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MATEMATIKA FANIDAN

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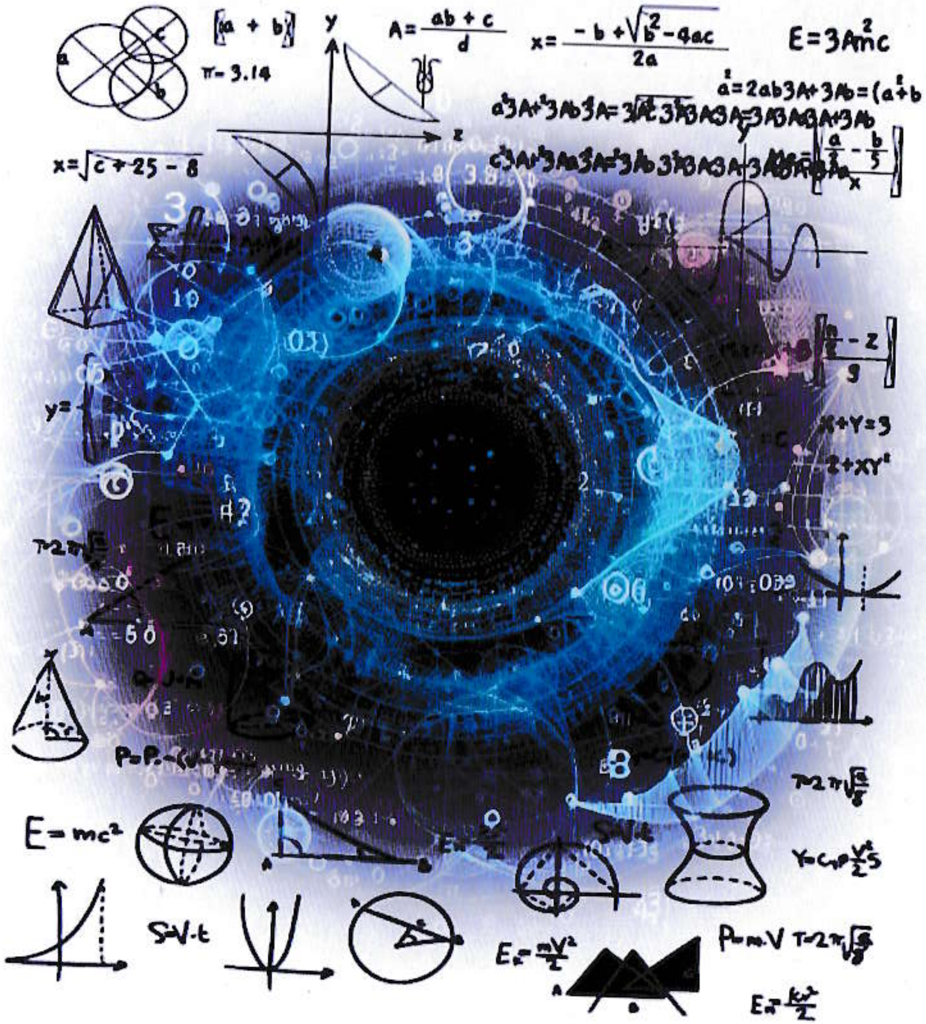

O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI
CHIRCHIQ DAVLAT PEDAGOGIKA UNIVERSITETI

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MATEMATIKA FANIDAN O'QUV QO'LLANMA

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Ushbu o'quv qo'llanma oliy ta'lim muassasalarining tabiiy fanlar yo'nalishlari bakalavrlari uchun «Oliy matematika» fan dasturi asosida yozilgan bo'lib, fanning chiziqli algebra elementlari, vektorli algebra elementlari, analitik geometriya, matematik analizga kirish, bir o'zgaruvchi funksiyasining differensial hisobi, bir o'zgaruvchi funksiyasining integral hisobi, oddiy differensial tenglamalar va ehtimollar nazariyasi bo'limlariga oid materiallarni o'z ichiga oladi. Qo'llanmada zarur nazariy tushunchalar, qoidalar, teoremalar va formulalar keltirilgan va ularning mohiyati misol va masalalar yechimlarida tushuntirilgan, talabalarning mustaqil ishlari uchun topshiriqlar berilgan. Har bir mustaqil ish topshirig'iga oid misol va masala namuna sifatida yechib ko'rsatilgan.

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SO'Z BOSHI

Oliy matematika fanini o'qitishda mutaxassislar oldida turgan muhim muammolardan biri ma'lumotni taqdim etishning darajasini tanlash, uzviylikni ta'minlash asosida talabalarning bilim ko'nikmalarida paydo bo'lishi mumkin bo'lgan bo'shliqni to'ldirish, mavzularni o'qitishdagi takomillashgan uslublarni ishlab chiqish, talabalarning mantiqiy tafakkurini rivojlantirish va zarur bilim, ko'nikmalarini hamda amaliy ko'nikmalarini shakllantirish o'rtasida to'g'ri muvozanatni o'rnatishga alohida e'tibor qaratishdan iboratdir.

Bugungi kunda oliy ta'lim muassasalari talabalarini matematika fanini o'zlashtirishlari uchun bir qator innovatsion metodlar yaratish, matematika fanini o'qitishda samaraga erishish usullari ishlab chiqish, fanning bo'limlari va mavzularini o'qitish metodlari ustida bir nechtalab ishlar amalga oshirilmoqda. Shu jumladan algebra va sonlar nazariyasi elementlari, vektorlar nazariyasi, analitik geometriya elementlari, matematik analiz kursiga kirish qismlari bilan qisqacha tanishtirish, hosila va integrallar, differensial tenglamalar mavzulari va extiomllr nazariyasi elementlari mavzularining boshqa fanlar bilan bog'liqligini o'rgatish, jumladan fizika, kimyo, tabiiy hamda texnika fanlar bilan uyg'unligini yangi innovatsion uslublar yordamida o'qitish masalalari bo'yicha yetarli darajada nazariy va amaliy natijalar olingan

Ushbu o'quv qo'llanma pedagogika oliy ta'lim muassasalari tabiiy fanlar va texnika yo'nalishlari 1-bosqich talabalari uchun mo'ljallangan bo'lib, quyidagi vazifalarning hal qilishga qaratilgan:

- chiziqli algebra elementlari va vektorli algebra elementlari mavzularini yangi pedagogik texnologiyalar yordamida o'qitishni takomillashtirish,
- analitik geometriya elementlari bilan tanishtirish metodikasini ishlab chiqish,
- matematik analizga kirish, bir o'zgaruvchi funksiyasining differensial hisobi, bir o'zgaruvchi funksiyasining integral hisobi mavzularini fanlararo bog'liqlikda o'qitishni tashkil qilish,
- oddiy differensial tenglamalar bo'limi mavzularini o'qitishda kasbga yo'naltirish ishlarini tashkil qilish,

- extimollar nazariyasi iqtisodiy va texnika masalalariga tadbiq qilingan holda o'qitishni takomillashtirish.

Bulardan tashqari ushbu o'quv qo'llanma o'quv dasturining kredit moodle dasturidagi asosiy vazifalardan, talabalarning mustaqil ta'limini tashkillashtirish vazifalari asosida shakllantirilgan.

Ushbu o'quv qo'llanma Davlat ta'lim standartlariga mos keladi va fanning o'quv dasturlariga to'la javob beradigan tarzda bayon qilingan.

Qo'llanmaning har bir bo'limi zarur nazariy tushunchalar, ta'riflar, teoremalar va formulalar bilan boshlangan, ularning mohiyati misol va masalalarning yechimlarida tushuntirilgan, shu bo'limga oid amaliy mashg'ulot darslarida va mustaqil uy ishlarida bajarishga mo'ljallangan ko'p sondagi mustahkamlash uchun mashqlar berilgan. Har bir bo'limning oxirida talabalarning mustaqil ishlari uchun topshiriqlar variantlari keltirilgan. Qo'llanmani yozishda davlat oliy ta'lim muassasalari va nodavlat oliy ta'lim muassasalarining pedagogika-psixologiya, maktabgacha ta'lim, boshlang'ich ta'lim, iqtisodiyot shu va shunga o'xshagan ta'lim yo'nalishlari bakalavrlari uchun oily matematika fanining amaldagi dasturida tavsiya qilingan adabiyotlardan hamda o'zbek tilida chop etilgan zamonaviy darslik va o'quv qo'llanmalardan keng foydalanilgan.

Qo'llanma haqida bildirilgan fikr va mulohazalar mamnuniyat bilan qabul qilinadi.

Muallif

I BOB CHIZIQLI ALGEBRA ELEMENTLARI VA VEKTORLI ALGEBRA ELEMENTLARI MAVZULARINI YANGI PEDAGOGIK TEXNOLOGIYALAR YORDAMIDA O'QITISHNI TAKOMILLASHTIRISH

1-mavzu: Arifmetik hisoblashga doir misollar va ularni yechish usullari.

Reja:

1) Haqiqiy sonlar ustidagi arifmetik amallarga oid misol va masalalarni yechish usullari.

2) Ko'pxadlar va ular ustida arifmetik amallarga oid misol va masalalar yechish.

3) Irratsional ifodalar va ular ustida arifmetik amallarga oid misol va masalalar yechish.

Matematika fanida bir nechta arifmetik amallar bo'lib, fanni o'rganilish boshlanishi bilan asosiy arifmetik amallar sifatida to'rtta amalaga oid, qo'shish, ayirish, ko'paytirish va bo'lishga oid misol va masalalar o'rganilgan. Fanning kengayishi bilan albatta mavzular va amallar ko'lami kengaygan holda teglamalar yechish, irratsional ifodalar ustida amallar bajarish kabi bir qator amallar bajarish ustida ish olib boriladi. Biz quyida bir nechta misollar yechilishlarini ko'rib chiqamiz.

1) $\dfrac{(7-6,35):6,5+9,9}{(1,2:36+1,2:0,25-1\frac{5}{16}):\frac{169}{24}}$ ushbu ifoda qiymatini hisoblang.

Javob:

$$\frac{(7-6,35):6,5+9,9}{(1,2:36+1,2:0,25-1\frac{5}{16}):\frac{169}{24}}=\frac{0,65:6,5+9,9}{(\frac{12}{10}\cdot\frac{1}{36}+\frac{12}{10}\cdot 4-\frac{21}{16})\cdot\frac{24}{169}}=$$

$$\frac{0,1+9,9}{(\frac{6}{5}(\frac{1}{36}+4)-\frac{21}{16})\cdot\frac{24}{169}}=\frac{10}{(\frac{6}{5}\cdot\frac{145}{36}-\frac{21}{16})\cdot\frac{24}{169}}=\frac{10}{(\frac{29}{6}-\frac{21}{16})\cdot\frac{24}{169}}=\frac{10}{\frac{338}{96}\cdot\frac{24}{169}}=20$$ bo'lib bu misolimiz haqiqiy sonlar ustida amallarga misol bo'ladi.

$$2)\ \left(\frac{3\frac{1}{3}+2,5}{2,5-1\frac{1}{3}}\cdot\frac{4,6-2\frac{1}{3}}{4,6+2\frac{1}{3}}\cdot 5,2\right):\left(\frac{0,05}{\frac{1}{7}-0,125}+5,7\right)=\left(\frac{\frac{10}{3}+\frac{5}{2}}{\frac{5}{2}-\frac{4}{3}}\cdot\frac{\frac{23}{5}-\frac{7}{3}}{\frac{23}{5}+\frac{7}{3}}\cdot\frac{26}{5}\right):\left(\frac{\frac{1}{20}}{\frac{1}{7}-\frac{1}{8}}+5,7\right)=$$

$$\left(\frac{\frac{35}{6}}{\frac{7}{6}}\cdot\frac{\frac{34}{15}}{\frac{104}{15}}\cdot\frac{26}{5}\right):\left(\frac{\frac{1}{20}}{\frac{1}{56}}+5,7\right)=\left(\frac{35\cdot 34}{7\cdot 104}\cdot\frac{26}{5}\right):\left(\frac{28}{10}+\frac{57}{10}\right)=\frac{17}{2}\cdot\frac{10}{85}=1.$$

