darajali funksiyaning integrali ekanligi ko'rinib turibdi. Demak $\int t^3 dt = \frac{t^4}{4} + C = \frac{\sin^4 x}{4} + C$ ekanligi kelib chiqadi va misol ishlandi.

2) Endi bu misolni ishlashning ikkinchi usuliga to'xtaladigan bo'lsak ikkinchi usulda xuddi shu qilingan ishlar bajariladi faqat differensiallash belgisi ostiga kiritilmaydi ya'ni, $t = \sin x$ belgilash kiritiladi va $dt = d(\sin x) = \cos x dx$ ekanligida bajargan belgilashlarimizni o'rniga qo'yib chiqamiz. $\int \sin^3 x \cos x dx = \int t^3 dt = \frac{t^4}{4} + C = \frac{\sin^4 x}{4} + C$ ekanligi kelib chiqadi. Bu ikki usul bir hil ma'noni anglatadi va farq qilmaydiganday tuyuladi lekin aslida ba'zida belgilash kiritganda ikkinchi usuli qo'l keladi. Sababi birinchi belgilash kiritish usulida integral ostidagi qaysi ifodani differensiallash belgisi ostiga kiritish kerakligini ajratib olish qiyin bo'lib qolishi mumkin.

Masalan. $\int \sqrt{1 - x^2} dx$ aniqmas integralni hisoblang.

Ushbu integralni hisoblashda x o'zgaruvchini $x = \sin t$ kabi belgilash kiritish qulayroq. Chunki irratsional ifodaning ostidan ma'lum bir ifoda chiqadi. Ya'ni $dx = d(\sin t) = \cos t dt$ belgilashlarni o'z o'rniga olib borib qo'yilsa u holda $\int \sqrt{1 - x^2} dx = \int \cos t \cdot \cos t dt = \int \cos^2 t dt$ ifodaga kelamiz va bu integralni trigonometrik funksiyalar xossalaridan foydalanib ya'ni daraja pasaytirib hisoblaymiz. $\int \cos^2 t dt = \int \frac{1 + \cos 2t}{2} dt$ bu ifodadan yuqoridagi xossalardan o'zgarmas sonni integral belgisi tashqarisiga chiqarish mumkinligidan, $\int \frac{1 + \cos 2t}{2} dt = \frac{1}{2} \int (1 + \cos 2t) dt = \frac{1}{2}(t + \frac{1}{2}\sin 2t + C)$ ekanligini topamiz. Oxirgi ifodada belgilash kiritilganligi uchun o'zgaruvchilarni o'z o'rniga qo'yishdan oldin soddalashtirib olamiz. $x = \sin t$ belgilash kiritilgan edi bundan $t = \arcsin x$ ekanligi kelib chiqadi. $\frac{1}{2}(t + \frac{1}{2}\sin 2t + C) = \frac{1}{2}t + \sin t \cos t + C = \frac{1}{2}\arcsin x + x\sqrt{1 - x^2} + C$ demak javob $\int \sqrt{1 - x^2} dx = \frac{1}{2}\arcsin x + x\sqrt{1 - x^2} + C$ ekanligi kelib chiqdi.

Ushbu usulni yana boshqa misollardagi talqinlarini ko'rib o'rganaylik.

1-Misol. $\int ctgx dx$ aniqmas integralni hisoblang.

Yechish: ushbu integralni hisoblash uchun quyidagicha belgilash va o'zgartirish kiritamiz:

$$\int ctgx dx = \int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} \cdot \cos x dx = \int \frac{1}{\sin x} d(\sin x) = \int \frac{1}{t} dt = \ln t + C =$$

$$\ln|\sin x| + C.$$

2-misol. $\int \frac{\arccos x}{\sqrt{1 - x^2}} dx$ aniqmas integralni hisoblang.

Yechish: bunday integrallarni hisoblashda qaysi qismini differensiallash belgisi ostiga kiritish kerakligini bilish lozim.

$$\int \frac{\arccos x}{\sqrt{1 - x^2}} dx = \int \arccos x \cdot \frac{1}{\sqrt{1 - x^2}} dx = \int \arccos x d(\arccos x) = \int t dt = \frac{t^2}{2} + C =$$

$$\frac{\arccos^2 x}{2}+C$$

3-misol. $\int\frac{\sqrt{1+\ln x}}{x}dx$ aniqmas integralni hisoblang.

Yechish:

$$\int\frac{\sqrt{1+\ln x}}{x}dx=\int\sqrt{1+\ln x}\cdot\frac{1}{x}dx=\int\sqrt{1+\ln x}d(1+\ln x)=\int\sqrt{t}dt=\int t^{\frac{1}{2}}dt=\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+C=$$

$$\frac{2}{3}t\sqrt{t}+C=\frac{2}{3}\ln x\sqrt{\ln x}+C.$$

4-misol. $\int\frac{dx}{(x+1)\sqrt{x}}$ aniqmas integralni toping.

Yechish:

$$\int\frac{dx}{(x+1)\sqrt{x}}=\int\frac{1}{(x+1)}\cdot\frac{1}{\sqrt{x}}dx=2\int\frac{d(\sqrt{x})}{(x+1)}=2\int\frac{dt}{(t^2+1)}=2arctgt+C=$$

$$2arctg\sqrt{x}+C.$$

Bo'laklab integrallash: faraz qilaylik $u=f(x)$ va $v=g(x)$ lar x ning ikkita funksiyasi va ular $u'=f'(x)$ va $v'=g'(x)$ uzluksiz hosilalarga ega bo'lsin. U vaqtda ikki funksiya ko'paytmani hosilasini olish qoidasiga ko'ra $(uv)'=u'v+uv'$ yoki $d(uv)=udv+vdu$ yoki $udv=d(uv)-vdu$ bo'ladi; $d(uv)$ ifoda uchun, shubhasiz, uv boshlang'ich funksiya bo'ladi: shuning uchun $\int udv=uv-\int vdu$ formula o'rinlidir.

Bu formula bo'laklab integrallash qoidasini ifodalaydi. Yuqoridagi keltirib chiqargan formulamiz bo'laklab integrallash formulasini beradi va bu formula ikki funksiya ko'paytmasining hosilasidan kelib chiqqan. Endi bu formulani misollarda tushuntirsak.

Masalan: $\int x\cos xdx$ aniqmas integralni hisoblang.

Bu integralni hisoblashda bo'laklab integrallash formulasidan foydalanamiz. Ya'ni $\cos x$ funksiyani differensiallash belgisi ostiga kiritamiz va integral quyidagi ko'rinishga keladi. $\int x\cos xdx=\int xd(\sin x)$ bu ifodada $u=x$, kabi belgilaymiz va yuqoridagi formuladan foydalanamiz. $\int xd(\sin x)=x\sin x-\int\sin xdx=x\sin x+\cos x+C$ ekanligi kelib chiqadi. Odatda integrallash belgisi ostidagi ifodada qaysi birini u funksiya va qaysi birini v funksichya sifatida qabul qilish mumkin kabi savollar paydo bo'ladi. Yuqoridagi kabi funksiyalarda odatda trigonometrik funksiyalarni differensiallash belgisi ostiga kiritib / funksiya siqatida olish mumkin. O'z-o'zidan ma'lumki darajasi oshib boruvchi funksiyalarni differensiallash belgisi ostiga kiritmagan ma'qul. Yani nomini o'zgartiruvchi yoki umuman o'zgarmaydigan funksiyalarni differensiallash belgisi ostiga kiritib / funksiya sifatida olgan ma'qul.

Misollar yordamida ko'rib chiqamiz.

1-misol. $\int\ln(x+8)dx$ aniqmas integralning hisoblang.

Yechish: $\int\ln(x+8)dx=\begin{cases}u=\ln(x+8) & du=\frac{1}{x+8}\\ dv=dx & v=x\end{cases}$ belgilashlar kiritamiz.

Endi bo'laklash formulasidan foydalanadigan bo'lsak, quyidagiga kelamiz.

$$\int\ln(x+8)dx=x\ln(x+8)-\int x\cdot d(\ln(x+8))=x\ln(x+8)-\int x\cdot\frac{1}{x+8}dx=x\ln(x+8)-$$

$$\int\frac{x+8-8}{x+8}dx=x\ln(x+8)-\int(1-\frac{8}{x+8})dx=x\ln(x+8)+8\ln(x+8)-x+C.$$

2-misol. $\int(x^2-3)\cos xdx$ aniqmas integralni hisoblang.

Yechish: ushbu misolni yechish uchun trigonometrik funksiyani differensiallash belgisi ostiga kiritib, keyin ikkita funksiyaga keltiriladi. $\int(x^2-3)\cos xdx=\int(x^2-3)d(\sin x)$ ko'rinishiga keltirib keyin quyidagicha belgilashlar kiritiladi.

$$\int(x^2-3)d(\sin x)=\begin{cases}u=x^2-3 & du=2xdx\\ dv=\cos xdx & v=\sin x\end{cases}$$

$\int (x^2-3)d(\sin x) = \sin x\cdot(x^2-3) - \int 2x\cdot\sin x dx$ integralimiz ko'rinishini o'zgartirdi endi integralning ikkinchi qismidagi integralni hisoblash uchun yana bir marta bo'laklash amalga oshiramiz. Bu bo'laklashda ham yuqoridagi kabi trigonometrik funksiyani differensiallash belgisi ostiga kiritamiz va yuqoridagi kabi belgilashni amalga oshiramiz.

$$\int 2x\cdot\sin x dx = -\int 2x d(\cos x) = -2x\cdot\cos x + 2\int \cos x dx = -2x\cos x + 2\sin x$$

ushbu topilgan oxirgi natija bilan oldingi natijani birlashtirib quyidagini topamiz:

$$\int (x^2-3)\cos x dx = \sin x\cdot(x^2-3) - (-2x\cos x + 2\sin x) = (x^2-3)\sin x + 2x\cos x -$$

$$2\sin x + C.$$

3-misol. $\int e^x \sin 2x dx$ aniqmas integralni hisoblang.

***Yechish*:** bu integralni topish uchun qaysi qunksiyani differensial belgisi ostiga kiritishning ahamiyati yo'q.

$\int e^x \sin 2x dx = \int \sin 2x de^x = \begin{cases} u = \sin 2x & du = 2\cos 2x \\ dv = e^x dx & v = e^x \end{cases}$ belgilash kiritgach bo'laklab integrallash formulasiga qo'yamiz:

$$\int \sin 2x de^x = \begin{cases} u = \sin 2x & du = 2\cos 2x \\ dv = e^x dx & v = e^x \end{cases} = e^x\cdot\sin 2x - 2\int e^x \cos 2x dx$$

Integral shu ko'rinishga kelgach integralning ikkinchi qismi uchun yana bo'laklash bajaramiz. $\int e^x \cos 2x dx = e^x\cos 2x + 2\int e^x \sin 2x dx$ yuqoridagi va keyingi natijalarni birlashtiradigan bo'lsak, quyidagi natijaga kelamiz. $\int \sin 2x de^x = e^x\cdot\sin 2x - 2(e^x\cos 2x + 2\int e^x \sin 2x dx) = e^x\cdot\sin 2x - 2e^x\cos 2x -$

$4\int e^x \sin 2x dx$ bundan ko'rinadiki integralishiz takrorlanuvchi yoki qaytariluvchi integrallardan ekan. Bunday integrallarni hisoblashda quyidagicha amal bajaramiz. $\int \sin 2x de^x = e^x\cdot\sin 2x - 2e^x\cos 2x - 4\int e^x \sin 2x dx$ aniqmas integralni biror noma'lum bilan belgilab tenglama ishlaymiz. $\int e^x \sin 2x dx = y$ va quyidagiga kelamiz.

$$y = e^x\cdot\sin 2x - 2e^x\cos 2x - 4y$$

$$5y = e^x\cdot\sin 2x - 2e^x\cos 2x \quad .$$

$$y = \frac{e^x\cdot\sin 2x - 2e^x\cos 2x}{5}$$

Демак, $\int e^x \sin 2x dx = \frac{e^x\cdot\sin 2x - 2e^x\cos 2x}{5} + C$ bo'ladi.

4-misol. $\int \frac{x}{\sin^2 x}dx$ aniqmas integralni hisoblang.

***Yechish*:** ushbu misolni yechishda ham bo'laklab integrallash formulasidan foydalanib ishlaymiz. $\int \frac{x}{\sin^2 x}dx = -\int x d(ctgx)$ integralni kabi ko'rinishga keltirib olamiz va quyidagicha belgilashlarini amalga oshiramiz. $-\int x d(ctgx) = \begin{cases} u = x, du = dx \\ v = ctgx, dv = -\frac{1}{\sin^2 x} \end{cases}$ belgilashlarni kiritganimizdan so'ng formulaga qo'yamiz $-\int x d(ctgx) = -xctgx + \int ctgx dx = -xctgx + \ln(\sin x) + C$.

Ba'zi trigonometrik funksiyalarni integrallash

Ba'zi trigonometrik funksiyalarni integrallashga oid misollarni ko'rib chiqaylik.

4-misol. $\int \sin 2x\cos 3x dx$ aniqmas integralni hisoblang.