Bu misol ham haqiqiy sonlar ustidagi amallarga misol bo'ladi. Endi biz quyida ko'phadlar ustida arifmetik amallarga oid misollar va ularni yechishni o'rganamiz.

$$3)\ \left(2-x+4x^2+\frac{5x^2-6x+3}{x-1}\right):\left(2x+1+\frac{2x}{x-1}\right)=\frac{4x^3-5x^2+3x-2+5x^2-6x+3}{x-1}\cdot\frac{x-1}{2x^2-x-1+2x}=$$

$$\frac{4x^3-3x+1}{2x^2+x-1}=\frac{4(x+1)(x^2-x+1)-3(x+1)}{(2x-1)(x+1)}=\frac{(x+1)(4x^2-4x+4-3)}{(2x-1)(x+1)}=\frac{(2x-1)^2}{(2x-1)}=2x-1.$$

$$4)\ \left(\frac{a}{b}+\frac{b}{a}+2\right)\left(\frac{a+b}{2a}-\frac{b}{a+b}\right):\left[\left(a-2b+\frac{b^2}{a}\right)\cdot\left(\frac{a}{a+b}+\frac{b}{a-b}\right)\right]=$$

$$\frac{(a+b)^2}{ab}\cdot\left(\frac{a^2+2ab+b^2-2ab}{2a(a+b)}\right):\left(\frac{(a-b)^2}{a}\frac{a^2+b^2}{(a-b)(a+b)}\right)=\frac{(a+b)^2}{ab}\ \frac{a^2+b^2}{2a(a+b)}\frac{a}{(a-b)^2}\ \frac{(a-b)(a+b)}{a^2+b^2}=\frac{(a+b)^2}{2ab(a-b}$$

Endilikda irratsional ifodalar qatnashgan ifodalarni soddalashtirishga oid misollarni yechimlarini topish bilan shug'ullanamiz.

$$5)\ \left(\frac{\sqrt[4]{8}+2}{\sqrt[4]{2}+\sqrt[3]{2}}-\sqrt[3]{4}\right):\left(\frac{\sqrt[4]{8}-2}{\sqrt[4]{2}-\sqrt[3]{2}}-3\sqrt[12]{128}\right)^{\frac{1}{2}}=\left(\frac{\sqrt[4]{2^3}+\sqrt[3]{2^3}}{\sqrt[4]{2}+\sqrt[3]{2}}-\sqrt[3]{4}\right):\left(\frac{\sqrt[4]{2^3}-\sqrt[3]{2^3}}{\sqrt[4]{2}-\sqrt[3]{2}}-3\sqrt[12]{128}\right)^{\frac{1}{2}}=$$

$$\left(\sqrt[4]{4}-\sqrt[12]{128}+\sqrt[3]{4}-\sqrt[3]{4}\right):\left(\sqrt[4]{4}+\sqrt[12]{128}+\sqrt[3]{4}-3\sqrt[12]{128}\right)^{\frac{1}{2}}=\sqrt[4]{2}(\sqrt[4]{2}-\sqrt[3]{2})\cdot\frac{1}{(\sqrt[4]{2}-\sqrt[3]{2})}=\sqrt[4]{2}$$

$$6)\ \sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\cdot\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}=$$

$$\sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{4-2-\sqrt{2+\sqrt{3}}}=\sqrt{2+\sqrt{3}}\cdot\sqrt{4-2-\sqrt{3}}=\sqrt{4-3}=1$$

Mustaqil yechish uchun misollar:

1-variant

1) $\dfrac{(7-6.35):6.5+9.9}{(1.2:36+1.2:0.25-1\frac{5}{16}):\frac{169}{24}}$ ifodaning qiymatini toping

2) $\dfrac{\sqrt{2}(x-a)}{2(1+\sqrt{a})}+\left[\left(\dfrac{\sqrt{x}}{\sqrt{2x}+\sqrt{a}}\right)^2+\left(\dfrac{\sqrt{2x}+\sqrt{a}}{2\sqrt{a}}\right)^{-1}\right]^{\frac{1}{2}}$; ifodani soddalashtiring va $a=0.32, x=0.08.$ qiymatini toping.

2-variant

1) $\left[\left(\frac{7}{9}-\frac{47}{72}\right):1.25+\left(\frac{6}{7}-\frac{17}{28}\right):(0.385-0.108)\right]\blacksquare 6-\frac{19}{25}$ ifodaning qiymatini toping

2) $\dfrac{\sqrt[2]{1+\frac{1}{4}\left(\sqrt{\frac{1}{t}}-\sqrt{t}\right)^2}}{\sqrt{1+\frac{1}{t}\left(\sqrt{\frac{1}{t}}-\sqrt{t}\right)^2-\frac{1}{2}\left(\sqrt{\frac{1}{t}}-\sqrt{t}\right)}}, t>0$; ifodani soddalashtirirng.

3-variant

1) $\dfrac{\left(0.5:1.25+\frac{7}{5}:1\frac{4}{7}-\frac{3}{11}\right)\blacksquare}{\left(1.5+\frac{1}{4}\right):18\frac{1}{3}}$ ifodaning qiymatini toping

2) $\dfrac{\frac{1}{a}-\frac{1}{b+c}}{\frac{1}{a}+\frac{1}{b+c}}\cdot\left(1+\dfrac{b^2+c^2-a^2}{2bc}\right):\dfrac{a-b-c}{abc}$, ifodani soddalshtiring va qiymatini $a=0.02, b=-11.05, c=1.07$ toping.

4-variant

1) $\left[\dfrac{(2.7:0.8)\blacksquare\frac{1}{3}}{(5.2-1.4):\frac{3}{70}}+0.125\right]:2\frac{1}{2}+0.43$ ifodaning qiymatini toping

2) $\dfrac{\dfrac{9}{b^{-\frac{4}{2}}}-a^{\frac{3}{2}}\cdot b^{-2}}{\sqrt{a^{\frac{3}{2}}b^{-2}+6a^{\frac{3}{4}}b^{-\frac{1}{3}}+9b^{\frac{4}{3}}}}\cdot\dfrac{b^{2}}{a^{\frac{3}{4}}-3b^{\frac{5}{3}}},a=3\sqrt{7},b=4$ ifodani soddalshtiring va qiymatini toping.

5-variant

1) $\dfrac{2\frac{3}{4}:1.1+3\frac{1}{3}}{2.5-0.4\blacksquare\frac{1}{3}}:\dfrac{5}{7}-\dfrac{\left(2\frac{1}{6}+4.5\right)}{2.75-1\frac{1}{2}}$ ifodaning qiymatini toping

2) $\left(\dfrac{\sqrt{a}}{2}-\dfrac{1}{2\sqrt{a}}\right)^{2}\cdot\left(\dfrac{\sqrt{a}-1}{\sqrt{a}+1}-\dfrac{\sqrt{a}+1}{\sqrt{a}-1}\right)$ ifodani soddalshtiring.

6-variant

1) $\dfrac{\left(13.75-9\frac{1}{6}\right)\blacksquare 2}{\left(10.3-8\frac{1}{2}\right)\blacksquare\frac{}{9}}+\dfrac{\left(6.8-3\frac{3}{5}\right)\blacksquare\frac{5}{6}}{\left(3\frac{2}{3}-3\frac{1}{6}\right)\blacksquare 6}-27\frac{1}{6}$ ifodaning qiymatini toping

2) $\dfrac{1-x^{-2}}{x^{\frac{1}{2}}-x^{-\frac{1}{2}}}-\dfrac{2}{x^{\frac{3}{2}}}+\dfrac{x^{-2}-x}{x^{\frac{1}{4}}-x^{-\frac{1}{2}}}$ ifodani soddalshtiring.

7-variant

1) $\dfrac{\left(\frac{1}{6}+0.1+\frac{1}{15}\right):\left(\frac{1}{6}+0.1-\frac{1}{15}\right)\blacksquare .52}{\left(0.5-\frac{1}{3}+0.25-\frac{1}{5}:\left(0.25-\frac{1}{6}\right)\blacksquare\frac{}{13}\right)}$ ifodaning qiymatini toping

2) $\left(\sqrt[4]{36mn^{2}p}+m\sqrt{\dfrac{3n}{m}}+\sqrt{3np}\right)\times\left(\sqrt[4]{36mn^{2}p}-\sqrt{3mn}-p\sqrt{\dfrac{3n}{p}}\right),$ ifodani $m>0,n>0,p>0.$ soddalshtiring.

8-variant

1) $\dfrac{0.4+8\left(5-0.8\blacksquare\frac{}{8}\right)-5:2\frac{1}{2}}{\left|1\frac{7}{8}\blacksquare-\left(8.9-2.6:\frac{2}{3}\right)\right|\blacksquare 4\frac{2}{5}}\blacksquare 0$ ifodaning qiymatini toping

2) $\left(\dfrac{a+2}{\sqrt{2a}}-\dfrac{a}{\sqrt{2a}+2}+\dfrac{2}{a-\sqrt{2a}}\right)\cdot\dfrac{\sqrt{a}-\sqrt{2}}{a+2}$ ifodani soddalashtiring.

9-variant

1) $\dfrac{\left(5\frac{4}{45}-4\frac{1}{6}\right):5\frac{8}{15}}{\left(4\frac{2}{3}+0.75\right)\blacksquare\frac{9}{13}}\blacksquare 4\frac{2}{7}+\dfrac{0.3:0.01}{70}+\dfrac{2}{7}$ ifodaning qiymatini toping

2) $1-\dfrac{\dfrac{1}{\sqrt{a-1}}-\sqrt{a+1}}{\dfrac{1}{\sqrt{a+1}}-\dfrac{1}{\sqrt{a-1}}}:\dfrac{\sqrt{a+1}\sqrt{a^{2}-1}}{(a-1)\sqrt{a+1}-(a+1)\sqrt{a-1}},a>1$ ifodani soddalshtiring.

10-variant

1) $\dfrac{\left(\frac{3}{5}+0.425-0.005\right):0.1}{30.5+\frac{1}{6}+3\frac{1}{3}}+\dfrac{6\frac{3}{4}+5\frac{1}{2}}{26:3\frac{5}{7}}-0.05$ ifodaning qiymatini toping

2) $\left(\dfrac{\sqrt[4]{a^{3}}-1}{\sqrt[4]{a}-1}+\sqrt[4]{a}\right)^{\frac{1}{2}}\cdot\left(\dfrac{\sqrt[4]{a^{3}}+1}{\sqrt[4]{a}+1}-\sqrt{a}\right)\cdot(a-\sqrt{a^{3}})^{-1},$ ifodani soddalshtiring.
$a>0,a\neq 1$

11-variant

1) $\dfrac{3\frac{1}{3}\blacksquare .9+19.5:4\frac{1}{2}}{\frac{62}{75}-0.16}:\dfrac{3.5+4\frac{2}{3}+2\frac{2}{15}}{0.5\left(1\frac{1}{20}+4.1\right)}$ ifodaning qiymatini toping

2) $\dfrac{a^3-a-2b-\frac{b^2}{a}}{\left(1-\sqrt{\frac{1}{a}+\frac{b}{a^2}}\right)\cdot(a+\sqrt{a+b})}:\left(\dfrac{a^3+a^2+ab+a^2b}{a^2-b^2}+\dfrac{b}{a-b}\right)$ $a=23, b=22.$ ifodani

soddalashtiring va qiymatini toping.

12-variant

1) $\dfrac{\left|1\frac{1}{5}:\left(\frac{17}{40}+0.6-0.005\right)\right|\blacksquare .7}{\frac{5}{6}+1\frac{1}{3}-1\frac{23}{30}}+\dfrac{4.75+7\frac{1}{2}}{33:4\frac{5}{7}}:0.25$ ifodaning qiymatini toping

2) $\dfrac{\left(\sqrt[5]{a^{\frac{4}{3}}}\right)^{\frac{3}{2}}}{\left(\sqrt[-2]{a^3\sqrt{a^2b}}\right)^4}:\left[\dfrac{\sqrt[5]{a^4}}{\left(\sqrt[4]{a\sqrt{b}}\right)^{-2}}\right]^3$ ifodani soddalshtiring.

13-variant

1) $\dfrac{\left(4.5\blacksquare\frac{2}{3}-6.75\right)\blacksquare .66...}{\left|3.(3)\blacksquare .3+0.(2)+\frac{4}{9}\right|:2\frac{2}{3}}+\dfrac{1\frac{4}{11}\blacksquare .22:0.3-0.96}{\left(0.2-\frac{3}{40}\right)\blacksquare .6}$ ifodaning qiymatini toping

2) $\dfrac{\sqrt[3]{x+\sqrt{2-x^2}}\cdot\sqrt[6]{1-x\sqrt{2-x^2}}}{\sqrt[3]{1-x^2}}, -1<x<1$ ifodani soddalshtiring.

14-variant

1) $\dfrac{\left(1.88+2\frac{3}{35}\right)\blacksquare\frac{3}{16}}{0.625-\frac{13}{18}:\frac{26}{9}}+\dfrac{\left(\frac{0.216}{0.15}+0.56\right):0.5}{\left(7.7:24\frac{3}{4}+\frac{2}{15}\right)\blacksquare .5}$ ifodaning qiymatini toping

2) $\dfrac{x(x^2-a^2)^{-\frac{1}{2}}+1}{a(x-a)^{-\frac{1}{2}}+(x-a)^{\frac{1}{2}}}:\dfrac{a^2\sqrt{x+a}}{x-(x^2-a^2)^{\frac{1}{2}}}+\dfrac{1}{x^2-ax};$ ifodani soddalashtiring.
$x>a>0;$

15-variant

1) $\left(16\frac{1}{2}-13\frac{7}{9}\right)\blacksquare\frac{18}{33}+2.2[0.(24)-0.(09)]+\frac{2}{11}$ ifodaning qiymatini toping

2) $\dfrac{\left(\sqrt[3]{(r^2+4)\cdot\sqrt{1+\frac{4}{r^2}}}-\sqrt[3]{(r^2-4)\cdot\sqrt{1-\frac{4}{r^2}}}\right)}{r^2-\sqrt{r^{\frac{1}{2}}-16}}, |r|\geq 2$ ifodani soddalshtiring.

16-variant

1) $\dfrac{0.128:3.2+0.86}{\frac{5}{6}\blacksquare .2+0.8}\blacksquare\dfrac{\left(1\frac{32}{63}-\frac{13}{21}\right)\blacksquare .6}{0.505\blacksquare\frac{2}{5}-0.002}$ ifodaning qiymatini toping

2) $\sqrt{\dfrac{\sqrt{2}}{a}+\dfrac{a}{\sqrt{2}}+2}-\dfrac{a^2\sqrt[4]{2}-2\sqrt{a}}{a\sqrt{2a}-\sqrt[4]{8a^4}};$ ifodani soddalshtiring.

17-variant

1) $\left[\left(1\frac{1}{7}-\frac{23}{49}\right):\frac{22}{147}-(0,6:3\frac{3}{4})2\frac{1}{2}+3,75:1\frac{1}{2}\right]:2,2$ ifodaning qiymatini toping

2) $\left\{\left[\frac{2^{\frac{3}{2}}+27y^{\frac{8}{5}}}{\sqrt{2}+3\sqrt[5]{y}}+3\sqrt[10]{32y^{2}}-2\right]\blacksquare^{2}\right\}^{5}$ ifodani soddalshtiring.

18-variant

1) $\left[2:3\frac{1}{5}+\left(3\frac{1}{4}:13\right):\frac{2}{3}+\left(2\frac{5}{18}-\frac{17}{36}\right)\cdot\frac{18}{65}\right]\cdot\frac{0,1(6)+0,(3)}{0,(3)+1,1(6)}$ ifodaning qiymatini toping

2) $\frac{x-1}{x^{\frac{3}{4}}+x^{\frac{1}{2}}}\blacksquare\frac{x^{\frac{1}{2}}+x^{\frac{1}{4}}}{x^{\frac{1}{2}}+1}\blacksquare^{\frac{1}{4}}+1$ ifodani soddalashtiring.

19-variant

1) $\frac{0,5+\frac{1}{4}+0,166666....+0,125}{0,(3)+0,4+\frac{14}{15}}+\frac{(3,75-0,625)\cdot\frac{48}{125}}{12,8\cdot 0,25}$ ifodaning qiymatini toping

2) $\left[\frac{\left(z^{\frac{2}{p}}+z^{\frac{2}{q}}\right)^{2}-4z^{\frac{2}{p}+\frac{2}{q}}}{\left(z^{\frac{2}{p}}-z^{\frac{2}{q}}\right)^{2}-4z^{\frac{1}{p}+\frac{1}{q}}}\right]$ ifodani soddalashtiring.

20-variant

1) $\left(26\frac{2}{3}:6,4\right)\cdot\left(19,2:3\frac{5}{9}\right)-\frac{8\frac{4}{7}:2\frac{26}{77}}{0,5:18\frac{2}{3}\cdot 11}-\frac{1}{18}$ ifodaning qiymatini toping

2) $\sqrt[m^{3}-mn^{2}]{y^{(m+n)^{2}-4mn}}\blacksquare\sqrt[+n]{y^{\frac{n}{m}}}$ ifodani soddalashtiring.

21-variant

1) $\frac{0,725+0,6+\frac{7}{40}+0,42(6)+0,12(3)}{0,128\cdot 6\frac{1}{4}-\left(0,035:\frac{3}{25}\right)}\cdot 0,25$ ifodaning qiymatini toping

2) $\frac{x-y}{x^{\frac{3}{4}}+x^{\frac{1}{2}}y^{\frac{1}{4}}}\blacksquare\frac{x^{\frac{1}{2}}y^{\frac{1}{4}}+x^{\frac{1}{4}}y^{\frac{1}{2}}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}\blacksquare\frac{x^{\frac{1}{4}}y^{-\frac{1}{4}}}{x^{\frac{1}{2}}-2x^{\frac{1}{4}}y^{\frac{1}{4}}+y^{\frac{1}{2}}}$ ifodani soddalashtiring.

22-variant

1) $\left[(520\cdot 0,43):0,26-217\cdot 2\frac{3}{7}\right]-\left(31,5:12\frac{3}{5}+114\cdot 2\frac{1}{3}+61\frac{1}{2}\right)$ ifodaning qiymatini toping

2) $\left(\frac{1}{\sqrt{a}+\sqrt{a+1}}+\frac{1}{\sqrt{-a}\sqrt{a-1}}\right):\left(1+\sqrt{\frac{a+1}{a-1}}\right)$ ifodani soddalshtiring.

23-variant

1) $\frac{(3,4-1,275)\cdot\frac{16}{17}}{\frac{5}{18}\cdot\left(1\frac{7}{85}+6\frac{2}{17}\right)}+0,5\cdot\left(2+\frac{12,5}{5,75+\frac{1}{2}}\right)$ ifodaning qiymatini toping

2) $\left(\frac{1+\sqrt{x}}{\sqrt{1+x}}-\frac{\sqrt{1+x}}{1+\sqrt{x}}\right)^{2}-\left(\frac{1-\sqrt{x}}{\sqrt{1+x}}-\frac{\sqrt{1+x}}{1-\sqrt{x}}\right)^{2}$ ifodani soddalshtiring.

24-variant

1) $\left(\frac{3,75+2\frac{1}{2}}{2\frac{1}{2}-1,875}-\frac{2\frac{3}{4}+1,5}{2,75-1\frac{1}{2}}\right)\cdot\frac{10}{11}$ ifodaning qiymatini toping

2) $t\blacksquare\frac{1+\frac{2}{\sqrt{t+4}}}{2-\sqrt{t+4}}+\sqrt{t+4}+\frac{4}{\sqrt{t+4}}$ ifodani soddalashtiring.

25-variant

1) $\left[(21,85:43,7+8,5:3,4):4,5\right]:1\frac{2}{5}+1\frac{11}{21}$ ifodaning qiymatini toping

2) $\frac{x-1}{x+x^{\frac{1}{2}}+1}:\frac{x^{0.5}+1}{x^{1.5}-1}+\frac{2}{x^{-0.5}}$ ifodani soddalashtiring.

2-mavzu: Matnli masalalarni turlari va ularni yechish usullari.

Reja:

1) Tenglamalar yordamida yechiladigan masalalar.
2) Foizlarga oid masalalar.
3) Aralashmaga oid masalalar.

Matnli masalalar mavzusida asosan matematika va kimyo fanlarini o'zaro bog'liqliklarini ko'rishimiz mumkin. Ya'ni shunday masalalar borki ushbu masalalar yordamida ba'zi kimyo faniga oid masalalarni yechish shu bilan birga aralashmaga va konsentratsiyani aniqlashga oid masalalarni yechish mumkin. Aslida esa matnli masalalarning bir nechta turi mavjud bo'lib, mantiqiy o'ylash orqali yechiladigan, bir nechta arifmetik amallar yordamida yechiladigan va tenglamalar yordamida yechiladigan masalalar shular jumlasidandir. Odarda matematikada ham kimyoda ham tenglamalar yordamida yechiladigan masalalar ko'riladi.

Endilikda biz quyida tenglamalar yordamida yechiladigan bir nechta matnli masalalarni ko'rib o'tamiz.

1) Tarkibida 85% suv bo'lgan 0,5t sellyuloza qorishmasidan 75%suv bo'lgan qorishma olish uchun necha kilogram suvni bug'lantirib yuborish kerak?

Javob: ushbu masalaning javobini topish uchun 500kg qorishmadagi sellyuloza miqdorini topishimiz kerak, ya'ni $500\cdot 0,15=75$ bo'lib bu qorishmadagi o'zgarmas sellyuloza miqdoriga teng bo'lib, keying 25% I 75 ga teng bo'lgan yangi qorishmaning kg miqdirini topamiz. Buning uchun $\begin{matrix} x\to & 100\% \\ 75\to & 25\% \end{matrix} \to x=\frac{75\cdot 100}{25}=300$ ga teng bo'lib, bu esa yangi qorishmaning kg miqdori bo'lib bundan esa 200kg suvni bug'lantirish lozimligi kelib chiqadi.

2) Ikki brigada bir vaqtda ishlab, yer uchastkasiga 12 soatda ishlov berib bo'lishdi. Agar brigadalarning ishlash tezliklarining nisbati 3:2 kabi bo'lsa, har bir brigadaning yolg'iz o'zi shu yer uchastkasiga necha soatda ishlov berib bo'ladi?

Javob: ushbu masalani yechish uchun tenglamalar sistemasi tuzib olamiz. Buning uchun brigadalarning ishlov bergan yer uchastkasini 1 ish deb olib, ikki brigadaning tezliklarini esa ikkita nomalum bilan belgilash kiritamiz. Bundan esa quyidagi tenglamalar sistemasiga kelamiz.

$$\begin{cases} \frac{1}{x+y}=12 \\ x=\frac{3}{2}y \end{cases} \Rightarrow \begin{cases} x+y=\frac{1}{12} \\ x=1,5y \end{cases} \Rightarrow \begin{cases} 2,5y=\frac{1}{12} \\ x=1,5y \end{cases} \Rightarrow \begin{cases} y=\frac{1}{30} \\ x=\frac{1}{20} \end{cases}$$

3) Molning narhini oldin 20% ga, keyin yangi narhini yana 15% g ava oxirgi xisobtdan keyin yana 10% ga arzonlashtirishdi molning birinchi baxosini hammasi bo'lib necha foizga arzonlashtirishgan.

Javob: molning narhini biror noma'lum bilan belgilab olamiz va $x-0,2x=0,8x$ bo'lib bu molning birinchi arzonlashtirilgandan keying baxosi bo'lib, $0,8x-0,15\cdot 0,8x=0,8x-0,12x=0,68x$ bo'lib bu esa ikkinchi marta arzonlashtirilgandan keying narx bo'lib uchunchi arzonlashtirishda $0,68x-0,1\cdot 0,68x=0,612x$ bo'lib bundan esa arzonlashgandan keying va arzonlashmasdan oldingi molning narxlarini solishtirsak u holda $0,388x$ bo'lib umumiy molni 38,8% ga arzonlashtirilgani kelib chiqadi.