***Yechish*:** bu integralni topish uchun trigonometriyaning formulalaridan biri bo'lgan ko'paytmani yig'indiga almashtirish formulasidan foydalanamiz. Ya'ni $\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$ formuladan foydalanib, integral ostidagi ifodani soddalashtiramiz. $\sin 2x\cos 3x = \frac{1}{2}(\sin 5x - \sin x)$ ekanligi topdik endi

$$\int \sin 2x \cos 3x dx = \int \frac{1}{2}(\sin 5x - \sin x)dx = \frac{1}{2}(-\frac{1}{5}\cos 5x + \cos x) + C$$

ekanligini topamiz va misolning javobi chiqdi.

5-misol. $\int \sin^4 x \cos^2 x dx$ aniqmas integralni hisoblang.

***Yechish*:** Ushbu integralni hisoblash uchun trigonametriyaning asosiy ayniyatlaridan biri bo'lgan daraja pasaytirish formulasidan foydalanib olding integral ostini soddalashtiramiz.

$$\sin^4 x \cos^2 x = \frac{(1-\cos 2x)^2}{4} \cdot \frac{1+\cos 2x}{2} = \frac{(1-\cos^2 2x)(1-\cos 2x)}{8} =$$

$$\frac{(1-\cos 4x)(1-\cos 2x)}{16} = \frac{1-\cos 4x - \cos 2x + \cos 4x \cos 2x}{8} =$$

$$\frac{1-\cos 2x - \cos 4x + \frac{1}{2}(\cos 6x + \cos 2x)}{16} = \frac{2-\cos 2x - 2\cos 4x + \cos 6x}{32}.$$

Ekanligidan $\int \sin^4 x \cos^2 x dx = \int \frac{2-\cos 2x - 2\cos 4x + \cos 6x}{32} dx =$

$$\frac{1}{32}(2x - \frac{1}{2}\sin 2x - \frac{1}{2}\sin 4x + \frac{1}{6}\sin 6x) + C.$$

6-misol. $\int \frac{dx}{\cos x}$ aniqmas integralni hisoblang.

***Yechish*:** ushbu misolni yechish uchun integral belgisi ostidagi funksiyaning surat va maxrajini $\cos x$ funksiyaga ko'paytiramiz va aniqmas integralni quyidagicha ko'rinishga keladi. $\int \frac{dx}{\cos x} = \int \frac{\cos x dx}{\cos^2 x}$ endi ushbu aniqmas integralni hisoblash uchun kasr suratidagi ifodani differensial belgisi ostiga kiritamiz va quyidagini hosil qilamiz,

$$\int \frac{\cos x dx}{\cos^2 x} = \int \frac{d(\sin x)}{1-\sin^2 x} = \int \frac{d(\sin x)}{(1-\sin x)(1+\sin x)} = \int \frac{dt}{(1-t)(1+t)} = -\int \frac{dt}{(t-1)(1+t)} = -\frac{1}{2}\int (\frac{1}{t-1} - \frac{1}{t+1})dt =$$

$$= -\frac{1}{2}\ln\frac{t-1}{t+1} + C = -\frac{1}{2}\ln\left|\frac{\sin x - 1}{\sin x + 1}\right| + C$$

bundan ko'rinadiki ushbu integralning javobi $\int \frac{dx}{\cos x} = -\frac{1}{2}\ln\left|\frac{\sin x - 1}{\sin x + 1}\right| + C$ га тенг bo'ladi.

Mustaqil yechish uchun misollar:

1-misol: Quyida berilgan aniqmas integrallarni hisoblang.

1.1 a) $\int (2x+1)^2 dx$ b) $\int ctg^2 x dx$

1.2 a) $\int \frac{x^2-1}{x} dx$ b) $\int \frac{dx}{1+\cos 2x}$

1.3 a) $\int (x + \frac{2}{\sqrt[3]{x}})dx$ b) $\int \frac{dx}{1-\cos 2x}$

1.4 a) $\int \frac{2x - 3\sqrt{x} + 3\sqrt[4]{x^3}}{x} dx$ b) $\int \sin\frac{x}{2}\cos\frac{x}{2} dx$

1.5 a) $\int \frac{\sqrt{x^4 + x^{-4} + 2}}{x^3} dx$ b) $\int \sin^2\frac{x}{4}\cos^2\frac{x}{4} dx$

1.6 a) $\int \sin^2\frac{x}{2} dx$ b) $\int e^x(3 + \frac{e^{-x}}{\cos^2 x})dx$

1.7 a) $\int \cos^2\frac{x}{2} dx$ b) $\int 4^x(3 + \frac{4^{-x}}{\sqrt[3]{x^2}})dx$

1.8 a) $\int(\sin\frac{x}{2}-\cos\frac{x}{2})dx$ b) $\int(2^{x+1}-5^{x-1})dx$

1.9 a) $\int(\sin\frac{x}{2}-\cos\frac{x}{2})(\sin\frac{x}{2}+\cos\frac{x}{2})dx$ b) $\int(2^x+3^x)^2dx$

2-misol: Quyida berilgan aniqmas integrallarni o'zgaruvchi almashtirish usulidan foydalanib toping.

2.1 a) $\int\frac{\cos 5x}{\sqrt{\sin^3 5x}}dx$ b) $\int\frac{dx}{\arcsin^4 x\sqrt{1-x^2}}$

2.2 a) $\int\frac{\sqrt{ctg4x}}{\sin^2 4x}dx$ b) $\int\frac{\sqrt{arctgx}}{1+x^2}dx$

2.3 a) $\int\frac{dx}{(1+x^2)\sqrt{arctgx}}$ b) $\int e^{\sin x}\cos xdx$

2.4 a) $\int e^{5-2x^2}xdx$ b) $\int 4^{\cos 5x}\sin 5xdx$

2.5 a) $\int\frac{2x-4}{x^2+16}dx$ b) $\int\frac{2^{\sqrt{x}}}{\sqrt{x}}dx$

2.6 a) $\int\frac{dx}{4+9x^2}$ b) $\int\sqrt[3]{3+5\cos x}\sin xdx$

2.7 a) $\int(2+5x)^9dx$ b) $\int\frac{x^2dx}{(8x^3+125)^{\frac{3}{4}}}$

2.8 a) $\int\frac{dx}{3x-1}$ b) $\int\frac{xdx}{9+x^4}$

2.9 a) $\int tgxdx$ b) $\int\frac{x^5}{\sqrt{x^{12}+3}}dx$

2.10 a) $\int ctgxdx$ b) $\int\frac{dx}{x\ln x}$

2.11 a) $\int\frac{xdx}{x^2+1}$ b) $\int\frac{\sqrt{1+\ln x}}{x}dx$

2.12 a) $\int\frac{\arccos x}{\sqrt{1-x^2}}dx$ b) $\int\cos\frac{1}{x}\frac{dx}{x^2}$

3-misol: Kuyida berilgan aniqmas integrallarni bo'laklab integrallash usulidan foydalanib toping.

3.1 a) $\int\ln(x+8)dx$ b) $\int\sqrt{25-x^2}dx$

3.2 a) $\int(x^2-x+1)\ln xdx$ b) $\int\sqrt{x^2+7}dx$

3.3 a) $\int(3x-2)\ln^2 xdx$ b) $\int\ln^2 xdx$

3.4 a) $\int(x+8)\sin 3xdx$ b) $\int\sqrt[3]{x}\ln^2 xdx$

3.5 a) $\int(x^2-3)\cos xdx$ b) $\int\frac{x}{\cos^2 x}dx$

3.6 a) $\int x\cos(x-4)dx$ b) $\int e^{2x}\cos xdx$

3.7 a) $\int\arcsin xdx$ b) $\int(5x-1)e^{2x}dx$

3.8 a) $\int arctg2xdx$ b) $\int xarctgxdx$

4-misol: Quyida berilgan ba'zi trigonometrik funksiyalarning aniqmas integralini toping.

4.1 a) $\int \sin 2x \cos 3x dx$ b) $\int tg^4 x dx$

4.2 a) $\int \sin^2 x \cos x dx$ b) $\int \sin^4 2x dx$

4.3 a) $\int \sin 5x \sin 2x dx$ b) $\int \frac{\sin^4 x}{\cos^2 x} dx$

4.4 a) $\int \cos 3x cos 4x dx$ b) $\int \frac{\cos^3 x}{\sin^2 x} dx$

4.5 a) $\int \cos x \cos 3x \cos 5x dx$ b) $\int \frac{dx}{\sin x}$

4.6 a) $\int \sin x \cos^3 x dx$ b) $\int \sin^2 x \cos^2 x dx$

4.7 a) $\int \sin^2 x dx$ b) $\int \cos^2 x dx$

Javoblar:

1.1 a) $\frac{3}{4}x^3 + 2x^2 + x + C$, **b)** $-ctgx - x + C$.

1.2 a) $\frac{x^2}{2} - \ln x + C$, **b)** $\frac{1}{2}tgx + C$.

1.3 a) $\frac{x^2}{2} + 3\sqrt[3]{x^2} + C$, **b)** $-\frac{1}{2}ctgx + C$.

1.4 a) $2x - 6\sqrt{x} + 4\sqrt[4]{x^3} + C$, **b)** $-\frac{1}{2}\cos x + C$.

1.5 $\ln x - \frac{1}{4x^4} + C$, $\frac{1}{8}x - \frac{1}{8}\sin x + C$.

1.6 a) $\frac{1}{2}x - \frac{1}{2}\sin x + C$, **b)** $3e^x + tgx + C$.

1.7 a) $\frac{1}{2}x + \frac{1}{2}\sin x + C$, **b)** $3\frac{4^x}{\ln 4} + 3\sqrt[3]{x} + C$.

1.8 a) $-2\cos\frac{x}{2} - 2\sin\frac{x}{2} + C$, **b)** $\frac{2^{x+1}}{\ln 2} - \frac{5^{x-1}}{\ln 5} + C$.

1.9 a) $-\sin x + C$, **b)** $\frac{4^x}{\ln 4} + \frac{6^x}{\ln 6} + \frac{9^x}{\ln 9} + C$.

2.1 a) $-\frac{2}{5\sqrt{\sin 5x}} + C$, **b)** $-\frac{1}{3\arcsin^3 x} + C$.

2.2 a) $-\frac{1}{6}\sqrt{ctg^3 4x} + C$, **b)** $\frac{2}{3}\sqrt{arctg^3 x} + C$.

2.3 a) $2\sqrt{arctgx} + C$, **b)** $e^{\sin x} + C$.

2.4 a) $-\frac{1}{4}e^{5-2x^2} + C$, **b)** $-\frac{4^{\cos 5x}}{5\ln 4} + C$.

2.5 a) $\ln(x^2 + 16) - arctg\frac{x}{4} + C$, **b)** $\frac{2^{\sqrt{x}+1}}{\ln 2} + C$.

2.6 a) $\frac{1}{6}arctg\frac{3}{2}x + C$, **b)** $-\frac{3}{20}\sqrt[3]{(3 + 5\cos x)^4} + C$.

2.7 a) $\frac{1}{50}(2 + 5x)^{10} + C$, **b)** $\frac{1}{6}(8x^3 + 125)^{\frac{1}{4}} + C$.

2.8 a) $\frac{1}{3}\ln(3x - 1) + C$, **b)** $\frac{1}{6}arctg\frac{x^2}{3} + C$.

2.9 a) $-\ln|\cos x| + C$.

2.10 a) $\ln|\sin x| + C$, **b)** $\ln\ln x + C$.

2.11 a) $\frac{1}{2}\ln(x^2 + 1) + C$, **b)** $\frac{2}{3}\sqrt{(1 + \ln x)^3} + C$.

2.12 **a)** $\frac{1}{2}\arccos^2 x + C$, **b)** $-\sin\frac{1}{x} + C$.

3.1 **a)** $x\ln(x+8) - x + 8\ln(x+8) + C$, **b)** $\frac{1}{2}(25\arcsin\frac{x}{5} + 2x\sqrt{25-x^2}) + C$.

3.2 **a)** $(\frac{x^3}{3} - \frac{x^2}{2} + x)\ln x - (\frac{x^3}{9} - \frac{x^2}{4} + x) + C$.

3.3 **a)** $\frac{3}{2}x^2 - 2x\ln^2 x - (\frac{3}{2}x^2 - 4x)\ln x + (\frac{3}{4}x^2 - 4x) + C$.

3.4 **a)** $-\frac{1}{3}(x+8)\cos 3x + \frac{1}{9}\sin 3x + C$, **b)** $\frac{3}{4}\sqrt[3]{x^4}\ln^2 x - \frac{9}{8}\sqrt[3]{x^4}\ln x + \frac{27}{32}\sqrt[3]{x^4} + C$.

3.5 **a)** $(x^2-3)\sin x + 2x\cos x - 2\sin x + C$, **b)** $xtgx + \ln|\cos x| + C$.

3.6 **a)** $x\sin(x-4) + \cos(x-4) + C$, **b)** $\frac{e^{2x}\sin x + 2e^{2x}\cos x}{5}$.

3.7 **a)** $x\arcsin x + \sqrt{1-x^2} + C$, **b)** $\frac{1}{2}e^{2x}(5x-1) - \frac{5}{4}e^{2x} + C$

4.1 **a)** $-\frac{1}{10}\cos 5x + \frac{1}{2}\cos x + C$, **b)** $\frac{tg^3 x}{3} - tgx + x + C$.