4) Proporsiya birinchi uchta hadining yig'indisi 58 ga teng. Uchunchi had, birinchi handing $\frac{2}{3}$ qismini, ikkinchisi esa uning $\frac{3}{4}$ qismini tashkil qiladi. Proporsiyaning to'rtinchi hadini toping va uni yozing.

Javob: $\begin{cases} x+y+z=58 \\ z=\frac{2}{3}x \\ y=\frac{3}{4}x \end{cases} \Rightarrow x+\frac{2}{3}x+\frac{3}{4}x=58 \Rightarrow \frac{29}{12}x=58 \Rightarrow x=24$ bo'lib proporsiyaning uchta hadi 24,16 va 18 lardan iborat. Proporsiyaning to'rtinchi hadini topish uchun o'rta hadlar ko'paytmasi chetki hadlar ko'paytmasiga ten kabi qoidadan foydalanib $16\cdot 18=24y \Rightarrow y=12$ bo'lib bu esa proporsiyaning to'rtinchi hadi 12 ga teng bo'lishini bildiradi.

Mustaqil yechish uchun misollar.

1-variant

1) Ikkita bir xil bassen bir vaqtda suv bilan to'ldirila boshlandi. Birinchi bassenga har soatda ikkinchisiga qaraganda 30 m ko'p ortiq suv keladi. Ma'lum bir vaqtda ikkala bassenga birgalikda ularning hajmi qancha bo'lsa, shuncha suv to'plandi. Shundan 2 soat-u 40 minut o'tgach birinchi bassen, yana 3 soat-u 20 minutdan keyin esa ikkinchi bassen suv bilan to'ldi. Har soatda har bir bassenga qanchadan suv kelgan?

2) Biror modda o'ziga namni tortib massasini orttiradi. 1400 kg namlikni tortishi uchun bu moddaning maydalanmaganidan maydalanganligiga qaraganda 300 kg ko'p olish kerak bo'ladi. So'rilgan namlik massasi maydalangan va maydalanmagan modda massaini qancha protsentini tashkil etishini aniqlang, bu son ikkinchi holatda birinchi holatdagidan 105 birlik kam.

2-variant

1) Montyorlar brigadasi elektr simi o'tkazishni soatiga 8 m dan bajarib, kunduzi soat 4 da tamomlashi mumkin edi. Topshiriqning yarmi bajarilgandan keyin bir ishchi brigadadan ketdi; shu munosabat bilan brigada soatiga 6 m dan sim torta boshladi va bir kunga planlashtirilgan butun ishni kech soat 6 da tamomladi. Necha metr sim tortilgan va necha soatda tortib bo'lingan?

2) Tarkibida mis protsenti turlicha bo'lgan ikki mis qotishmasi bor. Birinchi qotishma tarkibidagi mis protsentini ifodalovchi son ikkichi qotishmada tarkibida mis protsentini ifodalovchi sondan 40 ta kam. So'ngra bu ikkala qotishma eritib qo'yildi, shundan keyin mis bu qotishmada 36% ni tashkil etdi. Agar birinchi qotishmada 6 kg, ikkinchisida esa 12kg mis borligi ma'lum bo'lsa, birinchi va ikkinchi qotishma tarkibidagi mis protsentini aniqlang.

3-variant

1) Moddiy nuqtaga ikki kuch yo'naltirilgan bo'lib, ular orasidagi burchak esa 30° ga teng. Quyilgan kuchlardan birining kattaligi

ikkinchisidan $7\sqrt{3}$ marta ko'p, teng ta'sir etuvchi kuchning kattaligini aniqlang.

2) 500 kg rudaning tarkibida biror miqdorda temir bor. Bu rudadan tarkibida o'rtacha 12,5% temir bo'lgan 200kg aralashma chiqarib yuborilgandan keyin qolgan ruda tarkibidagi temir 20% ga ortdi. Rudaning tarkibida qancha temir qolganligini aniqlang.

4-variant

1) Ikki stanokdan birida bir to'p detallarga ikkinchisiga qaraganda 3 kun ko'p vaqt ichida ishlov beriladi. Agar berilgan detallarda 3 marta ko'p bo'lgan detallarga ikki stanok birgalikda ishlaganda 20 kunda ishlov berilgani ma'lum bo'lsa, berilgan detallarga har bir stanokning o'zida necha kunda ishlov beriladi?

2) Umumiy massasi 12 kg bo'lgan bir bo'lak mis bilan qalay qotishmasining tarkibida 45% mis bor. Yangi hosil bo'ladigan qotishmaning tarkibida 40% mis bo'lishi uchun berilgan qotishma bo'lagiga qancha toza qalay qushish kerak?

5-variant

1) Bir savxoz 1 gektardan o'rtacha 21 c dan qora bo'g'doy xosili oldi , ikkinchisi esa qorabo'g'doyga 12gektar kam yer ajratgan bo'lib , bir gektardan o'rtacha 25 c dan hosil olishga erishdi . Natijada ikkinchi savxozda birinchiga qaraganda 30 c ko'p qorabo'g'doy olindi . xar bir savxoz qanchadan qorabo'g'doy olgan?

2) Kiristall shakillanish davrida o'zining massasini bir tekis orttirib boradi . Ikki kristalning shakillanishini kuzatib quyidagi aniqlandi: Bir yil davomida birinchi Kristal massasi 4%ga , ikkinchisi esa 5% ga ortgan xamda shu davrda birinchi kristalning massasining 3 oydagi ortishi , ikkinchi kristal massasining 4 oylik ortishiga teng bo'lgan. Agar xar bir kristalning massasi 20 gramdan ortgandan so'ng birinchi kristalning massasi ikkinchi kristalning massasiga nisbati 1,5ga teng bo'lganligi malum bo'lsa, har bir kristallning dastlabgi massasi qancha bo'lgan?

6-variant

1) Labaratoriyada xap xil materiallardan yasalgan sterjinlar bo'ylab tovushning tarqalish tezligi o'lchandi. Birinchi tajribada shu narsa malum bo'ldiki , ketma-ket birlashtirilgan uchta sterjindan iborat masofani ***a*** sek da,ikkinchi va uchinchi sterjindan iborat masofani esa bitta birinchi sterjindagiga qaraganda ikki marta tezroq o'tgan. Boshqa bir tajribada ikkinchi sterjinni yangi sterjin bilan almashtirib, unda uchta sterjinni ketma ket tutashtirishddan hosil bo'lgan masofa bo'ylab tovush ***b*** sekundda o'tganligi ,birinchi va ikkinchi sterjinlarning birlashmasidan esa bitta uchinchi sterjindagidan ikki marta sekin o'tganligi aniqlandi. Agar yangi sterjinning uzunligi ***l*** ga teng bo'lsa, tovushning unda tarqalish tezligini toping

2) Massasi 36 kg bo'lgan mis rux qotishmasi bo'lagi tarkibida 45%mis bor. Tarkibida 60% mis bo'lgan yangi qotishma xosil qilish uchun shu bo'lakka qancha mis qo'shish kerak.

7-variant

1) Ikki brigada bir vaqtda ishlab, yer uchastkasiga 12 soatda ishlov berib bo'lishdi. Agar brigadalarning ishlash tezliklarining nisbati 3:2 kabi bo'lsa, har bir brigadaning yolg'iz o'zi shu yer uchastkasiga necha soatda ishlov berib bo'ladi?

2) Yangi qo'ziqorin og'irligining 90% ini suv tashkil qiladi , quritilgan qo'ziqorinda esa 12% suv bo'ladi. 22kg yangi qo'ziqorindan necha kg quritilgan qo'ziqorin olish mumkin?

8-variant

1) Zavod yanvar oyida tayyor mahsulot ishlab chiqarish oylik planini 105% qilib bajardi, fevralda esa yanvardagidan 4% ortiq maxsulot berdi. Zavod maxsulot chiqarish ikki oylik planini necha prosentga oshirib bajargan?

2) Tarkibida 85% suv bo'lgan 0.5 t sellyuloza qorishmasidan 75% suv bo'lgan qorishma olish uchun necha kilogram suvni bulg'antirib yuborish kerak?

9-variant

1) Kutubxonaning chet tillar bo'limida inglizcha, fransuzcha va nemischa kitoblar bor.Ingliz tilidagi kitoblar butun kitoblarning 36% ini, fransuzcha kitoblar ingliz tilidagi kitoblarning 75% ini tashkil qiladi, qolgan 185 ta kitob esa nemis tilida.Kutubxonada chet tillar kitobi nechta?

2) Dengiz suvi tarkibida o'rtacha 5% tuz bor. Eritmadi 1,5% tuz bo'lishi uchun 30kg dengiz suviga qancha chuchuk suv qo'shish kerak?

10-variant

1) Turist daryo bo'ylab qayiqda 90km suzdi va 10km piyoda yurdi. Bunda piyoda yurishga daryoda suzishga qaraganda 4 soat kam vaqt sarflandi. Agar turist daryoda suzishga qancha vaqt sarflagan bo'lsa , piyoda yurishga ham shuncha vaqt sarflaganda,piyoda yurishga qancha sarflagan bo'lsa, daryoda suzishga ham shunch vaqt sarflaganda edi, u holda o'tgan masofalar teng bo'lar edi. U qancha vaqt piyoda yurgan va qancha vaqt qayiqda daryo buylab suzgan?

2) Idishda 12 l xlorid kislota bor. Kislotaning bir qismi olinib, idishda shuncha suv quyildi. Agar idishda 25% li kislota eritmasi qolgan bo'lsa har gal qanchadan suyuqlik olingan?

11-variant

1) Ov miltig'ining poroxi selitra, oltingugurt va ko'mirdan iborat . Oltingugurt massasining selitra massasiga nisbati $\frac{1}{3}$:1,3 kabi bo'lishi kerak, ko'mirning massasi esa oltingugurt va selitraning birgalikdagi massasining $11\frac{1}{9}$ % ini tashkil qilishi lozim. 25 kg porox tayyorlash uchun xar bir moddadan qanchadan olish kerak.

2) Solishtirma og'irligi 20,88 bo'lgan bir bo'lak platina po'kak daraxtining (solishtirma og'irligi 0,24) bir bo'lagi bilan bog'lab quyilgan. Hosil bo'lgan sistemaning solishtirma og'irligi 0,48 ga teng. Agar platina bo'lagining og'irligi 87 g bo'lsa , daraxt bo'lagining og'irligi qancha? Jismning solishtirma og'irligi- bu uning hajm birligidagi og'irligidir.

12-variant

1) Uchta idishning har birida turli miqdorda suyuqlik bor. Ularni tenglashtirish uchun uch marta bir-biriga quyish bajarildi. Oldin birinchi idishdagi suyuqlikning uchdan bir qismi ikkinchi idishga keyin ikkinchi idishda bo'lgan suyuqlikda bo'lgan suyuqlikning to'rtdan bir qismi uchinchi idishga quyildi va nihoyat, uchinchi idishda bo'lgan suyuqlikning o'ndan bir qismi birinchi idishga quyildi. Shundan keyin har bir idishdagi suyuqlik 9 *l* dan bo'ldi.Oldin har bir idishda qanchadan suyuqlik bo'lgan?

2) Molning narxini oldin 20%ga, keyin yangi narxini yana 15%ga va oxirgi hisobotdan keyin yana 10% ga arzonlashtirishdi. Molning birinchi bahosini hammasi bo'lib necha prosentga arzonlashtirishgan?

13-variant

1) Omborda bo'lgan 300kg molni kilogrami 1,25 so'mdan ikki tashkilotga teng bo'lmagan miqdorda sotildi. Birinchi tashkilot sotib olgan molini 20 km masofaga, ikkinchisi esa 30 km masofaga tashib ketadi. 10 kg molni bir kilometr masofaga tashish 5 tiyinga tushadi. Ikkinchi tashkilot sotib olgan moliga va uni tashishga birinchi tashkilotga qaraganda 90 so'm ortiq to'laganini bilgan holda har bir tashkilot necha kilogrammdan mol olganligini hamda har bir olgan moli uchun va tashish uchun necha so'mdan sarflaganini aniqlang.

2) Ikki dvigatelni sinash davrida birinchisi 300 g, ikkinchisi esa 192 g benzin sarflangani aniqlandi, shu bilan birga ikkinchisi birinchisiga qaraganda 2 soat kam ishladi. Birinchi dvigatel bir soatda ikkinchisiga qaraganda 6 g benzin ko'p sarflaydi. Dvigatellarning har biri bir soatda qanchadan benzin sarflaydi?

14-variant

1) Institutning uch kafedrasidan laboratoriyaga qo'shimcha asbob-uskuna olish uchun talabnomalar tushdi. Birinchi kafedra talabnomasidagi asbob-uskanalarning bahosi ikkinchi kafedra talabnomasidagi bahoning 45% ini tashkil etadi, ikkinchi kafedra talabnomasidagi asbob-uskunalarning bahosi esa uchinchi kafedra talabnomasidagi bahoning

80% ini tashkil etadi. Uchinchi kafedra talabnomasidagi baho birinchi birinchi kafedraning talabnomasidagi bahodan 640 so'm ortiq. Uchala kafedra talabnomalaridagi asbob-uskuunalarning umumiy bahosi qancha?

2) Tarkibida xrom protsenti turlicha bo'lgan ikki sort chuyan eritib qo'yildi.Agar bir sortdan ikkinchisiga qaraganda 5 marta ortiq olinsa, u holda qotishmaning xrom tarkibi quyilayotgan bulaklarning tarkibida xromi kam protsentlisidan ikki marta ortiq bo'ladi. Agar ikkala sortdan bir xil miqdorda olinsa, qotishma tarkibida 8% xrom bo'ladi. Har bir sort chuyan tarkibidagi xrom protsentini aniqlang.

15-variant

1) Umumiy maydoni 110 ga bo'lgan ikki bog'ning har biri bir xil sondagi uchastkalarga bo'lingan. Har bir bog'dagi uchastkalarning yuzlari o'zaro teng, lekin bir bog' uchastkalarining yuzi ikkinchisinikidan farq qiladi. Agar birinchi bog' ikkinchi bog' uchastkasining yuziga teng bo'lgan uchastkalarga bo'linsa, u holda u 75 ta uchastkaga ega bo'lar edi, ikkinchi bog' huddi birinchidagi kabi qilib bo'linsa, u holda u 108 ta uchastkaga ega bular edi. Har bir bog'ning yuzini aniqlang.

2) Bir kitobning birinchi tomining 60 nusxasi va ikkinchi tomining 75 nusxasini narxi 270 so'mni tashkil qiladi. Haqiqatda esa hamma kitoblar uchun 237 so'm to'landi, chunki birinchi tom 15% ga, ikkinchi tom esa 10% ga arzonlashtirildi. Kitoblarning oldingi baholarini toping.

16-variant

1) Elektr lampalari sexining ishchilar brigadasi bir smenada 7200 ta detal yasashi kerak , shu bilan birga har bir ishchi bir xil miqdorda detal yasaydi. Ammo brigadaning uch ishchisi kasal bo'lib qolganligi sababli, butun normani bajarish uchun qolgan ishchilarni har biri yana 400 tadan qushimcha detal yasashiga tug'ri keldi. Brigadada necha ishchi bo'lgan?

2) Ikki idishdagi tuz eritmasi bug'lantirishga qo'yilgan. Tuzning har bir idishdan bug'lantirib olinadigankundalik miqdori bir xil. Birinchi idishdan 48 kg, undan 6 kun kam turgan ikkinchi idishdan esa 27 kg tuz olindi. Agar birinchi idish bug'lanishda ikkinchi idish qancha turgan bo'lsa ,

shuncha kun, ikkinchi idish esa birinchi idish necha kun turgan bo'lsa, shuncha kun turganida edi, u holda ikkala idishdan bir xil miqdorda tuz olingan bo'lar edi. Har bir eritma necha kundan turgan?

17-variant

1) Planga ko'ra korxona bir necha oy davomida 6000 ta nasos ishlab chiqarishi ko'zda tutilgan edi. Mehnat unumdorligini oshirib , korxona oyiga plandagiga ko'ra 70 tadan ortiq nasos ishlab chiqara boshladi. Natijada plan muddatidan biro y oldin va 30 ta nasos ortiq ishlab chiqarish bilan bajarildi. 6000 ta nasosni necha oy davomida ishlab chiqarish planlashtirilgan edi?

2) Hovuzdagi suv bir tekis bo'shata boshlangandan bir soat keyin unda $400m^3$, yana uch soatdan keyin esa $250m^3$ suv qoladi. Hovuzda qancha suv bo'lgan?

18-variant

1) Bir gruppa studentlar kanikul vaqtida shahar atrofiga safarga chiqishdi. Ular dastlabki 30km ni piyoda yurishdi, marshrutning qolgan qismining 20% ini sol bilan daryoda suzishdi, shundan keyin esa piyoda yurib, daryoda suzib o'tishgan masofadan 1,5 marta ko'p yo'lni bosib o'tishdi.Qolgan yo'lni 40km/soat tezlik bilan ketayotgan yo'lovchi mashinada 1 soat-u 30 minutda o'tishdi . Butun marshrutning uzuzligi qancha?

2) Berilgan to'rtda sondan birinchi uchtasi $\frac{1}{5}:\frac{1}{3}:\frac{1}{20}$ kabi nisbatda , to'rtinchisi esa ikkinchi sonning 15%ini tashkil qiladi .Agar ikkinchi son qolgan sonlarning yig'indisidan 8 birlikka ortiq ekanligi ma'lum bo'lsa , bu sonlarni toping.

19-variant

1) Og'irliklari bir xil bo'lgan ikki idishga suv quyilgan, shu bilan birga A idishning suvi bilan birgalikdagi og'irligi B idishning suvi bilan birgalikdagi og'irligining $\frac{4}{5}$ qismini tashkil qiladi. Agar B idishning suvi A

idishga quyilsa, u holda uning suvi birgalikdagi og'irliga B idishning og'irligidan 8 marta ortiq bo'ladi. B idishdagi suv A idishdagiga qaraganda $50g$ og'ir ekanligini bilgan holda har bir idishning va ulardagi suvning og'irligini aniqlang.

2) 30%li xlorid kislota eritmasini 10% li xlorid kislota bilan aralashtirib 600 g 15% li eritma olindi . Xar bir eritmadan necha giramdan olingan?

20-variant

1) Matematikadan kirish imtihonida kiruvchilardan 15%i birorta ham masalani yecha olmadi, 144 kishi masalalarni xatolar bilan yechdi, hamma masalalarni tug'ri yecha olmaganlar sonining masalalarni mutlaqo yecha olmaganlar soniga nisbati esa 5:3 kabi. Shu kuni matematikadan necha kishi imtihon topshirgan?

2) Bir tegirmon 19 sentner bo'g'doyni 3 soatda, ikkinchisi 32c ni 5 soatda , uchinchisi 10 c ni 2 soatda yanchib bo'ladi. Ishni bir vaqtda boshlab, bir vaqtda tamomlashlari uchun 133 tonna bo'g'doyni bu tegirmonlar orasida qanday taqsimlash kerak?

21-variant

1) Magazinga fizika va matematia darsliklari keltirildi. Matematika darsliklaridan 50% I va fizika darsliklaridan 20%i sotilgan kitoblarning umumiy soni 390 tani tashkil etib, matematika darsliklari fizika darsliklaridan 3 marta ko'p qoldi. Sotish uchun matematikadan nechta darsliklar kelgan.

2) Bir shtampovkalovchi press $3\frac{1}{2}$ soatda buyurtma detallarining 42% ini tayyorlay oladi. Ikkinchi press 9 soat ichida hamma detallarning 60% ini tayyorlay oladi, uchinchi pressning ish bajarish tezligi ikkinchisini ish bajarish tezligiga nisbati esa 6:5 kabi. Agar uchala press bir vaqtda ishlasa, barcha buyurtma qancha vaqtda bajariladi?

22-variant

1) Bir maxsulotning bir kilogrami bilan ikkinchi maxsulotning o'n kilogrami uchun 2 so'm to'langan. Agar narxlarning mavsumiy o'zgarishi bilan birinchi maxsulotning narxi 15% ga qimmatlashib, ikkinchi maxsulotning narxi 25% ga arzonlashsa, u holda huddi shunday miqdordagi bu maxsulotlar uchun 1 so'm 82 tiyin to'lanadi. Har bir maxsulotning bir kilogrami qanchadan turadi?

2) Umumiy bahosi 225 so'm bo'lgan ikkita qimmatbaho mo'ynali teri xalqaro auksionda(kim ochdi savdosida) 40 % foydasi bilan sotildi. Agar birinchi teriddan 25%, ikkinchisidan esa 50% foyda qilingan bo'lsa, har bir terining bahosini aniqlang.

23-variant

1) Ikki jism bir-biriga qarshi to'g'ri chiziq bo'ylab bir vaqtda harakat qila boshladi. Ulardan biri har bir minutda 7 m bosib o'tadi, ikkinchisi esa birinchi minutda 24 m o'tib, qolgan har minutda oldingisidan 4 m kam o'tadi. Agar harakatning boshlanishida jismlar orasidagi masofa 100 m bo'lgan bo'lsa, ular necha minutdan keyin uchrashadi?

2) Har xil sortli ikki bo'lak kabel bor. Birinchi bo'lakning massasi 65 kg , ikkinchisining massasi 120 kg bo'lib , uning uzunligi birinchi kabelning uzunligidan 3 m ga ortiq, har bir metrining massasi esa birinchi bo'lak har bir metrining massasidan 2 kg ortiq. Bu bo'laklarning uzunligini hisoblang.

24-variant

1) Uch ixtirochi o'zlarining ixtirolarini uchun miqdori 1410 so'm bo'lgan mukofot olishdi, shu bilan bigra ikkinchi ixtirochi birinchi ixtirochi olgan pulning $\frac{1}{3}$ qismini va yana 60 so'm, uchinchisi esa ikkinchisi olgan pulning $\frac{1}{3}$ qismini va yana 60 so'm oldi. Xar biri qanchadan mukofot olgan?

2) G'amlangan pichanning miqdori shundayki, hamma otlarga kuniga 96 kg pichan berish mumkin. Ikki otni qushni kolxozga berilgani uchun haqiqatda har bir otga kuniga beriladigan pichanni 4 kg ga orttirish mumkin bo'ldi. Oldin nechta ot bo'lgan?

25-variant

1) Nasos $7\frac{1}{2}$ minutda hovuzdagi suvning $\frac{2}{3}$ qismini bo'shatishi mumkin. Nasos 0,15 soat ishlaganidan so'ng to'xtab qoldi. Agar nasos to'xtagandan keyin hovuzda 25 m^3 suv qolgan bo'lsa, hovuzning sig'imini toping.

2) Bir hovuzda $200m^3$, ikkinchisida esa $112m^3$ suv bor. Basseynlarni to'ldiruvchi suv kranlari ochib quyilgan. Agar har soatda ikkinchi hovuzga birinchiga qaraganda 22 m^3 ko'p suv quyilsa, necha soatdan keyin hovuzlardagi suv miqdorlari teng bo'ladi?

3-mavzu: Determinantlar va ularning xossalari.

Determinantlar. Ikkinchi va uchinchi tartibli determinantlar. $a_{11}, a_{12}, a_{21}, a_{22}$ haqiqiy sonlar berilgan bo'lsin: *ikkinchi tartibli determinant (yoki aniqlovchi)* deb,

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

kabi jadval ko'rinishda yozilgan va

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

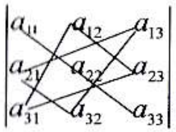
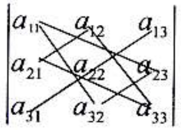
tenglik bilan aniqlandan songa aytiladi.