4.2 **a)** $\frac{\sin^3 x}{3} + C$, **b)** $\frac{3}{8}x - \frac{1}{8}\sin 8x + \frac{1}{64}\sin 8x + C$.

4.3 **a)** $\frac{1}{6}\sin 3x - \frac{1}{14}\sin 7x + C$, **b)** $tgx - 1{,}5x + \frac{1}{4}\sin 2x + C$.

4.4 **a)** $\frac{1}{2}\sin x + \frac{1}{14}\sin 7x + C$, **b)** $-\frac{1}{\sin x} - \sin x + C$.

4.5 **a)** $\frac{1}{4}(\sin x + \frac{1}{3}\sin 3x + \frac{1}{7}\sin 7x + \frac{1}{9}\sin 9x) + C$, **b)** $\frac{1}{2}\ln\left|\frac{\cos x - 1}{\cos x + 1}\right| + C$.

4.6 **a)** $-\frac{1}{4}\cos^4 x + C$, **b)** $\frac{1}{8}x - \frac{1}{32}\sin 4x + C$.

4.7 **a)** $\frac{1}{2}x - \frac{1}{4}\sin 2x + C$, **b)** $\frac{1}{2}x + \frac{1}{4}\sin 2x + C$

Mavzu: Aniq integralning ta'rifi. Aniq integralni hisoblash usullari.

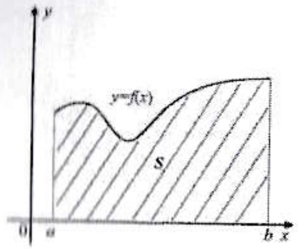
1. Aniq integralning ta'rifi. Uzluksiz funksiyalarning integrallanuvchanligi. Monoton funksiyalarning integrallanuvchanligi.
2. Nyuton – Leybnits formulasi. Aniq integralda o'zgaruvchilarni almashtirish va bo'laklab integrallash usullari.

Aniq integralning ta'rifi. Uzluksiz funksiyalarning integrallanuvchanligi. Monoton funksiyalarning integrallanuvchanligi.

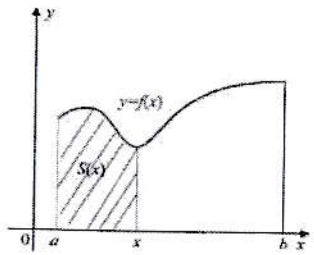
2-rasmda tasvirlangan shakl egri chiziqli trapetsiya deyiladi. Bu shakl yuqoridan $y = f(x)$ funksiyaning grafigi bilan, quyidan funksiyaning grafigi bilan, quyidan $[a,b]$ kesma bilan, yon tomonlardan esa $x = a$, $x = b$ to'g'ri chiziqlaming kesmalari bilan chegaralangan. $[a,b]$ kesma egri chiziqli trepetsiyaning asosi deyiladi.

Egri chiziqli trapetsiyaning yuzini qaysi formulaga ko'ra hisoblaymiz, degan savol tug'iladi.

Bu yuzni S deb belgilaylik. S yuzni $y = f(x)$ funksiyaning boshlang'ich funksiyasi yordamida hisoblash mumkin ekan. Shunga oid mulohazalami keltiramiz.



2-rasm.

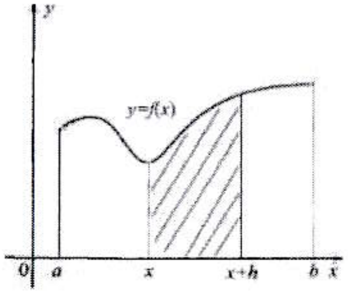


3-rasm.

$[a,x]$ asosli egri chiziqli trapetsiyaning yuzini asosli egri chiziqli trapetsiyaning yuzini $S(x)$ deb belgilaymiz (3-rasm), bunda x shu $[a,b]$

kesmadagi istalgan nuqta: $x=a$ bo'lganda $[a,x]$ kesma nuqtaga aylanadi, shuning uchun $S(a)=0$, $x=b$ da $S(b)=S$.

$S(x)$ ni $f(x)$ funksiyaning boshlang'ich funksiyasi bo'lishini, ya'ni $S'(x)=f(x)$ ekanini ko'rsatamiz.



$S(x+h)-S(x)$ ayirmani ko'raylik, bunda $h>0$ ($h<0$) hol ham xuddi shunday ko'riladi. Bu ayirma asosi $[x,x+h]$ bo'lgan egri chiziqli trapetsiyaning yuziga teng (4-rasm). Agar h son kichik bo'lsa, u holda bu yuz taqriban $f(x)\cdot h$ ga teng, ya'ni $S(x+h)-S(x)\approx f(x)\cdot h$. Demak, $\frac{S(x+h)-S(x)}{h}\approx f(x)$. Bu taqribiy tenglikning chap qismi $h\to 0$ da $S'(x)=f(x)$ tenglik hosil bo'ladi. Demak, $S(x)$ yuz $f(x)$ funksiya uchun boshlang'ich funksiyasi ekan.

Boshlang'ich funksiya $S(x)$ dan ixtiyoriy boshqa boshlang'ich $F(x)$ funksiya o'zgarmas songa farq qiladi, ya'ni

$$F(x)=S(x)+C.$$

Bu tenglikdan $x=a$ da $F(a)=S(a)+C$ ва $S(a)=0$ bo'lgani uchun $C=F(a)$. U holda (1) tenglikni quyidagicha yozish mumkin: $S(x)=F(x)-F(a)$. Bundan $x=b$ да $S(b)=F(b)-F(a)$ ekanini topamiz.

Demak, egri chiziqli trapetsiyaning yuzini (2-rasm) quyidagi formula yordam ida hisoblash mumkin:

$S(b)=F(b)-F(a)$.

Bunda $F(x)$ - berilgan $f(x)$ funksiyaning istalgan boshlang'ich funksiyasi.

Shunday qilib, egri chiziqli trapetsiyaning yuzini hisoblash $f(x)$ funksiyaning $F(x)$ boshlang'ich funksiyasini topishga, ya'ni $f(x)$ funksiyani integrallashga keltiriladi.

$F(b)-F(a)$ ayirma $f(x)$ funksiyaning $[a,b]$ kesmadagi aniq integrali deyiladi va bunday belgilanadi: $\int_a^b f(x)dx$.

$$\int_a^b f(x)dx=F(b)-F(a)$$

formula Nyuton-Leybnis formulasi deb ataladi.

1-misol. $\int_0^2 (6x^2-5)dx$ aniq integralni hisobolang.

Yechish: bu aniq integralni hisoblash uchun oldin aniqmas integral hisoblanadi.

$$\int_0^2 (6x^2-5)dx=\frac{6}{3}x^3-5x\Big|_0^2=2\cdot 8-5\cdot 2=6$$

2-misol. $\int_0^1 \frac{x^2}{x^2+1}dx$ aniq integralni hisoblang.

Yechish: $\int_0^1 \frac{x^2}{x^2+1}dx=\int_0^1 (1-\frac{1}{x^2+1})dx=x-arctgx\Big|_0^1=1-\frac{\pi}{4}$.

3-misol. $\int_{-\frac{\pi}{4}}^0 tg^2xdx$ aniq integralni hisoblang.

Yechish:

$$\int_{-\frac{\pi}{4}}^0 tg^2xdx=\int_{-\frac{\pi}{4}}^0 \frac{\sin^2 x}{\cos^2 x}dx=\int_{-\frac{\pi}{4}}^0 \frac{1-\cos^2 x}{\cos^2 x}dx=\int_{-\frac{\pi}{4}}^0 (\frac{1}{\cos^2 x}-1)dx=tgx-x\Big|_{-\frac{\pi}{4}}^0=$$

$$0-(-1+\frac{\pi}{4})=1-\frac{\pi}{4}.$$

4-misol. $\int_0^{\pi/2} \frac{dx}{1+\cos x}$ aniq integralni hisoblang.

Yechish: Ushbu misolni yechish uchun triganometriyaning asosiy ayniyatlari ya'ni yarim burchak formulalaridan foydalanamiz, ya'ni integral ostidagi ifodaning maxrajini soddalashtiramiz. $1+\cos x = 2\cos^2\frac{x}{2}$ ekanligidan quyidagiga kelamiz.

Aniq integrallarda o'zgaruvchi almashtirish va bo'laklash usullari

1-misol. $\int\limits_0^1 xe^{x^2}dx$ aniq integralni hisoblang.

Yechish: ushbu aniq integralni hisoblash uchun o'zgaruvchi almashtirish metodidan foydalanamiz.

$$\int\limits_0^1 xe^{x^2}dx = \frac{1}{2}\int\limits_0^1 e^{x^2}d(x^2) = \frac{1}{2}\int\limits_0^1 e^t dt = \frac{1}{2}e^t\Big|_0^1 = \frac{1}{2}e - \frac{1}{2}$$

2-misol. $\int\limits_1^2 \frac{e^{\frac{1}{x}}}{x^2}dx$ aniq integralni hisoblang.

Yechish: bu integralni hisoblash uchun ham o'zgaruvchi almashtirish metodidan foydalanamiz.

$$\int\limits_1^2 \frac{e^{\frac{1}{x}}}{x^2}dx = \int\limits_1^2 e^{\frac{1}{x}}\cdot\frac{1}{x^2}dx = \int\limits_1^2 e^{\frac{1}{x}}d(\frac{1}{x}) = \int\limits_1^2 e^t dt = e^t\Big|_1^2 = e^{\frac{1}{x}}\Big|_1^2 = e^{0.5} - e.$$

3-misol. $\int\limits_1^e x^2\ln x dx$ aniqmas integralni hisoblang.

Yechish: bu misolni hisoblash uchun bo'laklab integrallashdan foydalanamiz.

$$\int\limits_1^e x^2\ln x dx = \frac{1}{3}\int\limits_1^e \ln x d(x^3) = \frac{1}{3}x^3\ln x - \frac{1}{3}\int\limits_1^e x^3\cdot\frac{1}{x}dx = \frac{1}{3}x^3\ln x - \frac{1}{3}\int\limits_1^e x^2 dx =$$

$$(\frac{1}{3}x^3\ln x - \frac{1}{9}x^3)\Big|_1^e == \frac{1}{3}e^3 - \frac{1}{9}e^3 + \frac{1}{9} = \frac{2}{9}e^3 + \frac{1}{9}.$$

4-misol. $\int\limits_0^1 xe^{-x}dx$ aniq integralni hisoblang.

Yechish: integralni bo'laklab integrallash usuli bilan hisoblaymiz.

$$\int\limits_0^1 xe^{-x}dx = -\int\limits_0^1 xde^{-x} = -xe^{-x} + \int\limits_0^1 e^{-x}dx = -xe^{-x} - e^{-x}\Big|_0^1 = 1 - \frac{1}{e}.$$

5-misol. $\int\limits_{-3/2}^{2} \frac{(x-1)^2}{x^2+3x+4}dx$ aniq integralni hisoblang.

Yechish: $\int\limits_{-3/2}^{2} \frac{(x-1)^2}{x^2+3x+4}dx$ bu integralni hisoblash uchun integral belgisi ostidagi ifodaning surat qismini kavsni ochib chiqamiz.

$$\int\limits_{-3/2}^{2} \frac{(x+1)^2}{x^2+3x+4}dx = \int\limits_{-3/2}^{2} \frac{x^2+2x+1}{x^2+3x+4}dx = \int\limits_{-3/2}^{2} \frac{x^2+3x+4-x-3}{x^2+3x+4}dx =$$

$$\int\limits_{-3/2}^{2} 1 - \frac{x+3}{x^2+3x+4}dx = x - \frac{1}{2}\int\limits_{-3/2}^{2}\frac{3}{x^2+3x+4} + \frac{1}{2}\int\limits_{-3/2}^{2}\frac{d(x^2+3x+4)}{x^2+3x+4}dx =$$

$$x - \frac{1}{2}\int\limits_{-3/2}^{2}\frac{3dx}{(x+\frac{3}{2})^2+\frac{7}{4}} - \frac{1}{2}\int\limits_{-3/2}^{2}\frac{d(x^2+3x+4)}{x^2+3x+4} =$$

$$x - \frac{3}{\sqrt{7}}arcctg\frac{2}{\sqrt{7}}(x+\frac{3}{2}) - \ln(x^2+3x+4)\Big|_{-3/2}^{2} =$$

$$2 - \frac{3}{\sqrt{7}}arcctg2\sqrt{7} - \ln 14 + \frac{3}{2} + \ln\frac{7}{4}.$$

Ekanligini topamiz va oxirgi tenglikning natijasini soddalashtiramiz.

$$2 - \frac{3}{\sqrt{7}}arcctg2\sqrt{7} - \ln 14 + \frac{3}{2} + \ln\frac{7}{4} = 3{,}5 - \frac{3}{\sqrt{7}}arcctg2\sqrt{7} - \ln 14 : \frac{7}{4} =$$

$$3{,}5-\frac{3}{\sqrt{7}}arcctg2\sqrt{7}-\ln 14\cdot\frac{4}{7}=3{,}5-\frac{3}{\sqrt{7}}arcctg2\sqrt{7}-\ln 8.$$

bundan esa $\int\limits_{-3/2}^{2}\frac{(x-1)^2}{x^2+3x+4}dx=3{,}5-\frac{3}{\sqrt{7}}arcctg2\sqrt{7}-\ln 8$.

6-misol. $\int\limits_{1}^{e}\frac{dx}{x\sqrt{1+\ln x}}$ aniq integralni hisoblang.

***Yechish*:** $\int\limits_{1}^{e}\frac{dx}{x\sqrt{1+\ln x}}$ bu aniq integralni hisoblash uchun o'zgaruvchi kiritish usulidan foydalanamiz. Ya'ni integral ostidagi $\frac{1}{x}$ ifodani differensial belgisi ostiga kiritamiz.

$$\int\limits_{1}^{e}\frac{dx}{x\sqrt{1+\ln x}}=\int\limits_{1}^{e}\frac{1}{x}\cdot\frac{1}{\sqrt{1+\ln x}}dx=\int\limits_{1}^{e}\frac{1}{\sqrt{1+\ln x}}d(1+\ln x)=$$

$$\int\limits_{1}^{e}\frac{dt}{\sqrt{t}}=\int\limits_{1}^{e}t^{-1/2}dt=2t^{1/2}\Big|_{1}^{e}=2(1+\ln x)^{1/2}\Big|_{1}^{e}=2\sqrt{2}-2.$$

7-misol. $\int\limits_{1}^{4}\frac{dx}{x+x^2}$ aniq integralni hisoblang.

***Yechish*:** ushbu aniq integralni hisoblash uchun integral ostidagi ifodani ikkita kasr ayirmasi shaklida yozamiz ya'ni,

$$\int\limits_{1}^{4}\frac{dx}{x+x^2}=\int\limits_{1}^{4}\frac{dx}{x(1+x)}=\int\limits_{1}^{4}(\frac{1}{x}-\frac{1}{x+1})dx=(\ln x-\ln(x+1))\Big|_{1}^{4}=\ln\frac{x}{x+1}\Big|_{1}^{4}=\ln\frac{4}{5}-\ln\frac{1}{2}=$$

$$\ln\frac{4}{5}:\frac{1}{2}=\ln\frac{8}{5}.$$

8-misol. $\int\limits_{1}^{2}\sqrt{x}\ln x dx$ aniq integralni hisoblang.

***Yechish*:** Ushbu integralni hisoblash uchun aniq integrallarda bo'laklash usulidan foydalanamiz.

$$\int\limits_{1}^{2}\sqrt{x}\ln x dx=\frac{2}{3}\int\limits_{1}^{2}\ln x dx^{3/2}=\frac{2}{3}x^{3/2}\ln x-\frac{2}{3}\int\limits_{1}^{2}x^{3/2}\cdot\frac{1}{x}dx=\frac{2}{3}x^{3/2}\ln x-\frac{2}{3}\int\limits_{1}^{2}x^{1/2}dx=$$

$$\frac{2}{3}x^{3/2}\ln x-\frac{4}{9}x^{3/2}\Big|_{1}^{2}=\frac{4\sqrt{2}}{3}\ln 2-\frac{8\sqrt{2}}{9}+\frac{4}{9}.$$

$$\int\limits_{0}^{\pi/2}\frac{dx}{1+\cos x}=\int\limits_{0}^{\pi/2}\frac{dx}{2\cos^2\frac{x}{2}}=tg\frac{x}{2}\Big|_{0}^{\pi/2}=1-0=1.$$

Mustaqil yechish uchun misollar

1-misol: Quyida berilgan aniq integrallarni hisoblang.

1.1 a) $\int\limits_{0}^{1}(6x^2-5)dx$ b) $\int\limits_{1}^{2}(5x^2-\frac{3}{x^2})dx$

1.2 a) $\int\limits_{0}^{1}\frac{dx}{x^2-2x+2}$ b) $\int\limits_{0}^{\pi}\sin 3x\cos 2x dx$

1.3 a) $\int\limits_{-1/2}^{1/2}\frac{dx}{\sqrt{1-x^2}}$ b) $\int\limits_{-\pi/4}^{0}tg^2x dx$

1.4 a) $\int\limits_{0}^{\pi/2}\cos^2\frac{x}{2}dx$ b) $\int\limits_{0}^{2}(6x^2-5)dx$

1.5 a) $\int\limits_{0}^{1}\frac{x^2}{1+x^3}dx$.

2-misol: Quyidagi integrallarni belgilash kiritish va bo'laklab integrallash usullari yordamida hisoblang.

2.1 a) $\int_0^1 xe^{x^2}dx$ b) $\int_0^1 \frac{arctgx}{1+x^2}dx$

2.2 a) $\int_1^e \frac{\sin(\ln x)}{x}dx$ b) $\int_1^e x^2\ln x dx$

2.3 a) $\int_{-1/2}^{1/2} \frac{dx}{(1-x^2)\sqrt{1-x^2}}$ b) $\int_0^2 \frac{3x^5}{\sqrt{x^6+1}}dx$

2.4 a) $\int_0^{\pi/2} \frac{\cos ydy}{4+\sqrt{\sin y}}$ b) $\int_0^1 xe^{-x}dx$

2.5 a) $\int_0^{\pi/2} \sin^6 xdx$ b) $\int_1^2 \frac{e^x}{e^x-1}dx$.

2.6 a) $\int_1^2 \frac{e^{1/x}}{x^2}dx$. b) $\int_{\pi/6}^{\pi/2} \frac{\cos x}{\sin^3 x}dx$

2.7 a) $\int_0^{\pi} x\sin xdx$ b) $\int_0^{\ln 5} \frac{e^x\sqrt{e^x-1}}{e^x+3}dx$

2.8 a) $\int_{-3}^0 (x-2)e^{-x/3}dx$ b) $\int_{-1}^1 \frac{dx}{e^x+e^{-x}}$

2.9 a) $\int_0^{\pi} \sin x\cos^2 xdx$ b) $\int_0^{\sqrt{\pi}/2} \frac{xdx}{\cos^2 x^2}$

Javoblar:

1.1 a) 6, b) $\frac{61}{6}$.

1.2 a) $\frac{\pi}{4}$, b) $-\frac{4}{5}$.

1.3 a) $\frac{\pi}{3}$, b) $\frac{4-\pi}{4}$.

1.4 a) $\frac{\pi}{4}+\frac{\sqrt{3}}{4}$, b) 6.

1.5 a) $\frac{4-\pi}{4}$.

2.1 a) $\frac{1}{2}e-1$, b) $\frac{\pi^2}{32}$.

2.2 a) $1-\cos 1$, b) $\frac{2}{9}e^3+\frac{1}{9}$.

2.3 a) $\frac{2}{\sqrt{3}}$, b) $\sqrt{65}$.

2.4 a) $2-8\ln\frac{5}{4}$, b) $\frac{e-2}{e}$.

2.5 a) $\frac{\pi}{8}$, b) $\ln(e+1)$.

2.6 a) $e-\sqrt{e}$, b) 1,5

2.7 a) π, b) $4-arctg\sqrt{2}$.

2.8 a) $-3-6e$, b) $arctge-arctg\frac{1}{e}$.

2.9 a) $\frac{2}{3}$, b) 1

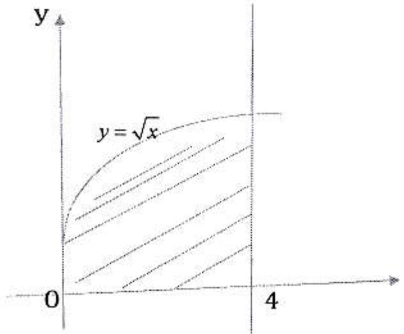
Mavzu: Aniq integral va ularning tadbiqlari

Endi esa biz aniq integrallarning tadbiqlarini ko'rib o'tamiz.

Birinchi tadbiqlaridan biri bu yoy uzunligini topish, ikkinchi tadbiqlaridan biri bu egri chiziqli trapetsiya yuzini topish yoki bir nechta funksiyalar bilan chegaralangan shaklning yuzini topish, uchinchi tadbiqlaridan biri esa aylanma jismlarning sirti yuzasini topish, to'rtinchi tadbiqlaridan biri o'zgaruvchi kuchning bajargan ishi va yana bir tadbiqlaridan inersiya momenti bo'lib biz aniq integrallarning yuza topishga doir misollarini ko'rib chiqamiz.

Misol. $y=\sqrt{x}$, $y=0$, $x=4$ chiziqlar bilan chegaralangan figuraning yuzini toping.

Yechish: ushbu misolni yechish uchun aniq integralning chegaralarini aniqlashtirib olishimiz kerak buning uchun $y=\sqrt{x}$, $y=0$ funksiyalarni tenglashtirib o'zgaruvchining qiymatlarini topamiz va yechim aniq integralning quyi chegarasi bo'ladi. $\int_0^4 \sqrt{x}dx=\frac{3}{2}x\sqrt{x}\Big|_0^4=\frac{3}{2}\cdot 4\cdot 2-0=12$



1-misol. $y=\sin x$, $y=0$, $x=\pi$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

Yechish: $\sin x=0$ tenglamani yechamiz va quyi chegarasi tenglamaning yechimi bo'ladigan aniq integral tuziladi. $\int_0^{\pi}\sin xdx=-\cos x|_0^{\pi}=-(-1)+0=1$.

Mustaqil yechish uchun misollar:

19.1 $y=\sqrt{x^3}$ funksiya grafigining $0\le x\le 4$ oraliq bilan chegaralangan qismining yuzini toping.

19.2. $y=\frac{x^3}{4}$, funksiya grafigining $0\le x\le 2$ oraliq bilan chegaralangan qismining yuzini toping.

19.3. $y=4-\frac{x^2}{2}$ funksiya grafigining $y\ge 0$ oraliq bilan chegaralangan qismining yuzini toping.

19.4. $y=\frac{4}{5}\sqrt[5]{x^4}$ funksiya grafigining $0\le x\le 9$ oraliq bilan chegaralangan qismining yuzini toping.

19.5. $y=\ln x$ funksiya grafigining $2\sqrt{2}\le x\le 2\sqrt{6}$ oraliq bilan chegaralangan qismining yuzini toping.

19.5. $y=\ln\cos x$ funksiya grafigining $0\le x\le\frac{\pi}{4}$ oraliq bilan chegaralangan qismining yuzini toping.

19.6. $y=e^x$ funksiya grafigining $0\le x\le\ln 7$ oraliq bilan chegaralangan qismining yuzini toping.

19.7. $y=\frac{x^2}{4}-\frac{1}{2}\ln x$ funksiya grafigining $1\le x\le e$ oraliq bilan chegaralangan qismining yuzini toping.

19.8. $y=2\ln\frac{4}{4-x^2}$ funksiya grafigining $-1\le x\le 1$ oraliq bilan chegaralangan qismining yuzini toping.

19.9. $y=\sqrt{2x-x^2}-1$ funksiya grafigining $\frac{1}{4}\le x\le 1$ oraliq bilan chegaralangan qismining yuzini toping.

19.10. $y=\frac{x}{4}\sqrt{2-x^2}$ funksiya grafigining $0\le x\le 1$ oraliq bilan chegaralangan qismining yuzini toping.

19.11. $y=2\sqrt{x}$ funksiya grafigining $1\le x\le 2$ oraliq bilan chegaralangan qismining yuzini toping.

19.12. $y=\dfrac{1}{x}$, $x=1$, $x=4$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

19.13. $y=\sqrt{x}$, $x=0$, $y=1$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

19.14. $y^2=6x$, $y=0$, $x=3$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

19.15. $y=x^2$, $y=\sqrt{x}$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

19.16. $y=3\sqrt{1-x^2}$, $y=1-x^2$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

19.17. $y=\dfrac{e^x+e^{-x}}{2}$, $y=0$, $x=-1$, $x=1$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

19.18. $y=\sqrt{x+1}$, $x+y=1$, $y=0$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

19.19. $y=\cos x$, $y=1-x$, $y=0$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

19.20. $y=4-x^2$, $x=0$, $y=0$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

Javoblar:

19.1 $\dfrac{64}{5}$.

19.2 $\dfrac{2}{3}$.

19.3 $\dfrac{32\sqrt{2}}{3}$.

19.4 $\dfrac{4}{9}3^{3,6}$.

19.5 $2\sqrt{6}(\ln 2\sqrt{6}-1)-2\sqrt{2}(\ln 2\sqrt{2}-1)$.

19.6 6.

19.7 $\dfrac{e^3-7}{12}$.

19.11 $\dfrac{8\sqrt{2}-4}{3}$.

19.12 $\ln 4$.

19.13 $\dfrac{2}{3}$.

19.14 $6\sqrt{2}$.

19.15 $\dfrac{1}{3}$.

19.16 $\dfrac{3\pi}{2}-\dfrac{4}{3}$.

19.17 $e-\dfrac{1}{e}$.

19.18 $\dfrac{7}{6}$.