Xuddi shu singari, berilgan $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}$ haqiqiy sonlardan tuzilgan, $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ kabi yozilgan,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

munosabat bilan aniqlangan songa *uchinchi tartibli determinant* deyiladi. Uchinchi tartibli determinantlarni uchburchaklar usulida, *"diagonal"* usulida hamda biror *satr yoki ustun elementlari bo'yicha yoyib* hisoblash mumkin.

1. Uchburchaklar usuli:

(+)  (-) 

$$a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}.$$

2. "diagonal":

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} -$$

$$-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}.$$

3. Birinchi ustun elementlari bo'yicha yoyib, hisoblash:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}.$$

Quyidagi misollarda determinantlarni hisoblanishiga e'tibor bering.

3-misol. $\begin{vmatrix} 3 & -4 \\ 2 & 5 \end{vmatrix} = 3\cdot 5 - (-4)\cdot 2 = 15 + 8 = 23.$

4-misol. $\begin{vmatrix} \sqrt{a} & -1 \\ a & \sqrt{a} \end{vmatrix} = \sqrt{a}\cdot\sqrt{a} - (-1)\cdot a = a + a = 2a.$

Uchinchi tartibli determinantlarni uchburchaklar usuli, Sarrius usuli hamda biror ixtiyoriy satr yoki ustun elementlari bo'yicha yoyib hisoblang.

5-misol. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 4 & -1 & -5 \end{vmatrix} = 1\cdot(-3)\cdot(-5) + 1\cdot 1\cdot 4 + 2\cdot(-1)\cdot 1 - 1\cdot(-3)\cdot 4 - 1\cdot 2\cdot(-5) - 1\cdot(-1)\cdot 1 =$

$= 15 + 4 - 2 + 12 + 10 + 1 = 40.$

6-misol. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 4 & -1 & -5 \\ 1 & 1 & 1 \\ 2 & -3 & 1 \end{vmatrix} = 15 - 2 + 4 + 12 + 1 + 10 = 40.$

7-misol. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 4 & -1 & -5 \end{vmatrix} = 1\begin{vmatrix} -3 & 1 \\ -1 & -5 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 4 & -5 \end{vmatrix} + 1\begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} = 16 + 14 + 10 = 40.$

Mustaqil yechish uchun misollar

1-variant

1) Ikki o'lchovli determinantni hisoblang. $\begin{vmatrix} (x+y)/x & 2x/(x-y) \\ (y-x)/(x^2-y^2) & (y-x)/(x^2-y^2) \end{vmatrix}$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang. $\begin{vmatrix} a & -a & a \\ a & a & -a \\ a & -a & -a \end{vmatrix}.$

3) Determinantni qulay usulda hisoblang $\begin{vmatrix} 7 & -3 & 0 & 4 \\ 2 & 1 & 1 & 5 \\ 3 & 6 & -1 & -3 \\ 8 & 1 & 1 & 1 \end{vmatrix}$

2-variant

1) Ikki o'lchovli determinantni hisoblang. $\begin{vmatrix} \sqrt{a}+\sqrt{b} & \sqrt{a}-\sqrt{b} \\ \sqrt{a}-\sqrt{b} & \sqrt{a}+\sqrt{b} \end{vmatrix}$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang. $\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}.$

3) Determinantni qulay usulda hisoblang $\begin{vmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ 2 & 3 & 0 & 1 \end{vmatrix}$

3-variant

1)Ikki o'lchovli determinantni hisoblang. $\begin{vmatrix} \sin 1^0 & \sin 89^0 \\ -\cos 1^0 & \cos 89^0 \end{vmatrix}$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang. $\begin{vmatrix} 2\cos^2\frac{\alpha}{2} & \sin\alpha & 1 \\ 2\cos^2\frac{\beta}{2} & \sin\beta & 1 \\ 1 & 0 & 1 \end{vmatrix}.$

3) Determinantni qulay usulda hisoblang $\begin{vmatrix} 1 & 2 & 0 & -3 \\ 3 & 1 & 0 & 4 \\ 1 & 5 & -1 & 7 \\ -2 & 1 & 0 & 1 \end{vmatrix}.$

4-variant

1)Ikki o'lchovli determinantni hisoblang. $\begin{vmatrix} \sqrt{5}-a^{\frac{1}{2}} & a^{\frac{1}{2}} \\ -a^{\frac{1}{2}} & \sqrt{5}+a^{\frac{1}{2}} \end{vmatrix}$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang. $\begin{vmatrix} 1+\cos\alpha & 1+\sin\alpha & 1 \\ 1-\sin\alpha & 1+\cos\alpha & 1 \\ 1 & 1 & 1 \end{vmatrix}.$

3) Determinantni qulay usulda hisoblang $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -9 & -9 & -9 & -9 \\ 4 & 3 & 2 & 1 \\ 1 & 0 & 1 & 0 \end{vmatrix}$

5-variant

1)Tenglamani yeching $\begin{vmatrix} x & 3 \\ 1 & 2x \end{vmatrix} + 3\begin{vmatrix} 0,(4) & x \\ 1 & 3 \end{vmatrix} = 0.$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang. $\begin{vmatrix} \sin\alpha & \cos\alpha & 1 \\ \sin\beta & \cos\beta & 1 \\ \sin\gamma & \cos\gamma & 1 \end{vmatrix}.$

3) Determinantni qulay usulda hisoblang $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 9 \\ 0 & 0 & 3 & 7 \\ -2 & -4 & -6 & 1 \end{vmatrix}.$

6-variant

1)Ikki o'lchovli determinantni hisoblang. $\begin{vmatrix} \sin^2 a & \cos^2 a \\ \sin^2 b & \cos^2 b \end{vmatrix}.$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang. $\begin{vmatrix} tg\alpha & ctg\alpha & 1 \\ tg\beta & ctg\beta & 1 \\ tg\alpha & \cos\alpha & 1 \end{vmatrix}.$

3) Determinantni qulay usulda hisoblang $\begin{vmatrix} 3 & -1 & 2 & -1 & 1 \\ 5 & 1 & -2 & 1 & 2 \\ 9 & -1 & 1 & 3 & 4 \\ 3 & 0 & 6 & -1 & 3 \\ 5 & 2 & 3 & -2 & 1 \end{vmatrix}.$

7-variant

1)Ikki o'lchovli determinantni hisoblang. $\begin{vmatrix} \sin 60^0 & \cos 45^0 \\ \sin 45^0 & tg30^0 \end{vmatrix}.$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.

$$\begin{vmatrix} a+x & x & x \\ x & b+x & x \\ x & x & c+x \end{vmatrix}.$$

3) Determinantni qulay usulda hisoblang

$$\begin{vmatrix} 1 & 2 & -3 & 1 \\ 3 & 0 & 1 & -1 \\ 2 & 0 & 4 & 1 \\ 5 & 1 & 2 & 1 \end{vmatrix}$$

8-variant

1)Ikki o'lchovli determinantni hisoblang.

$$\begin{vmatrix} tg\alpha & -1 \\ 4 & ctg\alpha \end{vmatrix}.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.

$$\begin{vmatrix} \cos\alpha & \sin\alpha\cos\alpha & \sin\alpha\sin\beta \\ -\sin\alpha & \cos\alpha\cos\beta & \cos\alpha\sin\beta \\ 0 & -\sin\beta & \cos\beta \end{vmatrix}.$$

3) Determinantni qulay usulda hisoblang

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$$

9-variant

1)Ikki o'lchovli determinantni hisoblang.

$$\begin{vmatrix} \sin 3\alpha & \cos 3\alpha & 1 \\ \sin 2\alpha & \cos 2\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \end{vmatrix}.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.

$$\begin{vmatrix} \cos\alpha & \sin\alpha\cos\alpha & \sin\alpha\sin\beta \\ -\sin\alpha & \cos\alpha\cos\beta & \cos\alpha\sin\beta \\ 0 & -\sin\beta & \cos\beta \end{vmatrix}.$$

3) Determinantni qulay usulda hisoblang

$$\begin{vmatrix} 0 & 6 & 3 & 5 & 1 \\ -3 & 2 & 4 & 1 & 0 \\ 5 & 1 & 4 & 3 & 2 \\ -3 & 8 & 7 & 6 & 1 \\ 1 & 0 & 3 & 4 & 0 \end{vmatrix}$$

10-variant

1)Quyidagini toping.

$A=\begin{vmatrix} 5 & 7 \\ 3 & 4 \end{vmatrix}$ $B=\begin{vmatrix} 1 & 4 \\ 2 & -9 \end{vmatrix}$ bo'lsa, $A \cdot B$ ni hisoblang.

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang va tenglik to'g'riligini ko'rsating

$$\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & c \\ \cos\alpha & \\ \cos\beta & c \end{vmatrix}$$

3) Determinantni qulay usulda hisoblang

$$\begin{vmatrix} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ 3 & -4 & -1 & -2 \\ 4 & 3 & 2 & -1 \end{vmatrix}.$$

11-variant

1)Tenglamani yeching.

$$(0.6)^x \cdot \left(\frac{25}{9}\right)^{\begin{vmatrix} x & 3 \\ 4 & x \end{vmatrix}} = \left(\frac{27}{125}\right)^3.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.

$$\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix}.$$

3) Determinantni qulay usulda hisoblang
$$\begin{vmatrix} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ 3 & -4 & -1 & -2 \\ 4 & 3 & 2 & -1 \end{vmatrix}$$

12-variant

1)Tengsizlikni yeching.
$$\begin{vmatrix} x & 1 \\ -4 & x \end{vmatrix} \le \begin{vmatrix} 5 & 2 \\ 1 & x \end{vmatrix}.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.
$$\begin{vmatrix} ax & a^2+x^2 & 1 \\ ay & a^2+y^2 & 1 \\ az & a^2+z^2 & 1 \end{vmatrix}.$$

3) Determinantni qulay usulda hisoblang
$$\begin{vmatrix} -1 & -1 & -1 & -1 \\ -1 & -2 & -4 & -8 \\ -1 & -3 & -9 & -27 \\ -1 & -4 & -16 & -64 \end{vmatrix}$$

13-variant

1)Tenglamani yeching.
$$\log_3 \frac{2}{\begin{vmatrix} 2 & x \\ 1 & 2 \end{vmatrix}} = \log_3 \begin{vmatrix} x & 2 \\ \frac{1}{2} & 1 \end{vmatrix}$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.
$$\begin{vmatrix} m+a & m-a & a \\ n+a & 2n-a & a \\ a & -a & a \end{vmatrix}.$$

3) Determinantni qulay usulda hisoblang
$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1-a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1-b \end{vmatrix}$$

14-variant

1)Tenglamani yeching.
$$(0.6)^x \cdot \left(\frac{25}{9}\right)^{\begin{vmatrix} x & 3 \\ 4 & x \end{vmatrix}} = \left(\frac{27}{125}\right)^3.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.
$$\begin{vmatrix} a & x & x \\ x & b & x \\ x & x & c \end{vmatrix}.$$

3) Determinantni qulay usulda hisoblang
$$\begin{vmatrix} 10 & 2 & 0 & 0 & 0 \\ 12 & 10 & 2 & 0 & 0 \\ 0 & 12 & 10 & 2 & 0 \\ 0 & 0 & 12 & 10 & 2 \\ 0 & 0 & 0 & 12 & 10 \end{vmatrix}$$

15-variant

1)Tenglamani yeching.
$$ln \frac{4}{\begin{vmatrix} 1 & x \\ x & 1 \end{vmatrix}} = lg \begin{vmatrix} 2 & x \\ 1 & 2 \end{vmatrix}.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.
$$\begin{vmatrix} \sin x & 0 & -1,5 \\ -2 & 1 & 4 \\ 0,5 & 0 & \cos x \end{vmatrix}.$$

3) Determinantni qulay usulda hisoblang
$$\begin{vmatrix} \sin^2 a & \cos 2a & \cos^2 a \\ \sin^2 b & \cos 2b & \cos^2 b \\ \sin^2 y & \cos 2y & \cos^2 y \end{vmatrix}$$