V BOB FUNKSIYA HOSILASI, ANIQMAS VA ANIQ INTEGRALLAR MAVZULARINI UMUMIY TAKRORLASH UCHUN MISOLLAR

Mavzu: Umumiy takrorlash uchun misollar

1-misol:

1.1. *Funksiyaning hosilasini toping*:

a) $f(x)=3x+5$;

b) $f(x)=10-2x$;

c) $f(x)=2x^3-x$;

d) $f(x)=-x^2+5x^4$;

e) $f(x)=\sqrt{x}+x^5$;

f) $f(x)=3sinx-cosx$;

g) $f(x)=3^x-e^x$;

h) $f(x)=\log_4 x-2^x$;

i) $f(x)=\text{tg}x+\text{ctg}x$;

j) $f(x)=3\sin x+2x^6$;

k) $f(x)=4arctgx$;

l) $f(x)=\ln x+\sqrt[5]{x}$;

m) $f(x)=\arccos x-\pi$;

n) $f(x)=tgx-\cos x$;

o) $f(x)=\arcsin x-\sin x$;

p) $f(x)=x^{0,5}-\sqrt[3]{x}$;

k) $f(x)=5arcctgx+e^2$;

r) $f(x)=5^x+2\ln x$.

1.2. Agar $f(x)=\frac{1}{3}x^3-2x^{\frac{1}{4}}$ bo'lsa, $f'(1)$ ni toping.

1.3. x ning qanday qiymatlarida quyidagi funksiyalarning hosilasi 0 ga teng bo'ladi:

1) $f(x)=3x^4-4x^3-12x^2$; 2) $f(x)=x^3+2x^2-7x+1$.

1.4. Quyidagilardan $f'(3)$ ni toping:

1) $f(x)=x^{\frac{3}{2}}-x^{-\frac{3}{2}}$; 2) $f(x)=\frac{3}{\sqrt{x}}-\frac{2}{x^3}$; 3) $f(x)=\sqrt{x}+\frac{1}{x}+3$.

1.5. x ning qanday qiymatlarida $f(x)$ funksiya hosilasi 0 ga teng bo'lishini aniqlang:

1) $f(x)=x-\cos x$; 2) $f(x)=x^2-6x-8\ln x$; 3) $f(x)=0,5x-\sin x$.

1.6. x ning $f(x)$ funksiya hosilasi musbat bo'ladigan qiymatlarini toping:

1) $f(x)=x-\ln x$; 2) $f(x)=x\ln 2-2^x$; 3) $f(x)=\sin x$.

1.7. $f(x)$ funksiya hosilasining x_0 nuqtadagi qiymatini toping:

1) $f(x)=2e^x+2\ln x-x$, $x_0=2$; 2) $f(x)=\log_2 x-2^x$, $x_0=1$.

1.8. x ning $f(x)$ funksiya hosilasi manfiy bo'ladigan qiymatlarini toping:

1) $f(x)=x^3-3\ln x$;\ 2) $f(x)=e^x-x$.

1.9. Agar 1) $f(x)=0,5x-\sin x$, 2) $f(x)=\sqrt{3}\cdot x-2\cos x$ bo'lsa, $f'(x)=0$ tenglamani yeching.

2-misol: Quyida berilgan Murakkab funksiyalarning hosilasini toping.

2.1. Funksiyaning hosilasini toping:

1) $f(x)=(4x+2)^2$;

2) $f(x)=4^{1-2x}+(5x-7)^3$;

3) $f(x)=(3x^2+1)^3$;

4) $f(x)=(3-x)^2+3^{-x}$;

5) $f(x)=\sqrt{4+3x}$;

6) $f(x)=\sqrt{x^2-5x}+1$;

7) $f(x)=3^{x^2}-e^{2-x}$;

8) $f(x)=(3+5x)^6$;

2.2. Funksiyaning hosilasini toping:

1) $f(x)=(2-5x)^3-e^{3x+1}$;

2) $f(x)=\log_7(3-5x)$;

3) $f(x)=tgx^2-\sqrt[3]{5x+1}$;

4) $f(x)=\ln(3x^2+x)$;

5) $f(x)=\cos^2 3x$;

6) $f(x)=5^{1+3x^2}+3$;

7) $f(x)=3\arcsin 2x$;

8) $f(x)=\sqrt[3]{2-3x}$ $f(x)=\sqrt[3]{2-3x}$;

2.3. Funksiyaning hosilasini toping:

1) $f(x)=3e^{2-5x}$;

2) $f(x)=tg^2 2x$;

3) $f(x)=ctg^2 4x$;

4) $f(x)=\ln(tg2x)$;

5) $f(x)=sin(5-3x)$;

6) $f(x)=\sin 4x+x$;

2.4. Agar: 1) $f(x)=\dfrac{x^3-3x^2+2}{x}$; 2) $f(x)=\dfrac{2}{x}-\dfrac{1}{x^2}$, $f'(1)$ ni toping.

2.5. $f(x)$ funksiya hosilasi nolga teng bo'ladigan xning qiymatlarini toping:

1) $f(x)=3^x+3^{-x}$;

2) $f(x)=\sin^2\dfrac{x}{2}$;

3) $f(x)=5^{2x}-2x\ln 5$;

4) $f(x)=\sqrt[3]{1+2x-x^2}$.

2.6. Agar $g(x)=\dfrac{3}{1+x+x^2}$ bo'lsa, $g'(x)\geq 0$ ni toping.

2.7. $f(x)=\cos(\cos x)$ bo'lsa, $f'\left(\dfrac{\pi}{2}\right)$ ni toping.

2.8. Funksiyaning hosilasini toping:

1) $f(x)=\dfrac{1}{(4-3x)^2}$;

2) $f(x)=\cos^2 4x-\sqrt[3]{2x^2+1}$;

3) $f(x)=e^{(2x+1)^4}$;

4) $f(x)=\dfrac{-1}{\sqrt[3]{x^2}}+\ln\sqrt{x^2-3}$.

2.9. $f(x)=(x-2)^x$ bo'lsa, $f'(1)$ ni toping.

2.10. Funksiyaning hosilasini toping:

1) $f(x)=x\ln x$;

2) $f(x)=x^3\ln x-e^{2x}$;

3) $f(x)=(2x-1)^2\lg x$;

4) $y=\dfrac{1-\cos 2x}{1+\cos 2x}$;

5) $f(x)=(3x+1)^3\cos x$;

6) $f(x)=(1-x)\sqrt{x}$;

7) $y=\dfrac{2x}{\sqrt{3x+1}}$;

8) $y=\dfrac{x}{1+e^x}$;

9) $y=\dfrac{12x\sqrt[3]{x}}{\sqrt{x}}$;

10) $f(x)=\dfrac{2x^2-3x+1}{1+2x}$.

2.11. $f(x)=e^{2x}(x^2-1)$ funksiya 0 ga teng bo'ladigan nuqtalarda uning hosilasining qiymatlarini toping.

2.12. $f(x)$ funksiya hosilasining x_0 nuqtadagi qiymatini toping:

1) $f(x)=2e^{3x-5}+\ln(x^2+x)$, $x_0=2$; 2) $f(x)=\log_3(2x^2+1)$, $x_0=1$.

2.13. $f(x)=|4-x^2|$ bo'lsa, $f'(7)$ ni toping.

2.14. $y=\dfrac{(11x+2)^2}{e^x}$ funksiya hosilasining $x_0=0$ nuqtadagi qiymatini toping.

2.15. $y=\left|x^2-3x+2\right|$ bo'lsa, $y'(-3)$ ni toping.

2.16. $f(x)$ funksiya hosilasining x_0 nuqtadagi qiymatini toping:

1) $f(x)=\dfrac{x^2}{1+x}$, $x_0=1$; 2) $f(x)=\sqrt{x^2+3x+4}$, $x_0=0$;

3) $f(x)=\ln(x^2+1)$, $x_0=2$; 4) $f(x)=(2x^2+3x)\cos 2x$, $x_0=0$.

2.17. Funksiyaning hosilasini toping:

1) $y=\sin^3(3x^2+2x)$; 2) $y=\ln(\cos 2x)$; 3) $y=\ln\dfrac{2tgx}{1-tg^2x}$

3-мисол: Quyida berilgan funksiyalarning o'sish va kamayish oraliqlarini toping.

3.1. Funksiyaning o'sish oralig'ini toping:

1) $y=2x^2-6x+5$;

2) $y=\dfrac{1}{3}x^3-2x^2+3x+1$;

3) $y=\dfrac{2x+3}{x-2}$;

4) $y=\dfrac{1+x}{\sqrt{x}}$;

5) $y=6x-2x^3$;

6) $f(x)=\sqrt{2}x-\sin^2 x$.

3.2. Funksiyaning kamayish oralig'ini toping:

1) $y=\dfrac{1}{4}x^4-\dfrac{1}{3}x^3-x^2$;

2) $y=(x+2)e^{-2x}$;

3) $y=0{,}5x-\sin x$;

4) $y=\ln(x^2-8x)$.

3.3. k ning qanday qiymatlarida $y=kx-\cos x$ funksiya aniqlanish sohasida kamayadi?

3.4. Quyidagi funksiyalardan qaysi biri monoton o'suvchi ?

1) $y=3+5x$;

2) $f(x)=2x^2-3x$;

3) $y=\dfrac{15}{x+1}$;

4) $y=\sqrt{x+5}$;

5) $y=3\sin x+1$;

6) $f(x)=3x^3+2x$.

3.5. k ning qanday qiymatlarida $y=\sin 2x+kx$ funksiya aniqlanish sohasida o'sadi?

3.6. $f(x)=\dfrac{1}{5}x^5-4x^2$ funksiyaning kamayish oralig'i uzunligini toping.

3.7. k ning qanday qiymatlarida $y=kx^3+3x^2-2x+1$ funksiya barcha haqiqiy sonlar to'plamida kamayadi ?

3.8. $y=2\cos 4x+4x$ funksiyaning o'sish oralig'ini toping.

3.9. Funksiyaning o'sish va kamayish oraliqlarini toping.

1) $y=x-\dfrac{4}{x}$; 2) $y=\dfrac{x}{1+x^2}$; 3) $y=x^3-6x^2+10$.

3.10. k ning qanday qiymatlarida $y=\sin x-kx$ funksiya aniqlanish sohasida kamayadi?

3.11. k ning qanday qiymatlarida $y=x^3-2x^2+kx+10$ funksiya barcha haqiqiy sonlar to'plamida o'sadi ?

4-misol: Quyidagi misollarda berilgan Funksiyalarning ekstremumlarini toping.

4.1. Funksiyaning statsionar nuqtalarini toping:

1) $y=2x^3-15x^2+36x$; 2) $y=\dfrac{x}{2}+\dfrac{8}{x}$; 3) $y=x+\sqrt{3-x}$.

4.2. Funksiyaning ekstremum nuqtalarini va uning shu nuqtalardagi qiymatlarini toping:

1) $y=x^3-6x^2+1$; 2) $y=e^{\sqrt{3-x^2}}$; 3) $y=(x-1)e^{3x}$.

4.3. $f(x)=(2x^2-34x+34)e^{x-34}$ funksiyaning minimum nuqtasini toping.

4.4. $y=11+6\sqrt{x}-2x\sqrt{x}$ funksiyaning maksimum nuqtasini toping.

4.5. $y=\sqrt{x^2-4x+13}$ funksiyaning qiymatlar sohasini toping.

4.6. $f(x)=x\sin x+\cos x-\frac{3}{4}\sin x$ funksiyaning $\left(0;\frac{\pi}{2}\right)$ kesmadagi minimum nuqtasini toping.

4.7. $f(x)=3x^5+5x^3+10$ funksiya nechta ekstremum nuqtaga ega?

4.8. Agar $4x+y=10$ bo'lsa, $x\cdot y$ ning eng katta qiymatini toping.

4.9. $f(x)=(21-x)e^{21-x}$ funksiyaning minimum nuqtasini toping.

4.10. $y=\frac{x^2+2x+5}{x^2+2x+3}$ funksiyaning eng katta qiymatini toping.

4.11. $y=x^2+px+q$ kvadrat funksiya $x=5$ da 1 ga teng minimumga ega bo'lishi uchun p va q koeffitsiyentlar qanday bo'lishi kerak?

4.12. Funksiyaning maksimumini toping:

1) $y=-x^3+3x$; 2) $y=(x+2)^3(x+3)^2$.

4.13. Funksiyaning minimumini toping:

1) $y=\frac{x^2+6x+3}{3x+4}$; 2) $y=x^4-4x^2+5$.

4.14. ax^2+bx+c ko'phad $x=2$ da 3 ga teng eng katta qiymatga va $x=5$ da -6 ga teng bo'lsa $a,\ b,\ c$ larni toping.

Javoblar:

1.1

A) $f'(x)=3$.

B) $f'(x)=-2$.

C) $f'(x)=6x^2-1$.

D) $f'(x)=-2x+20x^3$.

E) $f'(x)=\frac{1}{2\sqrt{x}}+5x^4$.

F) $f'(x)=3\cos x+\sin x$.

G) $f'(x)=3^x\ln 3-e^x$.

H) $f'(x)=\frac{1}{x\ln 4}-2^x\ln 2$.

I) $f'(x)=\frac{4}{\sin^2 2x}$.

J) $f'(x)=3\cos x+12x^5$.

K) $f'(x)=\frac{4}{1+x^2}$.

L) $f'(x)=\frac{1}{x}+\frac{1}{5\sqrt[5]{x^4}}$.

M) $f'(x)=\frac{1}{\sqrt{1-x^2}}$.

N) $f'(x)=\frac{1}{\cos^2 x}-\sin x$.

O) $f'(x)=\frac{1}{\sqrt{1-x^2}}-\cos x$.

P) $f'(x)=\frac{1}{2\sqrt{x}}-\frac{1}{3\sqrt[3]{x^2}}$.

Q) $f'(x)=\frac{5}{1+x^2}$.

R) $f'(x)=5^x\ln 5+\frac{2}{x}$.

1.2 $f'(1)=\frac{1}{2}$.

1.3 *1)* $x_1=-1,\ x_2=2$ *2)* $x_1=-\frac{7}{3},\ x_2=1$.

1.4 *1)* $f'(3)=\frac{14\sqrt{3}}{9}$, *2)* $f'(3)=\frac{4-9\sqrt{3}}{54}$, *3)* $f'(3)=\frac{9\sqrt{3}-6}{54}$.

1.5 **1)** $x=-\frac{\pi}{2}+2\pi n$, **2)** $x_1=-2,\ x_2=4$ **3)** $x=\pm\frac{\pi}{3}+2\pi n$.

1.6 **1)** $(-\infty;0)$ $(1;\infty)$, **2)** $(-\infty;0)$, **3)** $(-\frac{\pi}{2}+2\pi n;\frac{\pi}{2}+2\pi n)$

1.7 **1)** $f'(2)=2e^2$, **2)** $f'(1)=\dfrac{1-2\ln^2 2}{\ln 2}$.

1.8 **1)** $[0;1]$, **2)** $(-\infty;0)$.

1.9 **1)** $x=\pm\dfrac{\pi}{3}+2\pi n$, **2)** $x=(-1)^{n+1}\dfrac{\pi}{3}+\pi n$.

2.1

1) $f'(x)=16(2x+1)$,

2) $f'(x)=-2\cdot 4^{3-2x}\ln 4+15(5x-7)^3$

3) $f'(x)=18x(3x^2+1)^2$,

4) $f'(x)=-2(3-x)-3^{-x}\ln 3$,

5) $f'(x)=\dfrac{3}{2\sqrt{4+3x}}$,

6) $f'(x)=\dfrac{x}{\sqrt{x^2-5x}}$,

7) $f'(x)=2x\cdot 3^{x^2}\ln 3+e^{2-x}$,

8) $f'(x)=30(5x+6)^5$.

2.2

1) $f'(x)=-15(2-5x)^2-3e^{3x+1}$,

2) $f'(x)=-\dfrac{5}{(3-5x)\ln 7}$,

3) $f'(x)=\dfrac{2x}{\cos^2 x^2}-\dfrac{5}{3\sqrt[3]{(5x+1)^2}}$,

4) $f'(x)=\dfrac{6x+1}{3x^2+x}$,

5) $f'(x)=-3\sin 6x$,

6) $f'(x)=6x\cdot 5^{1+3x^2}\ln 5$,

7) $f'(x)=\dfrac{6}{\sqrt{1-4x^2}}$,

8) $f'(x)=-\dfrac{1}{\sqrt[3]{(2-3x)^2}}$.

2.3

1) $f'(x)=-15e^{2-5x}$,

2) $f'(x)=\dfrac{4\sin 2x}{\cos^3 4x}$,

3) $f'(x)=-\dfrac{8\cos 4x}{\sin^3 4x}$,

4) $f'(x)=\dfrac{4}{\sin 4x}$,

5) $f'(x)=-3\cos(5-3x)$, **6)** $f'(x)=4\cos 4x+1$.

2.4 **1)** $f'(1)=-9$, **2)** $f'(1)=0$.

2.5 **1)** $x=0$, **2)** $x=\pi n$, **3)** $x=0$, **4)**

2.6 $(-\infty;-0,5]$.

2.7 $f'(\frac{\pi}{2})=0$.

2.8

1) $f'(x)=\dfrac{6}{(4-3x)^3}$,

2) $f'(x)=-4\sin 8x-\dfrac{4x}{3\sqrt[3]{(2x^2+1)^2}}$,

3) $f'(x)=8(2x+1)^3\cdot e^{(2x+1)^4}$,

4) $f'(x)=\dfrac{2}{3\sqrt[3]{x^5}}+\dfrac{2x}{\sqrt{x^2-3}}$.

2.9 $\varnothing$.

2.10

1) $f'(x)=\ln x+1$,

2) $f'(x)=3x^2\ln x+x^2-2e^{2x}$,

3) $f'(x)=(4x-2)\lg x+\dfrac{(2x-1)^2}{x\ln 10}$,

4) $f'(x)=\dfrac{2\sin x}{\cos^3 x}$,

5) $f'(x)=(3x+1)^2(9\cos x-(3x+1)\sin x)$,

6) $f'(x)=\dfrac{-3x+1}{2\sqrt{x}}$,

7) $f'(x)=\dfrac{2}{(3x+1)\sqrt{3x+1}}$,

8) $f'(x)=\dfrac{1+e^x-xe^x}{(1+e^x)^2}$,

9) $f'(x)=\dfrac{10}{\sqrt[6]{x}}$,

10) $f'(x)=\dfrac{4x^2+4x-5}{(2x+1)^2}$.

2.11 $f'(1)=2e^2$, $f'(-1)=-2e^{-2}$.

2.12 $f'(2)=6e+\frac{5}{6}$, **2)** $f'(1)=\frac{4}{5\ln 3}$.

2.13 $f'(7)=-14$.

2.14 $f'(1)=-117$,

2.15 $y'(5)-y'(-3)=16$.

2.16

1) $f'(1)=\frac{3}{4}$,

2) $f'(0)=\frac{3}{4}$,

3) $f'(2)=\frac{4}{5}$,

4) $f'(0)=3$.

2.17

1) $y'=3(6x+2)\sin^2(3x^2+2x)\cos(3x^2+2x)$,

2) $y'=-2tg2x$,

3) $y'=\frac{4}{\sin 4x}$.

3.1

1) $[-1;0]\quad[2;\infty)$,

2) $(-\infty;1]\quad[3;\infty)$,

3) $(-\infty;\infty)$,

4) $[1;\infty)$,

5) $[-1;1]$,

6) $(-\infty;\infty)$.

3.2

1) $(-\infty;-1]\quad[0;2]$,

2) $[1,5;\infty)$,

3) $[-\frac{\pi}{3}+2\pi n;\frac{\pi}{3}+2\pi n]$,

4) $(-\infty;0]\quad[4;\infty)$.

3.3. $(-\infty;-1)$.

3.4

1) o'suvchi,

2) har doim ham o'suvchi emas,

3) kamayuvchi,

4) o'suvchi,

5) doim ham o'suvchi emas,

6) o'suvchi.

3.5 $[2;\infty)$.

3.6 2.

3.7 $(-\infty;-\frac{3}{2})$.

3.8 $(-\frac{7\pi}{6}+\frac{\pi}{2}n;\frac{\pi}{24}+\frac{\pi}{2}n)$.

3.9

1) $(-\infty;\infty)$ o'suvchi

2) $(-\infty;-1]\quad[1;\infty)$ kamayuvchi, $[-1;1]$ o'suvchi,

3) $(-\infty;0]\quad[4;\infty)$ o'suvchi, $[0;4]$kamayuvchi.

3.10 $k\in(1;\infty)$.

3.11 $k\in(\frac{4}{3};\infty)$.

4.1

1) $x_1=2$, $x_2=3$,

2) $x=\pm 4$, $x=2,75$.

4.2

1) $y_{min}(4)=-31, y_{max}(0)=1$,

2) $y_{max}(0)=e^{\sqrt{3}}$,

3) $y_{min}(\frac{2}{3})=-\frac{1}{3}e$.

4.3 15.

4.4. 1.

4.5 $[3;\infty)$.

4.6 0,75.

4.7 0 ta.

4.8 25.

4.10. 22.

4.11. 2.

4.12 $p=-10,\ q=26$.

4.13 $y_{\max}=2$.

4.14 $y_{\max}=0$.

Mavzu: Hosila va uning tadbiqlari. Funksiyaning oraliqdagi eng katta va eng kichik qiymatlari

1. $f(x)=x^2-\dfrac{1}{x}$ funksiyaning $[1;\ 2]$ kesmadagi eng katta qiymatini toping.

2. $y=2\sin x+\cos 2x$ va funksiyaning $\left[0;\dfrac{\pi}{2}\right]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

3. $y=9\ln(x+8)-9x+12$ funksiyaning $[-7,5;0]$ oraliqdagi eng katta qiymatini toping.

4. $f(x)=6tgx-6x+6$ funksiyaning $\left[\dfrac{-\pi}{4};0\right]$ kesmadagi eng katta qiymatini toping.

5. $y=\sqrt{16-3x^2}$ funksiyaning $y=\sqrt{16-3x^2}$ kesmadagi eng katta va eng kichik qiymatlari ayirmasini toping.

6. $f(x)=2\sqrt{2}\cos x+2x-\dfrac{\pi}{2}+5$ funksiyaning $\left[0;\dfrac{\pi}{2}\right]$ kesmadagi eng katta qiymatini toping.

7. $f(x)=2\cos x+\sin 2x$ funksiyaning $[0;\pi]$ kesmadagi eng kichik qiymatini toping.

8. $f(x)=(10-x)\sqrt{x+2}$ funksiyaning $[-1;7]$ kesmadagi eng katta qiymatini toping.

9. $M(2;\ 1)$ nuqtadan $y=3x-1$ to‘g‘ri chiziqqacha bo‘lgan eng qisqa masofani toping.

10. $f(x)=\ln x-x$ funksiyaning $[0,5\ ;\ 4]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

11. $f(x)=3\sqrt{3}\sin x\cdot\sin 2x$ funksiyaning $\left[-\dfrac{\pi}{2};\dfrac{\pi}{2}\right]$ kesmadagi eng katta qiymatini toping.

12. $y=\dfrac{2x^2+x+1}{2-x+3x^2}$ funksiyaning eng kichik qiymatini toping.

13. a ning qanday qiymatida $y=ax^2+3x-5$ funksiya $x=-3$ nuqtada eng kichik qiymatga ega bo'ladi ?

Urinma tenglamasi va urinmaning burchak koeffisiyenti.

14. $f(x)=x^2-3x$ funksiyaga $x=2$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

15. $f(x)=e^{x-1}$ funksiyaga $x=1$ nuqtada o'tkazilgan urinma tenglamasini tuzing.

16. $f(x)=\sqrt{4x+1}$ funksiya grafigining shunday nuqtasini topingki, bu nuqtada shu grafikka o'tkazilgan urinma $y=\dfrac{2}{3}x$ urinma to'g'ri chiziqqa parallel bo'lsin.

17. $y=\dfrac{3}{x}$ funksiyaga $M(1;\ 3)$ nuqtada o'tkazilgan urinma va koordinata o'qlari bilan chegaralangan uchburchakning yuzini toping.

18. $y=x^3-x+1$ funksiya grafigining Oy o'qi bilan kesishish nuqtasida unga o'tkazilgan urinmaning burchak koeffitsiyentini toping.

19. $y=2$ ordinatali nuqtada $y=3x^3-1$ funksiyaning grafigiga o'tkazilgan urinmaning burchak koeffitsiyentini toping.

20. $y=\dfrac{x}{1-x}$ funksiya grafigiga abssissasi $x_0=3$ bo'lgan nuqtasida o'tkazilgan urinmaning Ox *o'qi bilan tashkil etgan burchagi* α bo'lsa, $ctg2\alpha$ ni toping.

21.* $y=ax^2+bx+1$ funksiyaning $M(-2;2)$ nuqtasiga o'tkazilgan urinmasi $y=-1,5x-1$ bo'lsa, $a+b$ ni toping.

22.* $y=x^2-5x+6$ funksiyaning OX o'qi bilan kesishgan nuqtalaridan o'tuvchi urinmalari orasidagi burchakni toping.

23.* $y=2x+\dfrac{a}{x}$ funksiyaga $x_0=1$ nuqtada o'tkazilgan urinma $y=-4x+5$ to'g'ri chiziqqa parallel bo'lsa, a ni toping.

24.* $y=4x-1$ to'g'ri chiziq $y=x^2+c$ funksiyaning urinmasi bo'lsa, urinish nuqtasi ordinatasini toping.

25. $\dfrac{1}{3}x^3-2x$ funksiyaning grafigiga o'tkazilgan urinma Ox o'qining musbat yo'nalishi bilan 135° burchak tashkil qiladi. Urinish nuqtasining koordinatasini toping.

26. Qaysi nuqtada $y_1=3x^3-x^2+2x$ va $y_2=3x^3-4x+1$ funksiyalarning grafiklariga o'tkazilgan urinmalar o'zaro parallel bo'ladi.

27. $y=\ln 2x$ funksiya grafigining M nuqtasiga o'tkazilgan urinma og'ish burchagining tangensi $\sqrt{2}$ ga teng bo'lsa, M nuqtaning absissasini toping.

Hosilaning mexanik ma'nosi.

28. Jism to'gri chiziq bo'ylab $s(t)=5+2t+3t^2$ qonun bo'yicha harakatlanmoqda. Jismning harakat boshlanganidan 5 c dan keyingi tezligini toping.

29. To'g'ri chiziq bo'ylab $x(t)=\dfrac{1}{3}t^3-2t^2-5$ qonun bo'yicha harakatlanayotgan moddiy nuqta harakat boshlanganidan necha sekunddan keyin to'xtaydi ?

30. Moddiy nuqta $s(t)=6+4,5t^2-3t^3$ qonuniyat bo'yicha harakatlaniyapti. Uning tezlanishi 0 ga teng bo'lganda tezligi qanchaga teng bo'ladi.