16-variant

1)Tengsizlikni yeching.
$$\frac{1}{\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix}} < \frac{1}{3}.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$$

3) Determinantni qulay usulda hisoblang

$$\begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ c+a & b & 1 \end{vmatrix}$$

17-variant

1)Tenglamani yeching.

$$lg\frac{4}{\begin{vmatrix} 1 & x \\ x & 1 \end{vmatrix}} < lg\begin{vmatrix} 2 & x \\ 1 & 2 \end{vmatrix}$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.

$$\begin{vmatrix} 3^x & 2 & -1 \\ 9^x & 2^x & 0 \\ 2^x & 0 & 1 \end{vmatrix}.$$

3) Determinantlarni yig'indisini toping.

$$\begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 6 \\ 0 & 5 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & -2 \\ 0 & 5 & -4 \end{vmatrix}.$$

18-variant

1)Uch o'lchovli determinantni hisoblang

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.

$$\begin{vmatrix} 3^x & 2 & -1 \\ 9^x & 2^x & 0 \\ 2^x & 0 & 1 \end{vmatrix}.$$

3) Determinantlar yig'indisini toping

$$\begin{vmatrix} 1 & -3 & -5 \\ 4 & 2 & 1 \\ 7 & 6 & -6 \end{vmatrix} + \begin{vmatrix} 1 & 3 & -5 \\ 4 & -2 & 1 \\ 7 & 6 & -6 \end{vmatrix}$$

19-variant

1)Uch o'lchovli determinantni hisoblang

$$\begin{vmatrix} 5 & 3 & 2 \\ -1 & 2 & 4 \\ 7 & 3 & 6 \end{vmatrix}.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.

$$\begin{vmatrix} \sin^2 a & 1 & \cos^2 a \\ \sin^2 b & 1 & \cos^2 b \\ \sin^2 y & 1 & \cos^2 y \end{vmatrix}.$$

3) Determinantlar yig'indisini toping

$$A = \begin{vmatrix} 1 & -5 & 2 \\ -2 & 3 & 4 \\ 3 & 2 & 1 \end{vmatrix}$$

$$B = \begin{vmatrix} 1 & 5 & 2 \\ -2 & -1 & 4 \\ 3 & -2 & 1 \end{vmatrix}$$ bo'lsa,

$A + B$ ni hisoblang.

20-variant

1)Uch o'lchovli determinantni hisoblang

$$\begin{vmatrix} 1 & 2 & 3 \\ 8 & 1 & 4 \\ 2 & 1 & 1 \end{vmatrix}.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.

$$\begin{vmatrix} x & x & ax+bx \\ y & y & ay+by \\ z & z & az+bz \end{vmatrix}$$

3) Determinantlar yig'indisini toping
$$\begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 6 \\ 0 & 5 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & -2 \\ 0 & 5 & -4 \end{vmatrix}.$$

21-variant

1)Uch o'lchovli determinantni hisoblang
$$\begin{vmatrix} 3 & -1 & -2 \\ 1 & 2 & 5 \\ -4 & 1 & 6 \end{vmatrix}.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.
$$\begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ c+a & b & 1 \end{vmatrix}$$

3) Determinantni qulay usulda hisoblang
$$\begin{vmatrix} 10 & 2 & 0 & 0 & 0 \\ 12 & 10 & 2 & 0 & 0 \\ 0 & 12 & 10 & 2 & 0 \\ 0 & 0 & 12 & 10 & 2 \\ 0 & 0 & 0 & 12 & 10 \end{vmatrix}$$

22-variant

1)Uch o'lchovli determinantni hisoblang
$$\begin{vmatrix} 12 & 6 & -4 \\ 6 & 4 & 4 \\ 3 & 2 & 8 \end{vmatrix}.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.
$$\begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ x & 1 & -x \end{vmatrix}$$

3) Determinantni qulay usulda hisoblang
$$\begin{vmatrix} 2 & 3 & 4 \\ 2 & a+3 & b+4 \\ 2 & c+3 & d+4 \end{vmatrix}$$

23-variant

1)Uch o'lchovli determinantni hisoblang
$$\begin{vmatrix} 1 & 2 & -1 \\ 3 & 7 & 2 \\ 2 & 3 & -7 \end{vmatrix}.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.
$$\begin{vmatrix} 2 & 3 & 4 \\ 2 & a+3 & b+4 \\ 2 & c+3 & d+4 \end{vmatrix}$$

3) Determinantni qulay usulda hisoblang
$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1-a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1-b \end{vmatrix}$$

24-variant

1)Quyidagini toping.
$$A = \begin{vmatrix} 7 & 5 \\ 3 & 4 \end{vmatrix} \quad \text{va} \quad B = \begin{vmatrix} 2 & 9 \\ 1 & 7 \end{vmatrix}$$
bo'lsa, $A \cdot B$ ni hisoblang.

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.
$$\begin{vmatrix} 1 & b & 1 \\ 0 & b & 0 \\ b & b^2 & b \end{vmatrix}$$

3) Determinantni qulay usulda hisoblang
$$\begin{vmatrix} 2 & 3 & 4 \\ 2 & a+3 & b+4 \\ 2 & c+3 & d+4 \end{vmatrix}$$

25-variant

1)Uch o'lchovli determinantni hisoblang
$$\begin{vmatrix} 2 & -1 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -3 \end{vmatrix}.$$

2) Uch o'lchovli determinantni a) uchburchak usulda, b) Sarrius usulida, c) determinant xossalaridan foydalanib hisoblang.

$$\begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ -x & 1 & x \end{vmatrix}$$

3) Determinantni qulay usulda hisoblang

$$\begin{vmatrix} 1 & b & 1 \\ 0 & b & 0 \\ b & 0 & -1 \end{vmatrix}$$

4-mavzu: Matritsalar

Matritsalar ustida chiziqli amallar. Matritsalar ustida chiziqli amallarni bajarish deganda matritsalar ustida quyidagi arifmetil amallarni bajarishni tushinamiz: matritsalarni songa ko'paytirish, matritsalarni o'zaro qo'shish va ayirish hamda matritsalarni o'zaro ko'paytirish. Ushbu amallarni bajarish tartibiga va qoidalariga e'tibor qarating hamda eslab qoling.

1. Matritsani songa ko'paytirish uchun uning barcha elementlari shu songa ko'paytiriladi.

Masalan, bizga $k \neq 0$ soni hamda $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ matritsa berilgan bo'lsa, A matritsani k songa ko'paytirish $kA = \begin{pmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \end{pmatrix}$ kabi amalga oshiriladi.

2. O'lchamlari bir hil bo'lgan A va B matritsalarni qo'shish uchun ularning mos elementlari qo'shiladi, ya'ni $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ $B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$ bo'lsa, $A + B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{pmatrix}$ matritsa hosil bo'ladi.

3. O'lchamlari bir hil bo'lgan A va B matritsalarni ayirganda ularning mos elementlari ayriladi, ya'ni $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ $B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$ bo'lsa,

$$A - B = \begin{pmatrix} a_{11}-b_{11} & a_{12}-b_{12} & a_{13}-b_{13} \\ a_{21}-b_{21} & a_{22}-b_{22} & a_{23}-b_{23} \end{pmatrix}$$ matritsa hosil bo'ladi.

4. Matritsalarni matritsaga ko'paytirish. Agar A matritsaning ustunlari soni B matritsaning satrlar soniga teng bo'lsa A ni B ga ko'paytirish mumkin, nxm o'lchovli $A = (a_{ik})$ matritsani mxp o'lchovli $B = (b_{ik})$ matritsaga ko'paytirish natijasida hosil bo'ladigan nxp o'lchovli $C=(c_{ik})$ matritsa elementlari quyidagi formula bo'yicha aniqlanadi: $c_{ij}=\sum_{j=1}^{n} a_{ij}b_{jk}$.

Misollar ko'rib chiqaylik:

1-misol. $A=\begin{pmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{pmatrix}$ $B=\begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ berilgan bo'lsa, $A+B$ matritsani toping.

Yechish: $A+B=\begin{pmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}=\begin{pmatrix} 2+0 & 4+2 & 1+1 \\ -1+1 & 0+1 & 2+2 \end{pmatrix}=\begin{pmatrix} 2 & 6 & 2 \\ 0 & 1 & 4 \end{pmatrix}.$

2-misol. $A=\begin{pmatrix} 7 & -12 \\ -4 & 7 \end{pmatrix}$ $B=\begin{pmatrix} 26 & 45 \\ 15 & 26 \end{pmatrix}$ berilgan bo'lsa, $A\cdot B$ matritsani toping.

Yechish:

$$A\cdot B=\begin{pmatrix} 7 & -12 \\ -4 & 7 \end{pmatrix}\cdot\begin{pmatrix} 26 & 45 \\ 15 & 26 \end{pmatrix}=\begin{pmatrix} 7\cdot 26+(-12)\cdot 15 & 7\cdot 45+(-12)\cdot 26 \\ -4\cdot 26+7\cdot 15 & -4\cdot 45+7\cdot 26 \end{pmatrix}=\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}.$$

Matritsa rangini hisoblash. Teskari matritsani topish.

1. $A=\begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{pmatrix}$ berilgan bo'lsin A matritsaning rangi deb noldan farqli minorlarning eng katta tartibiga aytiladi va $rang(A)$ kabi ifodalanadi.

Matritsa rangi ikki usulda topiladi:

1. Matritsa rangi ta'rifga asoslangan "minorlar ajratish" usuli.

2. Elementar almashtirishlar bajarib diagonal matritsaga keltirishga asoslangan "Gauss algoritmi" usuli.

Misol. $A=\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}$ matritsa rangini hisoblang.

Yechish: A matritsa 3x5 tartibli, demak uning rangi 3 dan yuqori bo'lmaydi. Uchinchi tartibli minorlarni hisoblaymiz:

$$M_1=\begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 5 \\ 2 & -1 & 1 \end{vmatrix}=-4-10-12+12+4+10=0,$$

$$M_2=\begin{vmatrix} 2 & -1 & -2 \\ 4 & -2 & 1 \\ 2 & -1 & 8 \end{vmatrix}=-32-2+8-8+32+2=0,$$

$$M_3=\begin{vmatrix} 2 & -1 & 4 \\ 4 & -2 & 7 \\ 2 & -1 & 2 \end{vmatrix}=-8-14-16+16+8-14=0,$$

$$M_4=\begin{vmatrix} -1 & 3 & -2 \\ -2 & 5 & 1 \\ -1 & 1 & 8 \end{vmatrix}=-40-3+4-10+48+1=0,$$

$$M_5=\begin{vmatrix} -1 & 3 & 4 \\ -2 & 5 & 7 \\ -1 & 1 & 2 \end{vmatrix}=-10-21-8+20+12+7=0,$$

$$M_6=\begin{vmatrix} 3 & -2 & 4 \\ 5 & 1 & 7 \\ 1 & 8 & 2 \end{vmatrix}=6-14+160-4+20-168=0,$$

$$M_7=\begin{vmatrix} -1 & -2 & 4 \\ -2 & 1 & 7 \\ -1 & 8 & 2 \end{vmatrix}=-2+14-64+4+56-8=0,$$

$$M_8=\begin{vmatrix} 2 & 3 & 4 \\ 4 & 5 & 7 \\ 2 & 1 & 2 \end{vmatrix}=20+42+16-40-14-24=0,$$

$$M_9 = \begin{vmatrix} 2 & 3 & -2 \\ 4 & 5 & 1 \\ 2 & 1 & 8 \end{vmatrix} = 80 + 6 - 8 + 20 - 2 - 96 = 0,$$

$$M_{10} = \begin{vmatrix} 2 & -2 & 4 \\ 4 & 1 & 7 \\ 2 & 8 & 2 \end{vmatrix} = 4 + 128 - 28 - 8 + 16 - 112 = 0.$$

Barcha uchinchi tartibli minorlar nolga teng. Endi ikkinchi tartibli minorlarni hisoblaymiz: $M_{1(11)} = \begin{vmatrix} -1 & 3 \\ -2 & 5 \end{vmatrix} = -5 + 6 = 1$ $M_{1(11)} \neq 0$, demak, $r(A) = 2$.