31. $s(t)=3t^2-\dfrac{1}{3}t^3$ qonuniyat bo'yicha harakatlaniyotgan jismning eng katta tezligini toping.

32. $x(t)=\dfrac{1}{3}t^3-\dfrac{1}{2}t^2-6t$ qonuniyat bo'yicha to'gri chiziq bo'ylab harakatlanayotgan nuqta harakat boshlanganidan necha sekunddan keyin to'xtaydi ?

33. $s(t)=6+4,5t^2-3t^3$ qonuniyat bo'yicha harakatlaniyotgan jismning tezligi harakat boshlaganidan keyin qancha vaqtdan keyin 1 $м/c$ ga teng bo'ladi?

34. Ikkita nuqta $s_1(t)=8-9t+3t^2$ va $s_2(t)=-5+3t+0,5t^2$ qonuniyat bilan harakatlaniyapti . Vaqtning qaysi paytida ikkinchi nuqtaning tezligi birinchi nuqtaning tezligidan 3 marta kichik bo'ladi?

35. Jism $s(t)=6t^2-t^3$ qonuniyat bo'yicha harakatlaniyotgan jismning eng katta tezligini toping.

Hosila bo'yicha umumiy takrorlash.

36. $f(x)=x^2+6|x|$ bo'lsa, $f'(-3)$ ni toping.

37. $y=x^2-x+1$ funksiya grafigining qaysi nuqtasida o'tkazilgan urinma $y=3x-1$ to'g'ri chiziqqa parallel bo'ladi ?

38. $f(x)=\dfrac{3x-4}{x+7}$ бўлса, $f'(x)=25$ tenglamani yeching.

39.. a ning qanday qiymatida $y=x^2+ax+4$ funksiya grafigi Ox o'qqa urinadi?

40. $y=x^2+2$ funksiya grafigiga o'tkazilgan urinma $y=4x-1$ to'g'ri chiziqqa parallel. Urinish nuqtasining ordinatasini toping.

41. $y=\dfrac{\sin 6x}{2\sqrt{3}}$ funksiya grafigi Ox o'qni $(0;0)$ nuqtada qanday burchak ostida kesib o'tadi ?

42. Abssissasi qanday nuqtada $y_1=x^3-3x^2+8$ va $y_2=x^3-6\ln x$ funksiyalarning grafiklariga o'tkazilgan urinmalar o'zaro parallel bo'ladi?

43. $y_2=x^3-6\ln x$ parabolaga $x=0$ abssissali nuqtada o'tkazilgan urinmaning tenglamasi $y=4x-3$ bo'lsa, $b-c$ ni toping.

44. $y=\cos(\cos x)$ bo'lsa, $y'(\frac{\pi}{2})$ ni toping.

45. $y=x^2\cdot e^{-x}$ funksiya grafigiga $x=2$ abssissali nuqtada o'tkazilgan urinma tenglamasini toping.

46. Агар $f(x)=\dfrac{9x^2+bx+c}{x^3}$ funksiya grafigi $(3;0)$ nuqtada Ox o'qqa urinib o'tsa, $\sqrt{b+c}$ ni toping.

47. $y=\dfrac{x^2-3x+2}{x^2+3x+2}$ funksiyaning ekstremum nuqtalarini toping.

48. $y=x\sqrt[3]{4-x}$ funksiyaning $(0;4)$ oraliqdagi eng katta qiymatini toping.

49. $y=(x-2)^2+3$ funksiya grafigiga $y=-2x+5$ to'g'ri chiziqqa parallel qilib urinma o'tkazilgan. Urinish nuqtasidan koordinata boshigacha bo'lgan masofani toping.

50. $f(x)=\dfrac{7}{\sqrt{x^2+24}}$ funksiyaning $[-5;5]$ kesmadagi eng katta va eng kichik qiymatlari ko'paytmasini toping.

51. ax^2+bx+c kvadrat uchhad $x=8$ da nolga aylanishi va $x=6$ da -12 ga teng eng kichik qiymatni qabul qilishi ma'lum bo'lsa, $\sqrt{a+b+c}$ ni toping.

52. $f(x)=\dfrac{x}{\ln x}$ funksiyaning o'sish oralig'ini toping.

53. $s(t)=2t^3-3t+4$ qonuniyat bo'yicha to'gri chiziq bo'ylab harakatlanayotgan jismning $t=2$ vaqtdagi tezligi va tezlanishini toping.

54. Koordinata o'qlari va $y=\dfrac{10x}{4x-2}$ funksiyaga $x_0=1$ nuqtada o'tkazilgan urinma bilan chegaralangan uchburchakning yuzini toping.

55. $y=\dfrac{x}{1+x}$ funksiya grafigiga abssissasi $x_0=-3$ bo'lgan nuqtasida o'tkazilgan urinmaning Ox o'qi bilan tashkil etgan burchagi α bo'lsa, $\sin 2\alpha$ ni toping.

Javoblar:

1. 3,5.

2. 1,5 ва 1.

3. 75.

4. 6.

5. 3.

6. 7.

7. $0,4\sqrt{10}$

8. 16.

9. $0,4\sqrt{10}$.

10. $\ln 4-4$, -1.

11. 4.

12. $\frac{7}{23}$.

13. $a=0,5$.

14. $y=x-4$.

15. $y=x$.

16. $x_0=2$.

17. $\sqrt{40}$.

18. $k=-1$.

19. $k=9$.

20. $\frac{15}{8}$.

21. 1.

22. 90°.

23. 6.

24. 7.

25. $x=\pm 1$.

26. $x=3$.

27. $x=\frac{1}{\sqrt{2}}$.

28. $s'(5)=32$.

29. $t=4$.

30. $v(\frac{1}{2})=\frac{9}{4}$.

31. $v(3)=9$.

32. $t=3$.

33. $t=3$.

34. $t=6$.

35. $v_{max}=12$.

36. -12.

37. $(2;3)$.

38. -8 ва -6.

39. $a=\pm 4$.

40. 6.

41. 60°.

42. 1.

43. 7.

44. 0.

45. $y=\frac{4}{e^2}$.

46. $3\sqrt{3}$.

47. $x_{min}=\sqrt{2}$, $x_{max}=-\sqrt{2}$.

48. 3.

49. $\sqrt{17}$.

50. $\frac{7\sqrt{6}}{2}$.

51. $\sqrt{63}$.

52. $x\ge e$.

53. $v=21$, $a=24$.

55. $\frac{8}{17}$.

Mavzu: Boshlang'ich funksiya va aniqmas integrallarni hisoblash bo'yicha misol va masalalar

1. Funksiyalarning boshlang'ich funksiyalarini toping.

a) $2x^5-3x^2$;

b) $5x^4+2x^3$;

c) $\frac{2}{x}+\frac{3}{x^2}$;

d) $\frac{2}{x^3}-\frac{3}{x}$;

e) $\sqrt{x}+2\sqrt[3]{x}$;

f) $4\sqrt[3]{x}-6\sqrt{x}$;

g) $3x^3+2x-1$;

h) $6x^2-4x+3$;

i) $2x^2-3x+1$;

2. Funksiyalarning boshlang'ich funksiyalarini toping.

a) $3\cos x-4\sin x$;

b) $5\sin x+2\cos x$

c) $e^x-2\cos x$;

d) $3e^x-\sin x$;

e) $5-e^{-x}+3\cos x$;

f) $1+3e^x-4\cos x$;

g) $6\sqrt[3]{x}-\frac{2}{x}+3e^x$

h) $\frac{4}{\sqrt{x}}+\frac{3}{x}-2e^{-x}$

3. Quyidagilarning boshlang'ich funksiyalarini toping.

a) $(x+1)^4$

b) $(x-2)^3$

c) $\frac{2}{\sqrt{x-2}}$

d) $\frac{3}{\sqrt[3]{x+3}}$

e) $\frac{1}{x-1}+4\cos(x+2)$

f) $\frac{3}{x-3}-2\sin(x-1)$

4. Funksiyalarning boshlang'ich funksiyalarini toping.

a) $\sqrt[3]{\frac{x}{4}}-3\cos(6x-1)$

b) $\sqrt{\frac{x}{5}}+4\sin(4x+1)$

c) $\frac{3}{\sqrt[3]{2x-1}}+\frac{2}{1-x}$

f) $\frac{2x^4-4x^3+x}{3}$

d) $\frac{4}{\sqrt{3x+2}}-\frac{3}{2x-5}$

g) $\frac{6x^3-3x+2}{5}$

5. $f(x)$ funksiya uchun grafigi M nuqta orqali o'tadigan boshlang'ich funksiyani toping.

a) $f(x)=x^2$, $M(1;2)$

b) $f(x)=x$, $M(-1;3)$

c) $f(x)=\frac{1}{x^2}$, $M(1:1)$

d) $f(x)=\sqrt{x}$, $M(9;10)$

6. Agar $F(\frac{3}{2})=1$ bo'lsa, $f(x)=\frac{6}{(4-3x)^2}$ funksiyaning boshlang'ich funksiyasi $F(x)$ ni toping.

7. Agar $F(-1)=3$ bo'lsa, $f(x)=6x^2-3x-2{,}5$ funksiyaning boshlang'ich funksiyasi $F(x)$ ning $x=-2$ nuqtadagi qiymatini toping.

8. Agar $F(0)=3$ bo'lsa, $f(x)=e^{\frac{x}{2}}+\frac{1}{2x+1}$ funksiyaning boshlang'ich funksiyasi $F(x)=?$

9. $f(x)=3x^2+2x-3$ funksiya uchun grafigi $M(1;-2)$ nuqtadan o'tuvchi boshlang'ich funksiyani toping.

10. Hisoblang.

a) $\int\frac{2(1+x^2)-\sqrt{1-x^2}}{(1+x^3)\sqrt{1-x^2}}dx$;

b) $\int\left(\frac{5}{\sin^2 x}+\frac{6}{\cos^2 x}\right)dx$

Quyidagi funksiyalarning boshlang'ich funksiyasining umumiy ko'rinishini toping.

11.

a) $y=3x^2-8x+1$

b) $y=4x^3-12x^{-2}+5$

c) $y=\frac{2}{x}-\frac{1}{x^2}$

d) $y=6\sqrt{x}-24\sqrt[3]{x}$

e) $y=\frac{1}{\sqrt{x}}+x\cdot\sqrt[3]{x}+2$

f) $y=15x^4+\frac{1}{x}-\frac{12}{\sqrt[3]{x}}$

12.

a) $y=\sin x-3\cos x-10$

b) $y=\cos 3x-\frac{1}{tgx}$

c) $y=1-2\sin^2 x$

d) $y=-\frac{3}{e^x}$

e) $y=e^{2-3x}$

f) $y=\frac{1}{\cos^2\left(\frac{x}{4}+1\right)}$

13. $f(x)=-x+\frac{x^2}{2}$ funksiyaning $(6;2)$ nuqtadan o'tuvchi boshlang'ich funksiyasini toping.

14. $F'(x)=x-4$ va $F(-2)=0$ bo'lsa, $F(x)$ funksiyani aniqlang.

15. $y=x+\frac{1}{\sin^2 2x}+1$ ushbu funksiyaning boshlang'ich funksiyasining umumiy ko'rinishini toping.

16. Ushbu funksiyalardan $f(x)$ ni toping

a) $F(x)=tgx-4x^2+5x+C$

b) $F(x)=c\,tgx+tgx-2x+C$

c) $F(x)=\sin x+\frac{1}{2}x^2-x+C$

d) $F(x)=tgx+\frac{1}{2}\cos 2x-e^x+C$

17. $4y'=y$ tenglamani yeching.

18. Agar $y=F(x)$ funksiya $y=f(x)$ funksiyaning boshlang'ich funksiyasi bo'lsa, $y=-4f(-2x)$ funksiyaning boshlang'ich funksiyasini toping.

19. Agar $F'(x)=\cos x$ va $F(1)=3$ bo'lsa, $F(x)$ ni toping.

20. $f(x)=(x-1)x^3+3^{3x}-x^2+\frac{1}{3x}$ ushbu funksiyaning boshlang'ich funksiyasini toping.

Javoblar:

1.

a) $\frac{x^6}{3}-x^3+C$

b) $x^5+\frac{x^4}{2}+C$

c) $2\ln|x|-\frac{3}{x}+C$

d) $-\frac{1}{x^2}-3\ln|x|+C$

e) $\frac{2\sqrt[3]{x^2}}{3}+\frac{3x\sqrt[3]{x}}{2}+C$

f) $3x^{\frac{4}{3}}-4x^{\frac{3}{2}}+C$

g) $\frac{3x^4}{4}+x^2-x+C$

i) $\frac{2x^3}{3}-\frac{3x^2}{2}+x+C$

h) $2x^3-2x^2+3x+C$

2.

a) $3\sin x+4\cos x+C$

b) $-5\cos x+2\sin x+C$

c) $e^x-2\sin x+C$

d) $3e^x+\cos x+C$

e) $5x+e^x+3\sin x+C$

f) $x+3e^x-4\sin x+C$

g) $4{,}5x^{\frac{4}{3}}-2\ln|x|+3e^x$

h) $8\sqrt{8}+3\ln|x|+2e^x+C$

3.

a) $\frac{(x+1)^5}{5}+C$

b) $\frac{(x-2)^4}{4}+C$

c) $4(x-2)^{\frac{1}{2}}+C$

d) $4{,}5(x+3)^{\frac{2}{3}}+C$

e) $\ln|x-1|+4\sin(x+2)+C$

f) $3\ln|x-3|+2\cos(x-1)+C$

4.

a) $\frac{3x\sqrt[3]{x}}{4\sqrt[3]{4}}-\frac{\sin(6x-1)}{2}+C$

b) $\frac{2x\sqrt{x}}{3\sqrt{5}}-\cos(4x+1)+C$

c) $\frac{9(2x-1)^{\frac{2}{3}}}{4}-2\ln|1-x|+C$

d) $\frac{8\sqrt{3x+2}}{3}-\frac{3\ln|2x-5|}{2}+C$

f) $\frac{2x^5}{15}-\frac{x^4}{3}+\frac{x^2}{6}+C$

g) $\frac{3x^4}{10}-\frac{3x^2}{10}+\frac{2x}{5}+C$

5.

a) $\frac{x^3}{3}+\frac{5}{3}$

b) $\frac{x^2}{2}+\frac{5}{2}$

c) $-\frac{1}{x}+2$

d) $\frac{2x\sqrt{x}}{3}-8$

6. $F(x)=\frac{2}{4-3x}-5$

7. -23

8. $F(x)=2e^{\frac{x}{2}}+\frac{\ln|2x+1|}{2}+1$

9. $F(x)=x^3+x^2-3x-1$

10.

a) $2\arcsin x-\mathrm{arctg}\,x+C$,

b) $-5\,\mathrm{ctg}\,x+6\,\mathrm{tg}\,x+C$.

11.

a) $Y=x^3-4x^2+x+C$,

b) $Y=x^4+12x^{-1}+5x+c$,

c) $Y=2\ln x+\frac{1}{x}+C$,

d) $Y=4x\sqrt{x}-18x\sqrt[3]{x}+C$,

e) $Y=2\sqrt{x}+\frac{3}{7}x^2\sqrt[3]{x}+2x+C$,

f) $Y=3x^5+\ln x-18\sqrt[3]{x^2}+C$.

12

a) $Y=-\cos x-3\sin x-10x+C$,

b) $Y=\frac{1}{3}\sin 3x-\ln|\sin x|+C$,

c) $Y=\frac{1}{2}\sin 2x+C$,

d) $Y=\frac{3}{e^x}+C$,

e) $Y=-\frac{1}{3}e^{2-3x}+C$,

f) $Y=4tg\left(\frac{x}{4}+1\right)+C$.

13 $F(x)=-\frac{x^2}{2}+\frac{x^3}{6}-16$.

14 $F(x)=\frac{x^2}{2}-4x-10$.

15 $Y=\frac{x^2}{2}-\frac{ctg2x}{2}+x+C$.

16

a) $f(x)=\frac{1}{\cos^2 x}-8x+5$,

b) $f(x)=\frac{4\cos 2x+2\sin^2 2x}{\sin^2 2x}$,

c) $f(x)=\cos x+x-1$,

d) $f(x)=\frac{1}{\cos^2 x}-\sin 2x-e^x$.

17 $y=e^{\frac{1}{4}x}\cdot C$.

18 $Y=2F(-2x)+C$.

19 $\sin x-\sin 1+3$.

20 $F(x)=\frac{x^5}{5}-\frac{x^4}{4}+\frac{e^{3x}}{3}-\frac{x^3}{3}+\frac{1}{3}\ln x+C$.

Mavzu: Aniq integral va uning tadbiqlariga oid umumiy takrorlash misollari

1. Integralni hisoblang.

a) $\int_0^1 xdx$;

b) $\int_0^3 x^2dx$;

c) $\int_{-1}^2 3x^2dx$;

d) $\int_{-2}^1 2xdx$;

e) $\int_2^3 \frac{1}{x^2}dx$;

f) $\int_1^3 \frac{1}{x^3}dx$;

g) $\int_1^7 \sqrt{x}dx$;

h) $\int_4^9 \frac{1}{\sqrt{x}}dx$;

2. Integralni hisoblang.

a) $\int_1^e \frac{1}{x}dx$;

b) $\int_0^{\ln 2} e^x dx$;

c) $\int_{-\pi}^{2\pi} \cos x\, dx$;

d) $\int_{-2\pi}^{\pi} \sin xdx$;

e) $\int_{-2\pi}^{\pi} \sin 2xdx$;

f) $\int_{-3\pi}^{0} \cos 3xdx$;

g) $\int_0^1 (e^x-2)dx$;

h) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos xdx$.

3. Integralni hisoblang.

a) $\int_{-1}^{2} (2x-3)dx$;

b) $\int_{-2}^{-1} (5-4x)dx$;

c) $\int_{-1}^{2}(1-3x^2)dx$.

d) $\int_{-1}^{1}(x^2+1)dx$;

e) $\int_{-2}^{-1}(6x^2+2x-10)dx$;

f) $\int_{0}^{2}(2x^2-5x+3)dx$.

4. Integralni hisoblang.

a) $\int_{0}^{9}(x-3\sqrt{x})dx$;

b) $\int_{1}^{9}\left(2x-\frac{3}{\sqrt{x}}\right)dx$;

c) $\int_{0}^{2}e^{3x}dx$;

d) $\int_{1}^{2}4e^{4x}dx$.

5. $\int_{1}^{b}(b-4x)dx \geq 6-5b$ tegsizlik bajariladigan $b>1$ sonlarni toping.

6. b ning qanday qiymatlarida $\int_{b/2}^{b}\frac{1+2x}{4}dx=\frac{5}{2}$ tenglik bajariladi.

7. a ning qanday qiymatida $\int_{a/2}^{a}\frac{1-2x}{3}dx=-\frac{4}{3}$ tenglik o'rinli bo'ladi.

8. Integralni hisoblang.

a) $\int_{-2}^{0}x(x+3)(2x-3)dx$;

b) $\int_{-1}^{0}(x-1)(x^2-5)dx$;

c) $\int_{1}^{3}\left(x+\frac{1}{x}\right)^3 dx$.

9. Integralni hisoblang.

a) $\int_{1}^{3}\frac{3x-1}{\sqrt[3]{x}}dx$;

b) $\int_{1}^{4}\frac{x+1}{\sqrt{x}}dx$;

c) $\int_{0}^{4}\frac{5}{\sqrt{2x+1}}dx$;

d) $\int_{0}^{7}\frac{4}{\sqrt{x+2}}dx$.

10. Integralni hisoblang.

a) $\int_{-\pi}^{\pi}\sin^2 x dx$;

b) $\int_{0}^{\frac{\pi}{2}}\sin x\cos x\, dx$

c) $\int_{0}^{\frac{\pi}{4}}(\cos^2 x-\sin^2 x)dx$

d) $\int_{0}^{\pi}(\sin^4 x+\cos^4 x)dx$.

11. Integralni hisoblang.

a) $\int_{0}^{\log_3 2}3^{0,5x}dx$;

b) $\int_{0}^{\log_2 3}2^{3x}dx$;

c) $\int_{0,5b}^{b}\frac{1}{2\ln x}dx$.

12. a ning qanday qiymatida $\int_{0,5a}^{a}e^{2x}dx=1$ tenglik o'rinli bo'ladi.

13. $\int_{1}^{a}2xdx>3$ tengsizlikni qanoatlantiruvchi barcha a larni toping.

14. $\int_{2}^{a}2xdx<5$ tengsizlikni qanoatlantiruvchi barcha a larni toping.

15. Hisoblang.

a) $\int_{-\pi/2}^{\pi}|-2\cos x|dx$;

b) $\int_{-1}^{0}(|x|+2)dx$;

c) $\int_{1}^{4}|x-3|dx$;

d) $\int_{1}^{4}|x^2-1|dx$;

e) $\int_{-1}^{0}|3^x-3^{-x}|dx$;

f) $\int_{-1}^{1}x^2|x|dx$;

g) $\int_{-2}^{1}|x^2+2x+5|dx$.

16. Bo'laklab hisoblang.

a) $\int x\sin 2x dx$; e) $\int \ln x\, dx$;

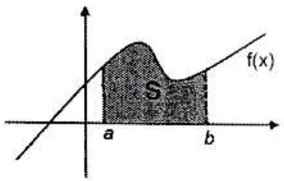
b) $\int x\cos x dx$; f) $\int x\ln x\, dx$;

c) $\int xe^{-x}dx$ g) $\int e^{x}\cos x dx$;

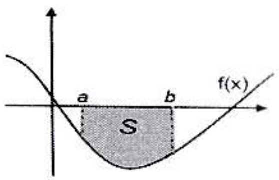
d) $\int x\cdot 3^{x}dx$; h) $\int x^{2}\cdot e^{x}dx$.

Egri chiziqli trapetsiyaning yuzi

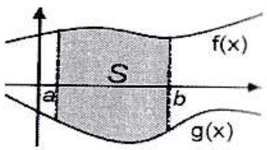
$x\in[a;b]$ da $y=f(x)$ funksiyaning grafigi va Ox o'qi bilan chegaralangan yuza - $S=\int_a^b f(x)dx$.



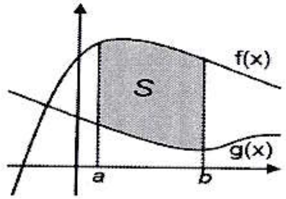
1. Agar $y=f(x)$ funksiyaning grafigi $x\in[a;b]$ da Ox o'qidan pastda joylashgan bo'lsa, u holda yuza - $S=-\int_a^b f(x)dx$



2. $x\in[a;b]$ da $y=f(x)$ va $y=g(x)$ funksiyalar grafiglari bilan chegaralangan yuza - $S=\int_a^b (f(x)+g(x))dx$.

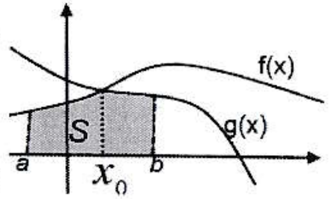


3. $x\in[a;b]$ da $y=f(x)$ va $y=g(x)$ funksiyalar grafiklari bilan chegaralangan yuza - $S=\int_a^b (f(x)-g(x))dx$.

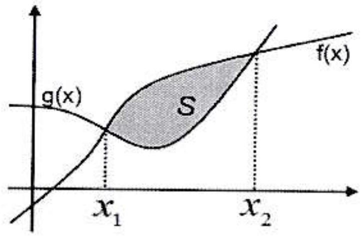


4. $x\in[a;b]$ da $y=f(x)$ va $y=g(x)$ funksiyalar grafiklari va Ox o'qi bilan chegaralangan yuza - $S=\int_a^{x_0} (f(x)-g(x))dx$

Бу ерда x_0 - $f(x)=g(x)$ tenglamaning $x\in[a;b]$ dagi ildizi.



$y=f(x)$ va $y=g(x)$ funksiyalar grafiklari bilan chegaralangan yuza - $S=\int_{x_1}^{x_2} (f(x)-g(x))dx$



Bu yerda x_1 va x_2 lar $f(x)=g(x)$ tenglamaning $x\in[a;b]$ dagi ildizlari.

17. $y=\sqrt{x}$, $y=0$ va $x=3$ chiziqlar bilan chegaralangan figuraning yuzini toping.

18. $y=-2x+5$, $y=0$ ва $x=2$ chiziqlar bilan chegaralangan yuzani hisoblang.

19. Ushbu chiziqlar bilan chegaralangan figuraning yuzini hisoblang. $y=\sin 2x$, $y=0$, $x=0$ ва $x=\frac{\pi}{4}$

20. Quyidagi chiziqlar bilan chegaralangan yuzani hisoblang.

$y=-\frac{x}{2}$, $y=0$ va $x=5$

21. Ushbu $y=-3x^2$, $y=0$, $x=1$ ва $x=2$ chiziqlar bilan chegaralangan figuraning yuzini toping.

22. Ushbu $y=4x^2$, $y=\frac{3}{x}$, va $x=e$ chiziqlar bilan chegaralangan figuraning yuzini toping.

23. $y=5x^2$, $y=2x+1$ chiziqlar bilan chegaralangan sohaning yuzini toping.

24. $y=3x^3$ va $y=4\sqrt{x}$ chiziqlar bilan chegaralangan shaklning yuzini toping.

25. $4x-6y+10=0$, $y=0$, $x=4$ va $x=5$ chiziqlar bilan chegaralangan figuraning yuzini toping.

26. $y=x$, $y=\frac{4}{x}$, $y=0$ va $x=e$ chiziqlar bilan chegaralangan figuraning yuzini toping.

27. $x=0$, $y=16-x^2$ va $y=x^2+3$ chiziqlar bilan chegaralangan sohaning yuzini toping.

28. $y=2-|x-3|$ va $y=|x-3|$ chiziqlar bilan chegaralangan sohaning yuzini toping.

29. $x\in[0;\pi]$ da $y=\cos x$ funksiyaning grafigi va / o'qi bilan chegaralangan yuzani toping.

30. $y=16-x^2$ va $y=2x^2-8$ chiziqlar bilan chegaralangan sohaning yuzini toping.

31. $y=\sqrt{x+2}$, $y=x-4$ va $y=0$ chiziqlar bilan chegaralangan shaklning yuzini toping.

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MUNDARIJA.

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