Bu usul qo'llanganda noldan farqli minor topilgunga qadar hisoblashlar davom etadi. Shuning uchun bu usul tartibi kattaroq matritsa rangini hisoblash bir muncha qiyinchiliklarga olib keladi.

Misol. $A = \begin{pmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{pmatrix}$ matritsa rangini hisoblang.

Yechish: Matritsa rangini elementar almashtirishlar yordamida hisoblaymiz:

$$A = \begin{pmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{pmatrix} \text{π} \begin{pmatrix} 25 & 31 & 17 & 43 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 5 \\ 0 & 1 & 3 & 5 \end{pmatrix} \text{π} \begin{pmatrix} 25 & 31 & 17 & 43 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

bu matritsaning rangi $\begin{pmatrix} 31 & 17 & 43 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$ matritsa rangiga teng.

$$\begin{vmatrix} 31 & 17 & 43 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{vmatrix} = 40 \neq 0 \qquad r\begin{pmatrix} 31 & 17 & 43 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix} = 3.$$

Demak, berilgan matritsaning rangi ham 3 ga teng. $r(A) = 3$.

A kvadrat matritsa uchun $\det A \neq 0$ bo'lsa, teskari matritsa 2 usulda: Klassik usulida yoki Jordan usulida topiladi.

Misol. $A = \begin{pmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 1 & -2 & -1 \end{pmatrix}$ matritsaga teskari bo'lgan A^{-1} matritsani ta'rifga ko'ra toping.

Yechish: Ta'rifga ko'ra teskari matritsa $A^{-1} = \frac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$

formula bo'yicha aniqlanadi. Bu yerda $|A|$ berilgan kvadtat matritsa determinanti. $A_{ij}(i = 1, 2, 3; j = 1, 2, 3)$ berilgan matritsa elementlarining algebraik to'ldiruvchilari.

$$|A| = \begin{vmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 1 & -2 & -1 \end{vmatrix} = -2 + 12 - 20 - 2 + 15 + 16 = 43 - 24 = 19 \neq 0.$$

Demak, A xosmas matritsa va A^{-1} teskari matritsasi mavjud.

A matritsa elementlarining algebraik to'ldiruvchilarini hisoblaymiz:

$$A_{11} = \begin{vmatrix} 1 & 4 \\ -2 & -1 \end{vmatrix} = -1 + 8 = 7, \qquad A_{22} = \begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix} = -2 - 2 = -4,$$

$$A_{21} = -\begin{vmatrix} 3 & 2 \\ -2 & -1 \end{vmatrix} = -(-3 + 4) = -1, \qquad A_{32} = -\begin{vmatrix} 2 & 2 \\ 5 & 4 \end{vmatrix} = -(8 - 10) = 2,$$

$$A_{31} = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 12 - 2 = 10, \qquad A_{13} = \begin{vmatrix} 5 & 1 \\ 1 & -2 \end{vmatrix} = -10 - 1 = -11,$$

$$A_{12} = -\begin{vmatrix} 5 & 4 \\ 1 & -1 \end{vmatrix} = -(-5 - 4) = 9, \qquad A_{23} = -\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -(-4 - 3) = 7,$$

$$A_{33} = \begin{vmatrix} 2 & 3 \\ 5 & 1 \end{vmatrix} = 2 - 15 = -13.$$

A_{ij} larni teskari matritsa formulasiga qo'yamiz:

$$A^{-1}=\frac{1}{19}\begin{pmatrix}7 & -1 & 10\\ 9 & -4 & 2\\ -11 & 7 & -13\end{pmatrix}$$ teskari matritsaning to'g'ri topilganini

$AA^{-1}=E$ formula bo'yicha tekshiramiz:

$$\begin{pmatrix}2 & 3 & 2\\ 5 & 1 & 4\\ 1 & -2 & -1\end{pmatrix}\cdot\frac{1}{19}\begin{pmatrix}7 & -1 & 10\\ 9 & -4 & 2\\ -11 & 7 & -13\end{pmatrix}=\frac{1}{19}\cdot\begin{pmatrix}14+27-22 & -2-12+14 & 20+6-26\\ 35+9-44 & -5-4+28 & 50+2-52\\ 7-18+11 & -1+8-7 & 10-4+13\end{pmatrix}=$$

$$=\frac{1}{19}\cdot\begin{pmatrix}19 & 0 & 0\\ 0 & 19 & 0\\ 0 & 0 & 19\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=E.$$ Demak, A^{-1} teskari matritsa to'g'ri topilgan.

Misol. $A=\begin{pmatrix}1 & 2 & 1\\ -1 & -1 & -3\\ 4 & 3 & -2\end{pmatrix}$ uchun Jordan usulida teskari matritsani toping.

Yechish: $|A|=16\neq 0$ teskari matritsa mavjud. Teskari matritsani Jordan usulida topamiz. Berilgan matritsani birlik matritsa hisobida kengaytirib, faqat satrlar ustida elementar almashtirishlar bajaramiz, bu usulni to chap tomonda A matritsa o'rnida birlik matritsa hosil bo'lguncha davom ettiramiz, o'ng tomonda hosil bo'lgan matritsa berilgan matritsaga nisbatan teskari matritsa bo'ladi.

$(A|E)\sim(E|A^{-1})$ - Jordan usuli algoritmi.

$$\left(\begin{array}{ccc|ccc}1 & 2 & 1 & 1 & 0 & 0\\ -1 & -1 & -3 & 0 & 1 & 0\\ 4 & 3 & -2 & 0 & 0 & 1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1 & 2 & 1 & 1 & 0 & 0\\ 0 & 1 & -2 & 1 & 1 & 0\\ 0 & 5 & -6 & -4 & 0 & 1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1 & 2 & 1 & 1 & 0 & 0\\ 0 & 1 & -2 & 1 & 1 & 0\\ 0 & 0 & -16 & 1 & 5 & 1\end{array}\right)\sim$$

$$\left(\begin{array}{ccc|ccc}1 & 2 & 1 & 1 & 0 & 0\\ 0 & 1 & -2 & 1 & 1 & 0\\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1 & 0 & 5 & -1 & -2 & 0\\ 0 & 1 & 0 & 14/16 & 6/16 & -2-16\\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16\end{array}\right)\sim$$

$$\left(\begin{array}{ccc|ccc}1 & 0 & 0 & -11/16 & -7/16 & 5/16\\ 0 & 1 & 0 & 14/16 & 6/16 & -2-16\\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16\end{array}\right)\qquad A^{-1}=\frac{1}{16}\cdot\begin{pmatrix}-11 & -7 & 5\\ 14 & 6 & -2\\ -1 & -5 & -1\end{pmatrix}$$ teskari matritsa to'g'ri topilganini yana formulaga qo'yib, tekshiramiz:

$$AA^{-1}=\frac{1}{16}\cdot\begin{pmatrix}1 & 2 & 1\\ -1 & -1 & -3\\ 4 & 3 & -2\end{pmatrix}\cdot\begin{pmatrix}-11 & -7 & -5\\ 14 & 6 & -2\\ -1 & -5 & -1\end{pmatrix}=\frac{1}{16}\cdot\begin{pmatrix}-11+28-1 & -7+12-5 & 5-4-1\\ 11-14+3 & 7-6+15 & -5+2+3\\ -44+42+2 & -28+18+10 & 20-6+2\end{pmatrix}=$$

$$=\frac{1}{16}\cdot\begin{pmatrix}16 & 0 & 0\\ 0 & 16 & 0\\ 0 & 0 & 16\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}.$$ Demak, teskari matritsa to'gri topilgan.

Misol. $A=\begin{pmatrix}2 & 4 & 1\\ 3 & 5 & 7\end{pmatrix}$ matritsa normasini toping.

Yechish: $N(A)=\sqrt{2^2+3^2+4^2+5^2+1^2+7^2}=\sqrt{104}$.

Mustaqil yechish uchun mashqlar

1-variant

1)Matritsalar ustida amallarni bajaring $A=\begin{pmatrix}1 & 5\\ 2 & -4\end{pmatrix}$ $B=\begin{pmatrix}3 & 2\\ 4 & 1\end{pmatrix}$, $2A-B=?$

2)Berilgan matritsaning rangini toping $\begin{pmatrix}1 & 2 & 3 & 6\\ 2 & 3 & 1 & 6\\ 3 & 1 & 2 & 6\end{pmatrix}$

3)Berilgan matritsaning teskari matritsasini toping. $\begin{pmatrix}1 & 5 & 7\\ 3 & 1 & 1\\ 2 & 3 & 4\end{pmatrix}$

2-variant

1)Matritsalar ustida amallarni bajaring $A=\begin{pmatrix}1 & -1 & -3\\ 2 & 1 & 5\end{pmatrix}$ $B=\begin{pmatrix}0 & 3 & 2\\ -1 & 4 & 1\end{pmatrix}$, $3A-2B=?$

2)Berilgan matritsaning rangini toping $\begin{pmatrix}1 & 2 & 3 & 6\\ 2 & 3 & 1 & 6\\ 3 & 1 & 2 & 6\end{pmatrix}$

3)Berilgan matritsaning teskari matritsasini toping. $\begin{pmatrix}2 & -1 & 7\\ 5 & 3 & 2\\ 1 & 4 & 3\end{pmatrix}$

3-variant

1)Matritsalar ustida amallarni bajaring $\begin{pmatrix}7 & 0\\ 3 & 1\\ -1 & 2\end{pmatrix}-3\begin{pmatrix}2 & \sqrt{2}\\ 1 & -1\\ -1 & 0\end{pmatrix}+\begin{pmatrix}1 & \sqrt{18}\\ 4 & -5\\ 3 & 1\end{pmatrix}$.

2)Berilgan matritsaning rangini toping $\begin{pmatrix}0 & 2 & 0 & 0\\ 1 & 0 & 0 & 4\\ 0 & 0 & 3 & 0\end{pmatrix}$

3)Berilgan matritsaning teskari matritsasini toping. $\begin{pmatrix}1 & 0 & -2\\ 3 & 1 & 0\\ -1 & 2 & 4\end{pmatrix}$

4-variant

1)Matritsalar ustida amallarni bajaring $C=(1\ 2\ 3)$, $F=\begin{pmatrix}4 & -3\\ 1 & 2\\ 0 & 2\end{pmatrix}$, $C\cdot F=?$

2)Berilgan matritsaning rangini toping $\begin{pmatrix}4 & 5 & 2 & 1 & -3\\ 0 & 2 & 1 & 1 & 2\\ 4 & 7 & 3 & 3 & -1\\ 8 & 12 & 5 & 3 & -4\end{pmatrix}$

3)Berilgan matritsaning teskari matritsasini toping. $\begin{pmatrix}3 & 2 & 2\\ 1 & 3 & 1\\ 5 & 3 & 4\end{pmatrix}$

5-variant

1)Matritsalar ustida amallarni bajaring $A=\begin{pmatrix}2 & 1 & -1\\ 0 & 1 & 0\\ 0 & 0 & -1\end{pmatrix}$, $B=\begin{pmatrix}1 & -1\\ 0 & 1\\ 1 & 0\end{pmatrix}$, $A\cdot B=?$

2)Berilgan matritsaning rangini toping $\begin{pmatrix}2 & -4 & 3 & 1 & 0\\ 1 & -2 & 1 & -4 & 2\\ 0 & 1 & -1 & 3 & 1\\ 4 & -7 & 4 & -4 & 5\end{pmatrix}$

3)Berilgan matritsaning teskari matritsasini toping. $\begin{pmatrix}tg\alpha & 1\\ 2 & ctg\alpha\end{pmatrix}$

6-variant

1)Matritsalar ustida amallarni bajaring $A=\begin{pmatrix}1 & -1 & 2\\ 2 & 3 & 4\\ -4 & 5 & 1\end{pmatrix}$, $B=\begin{pmatrix}3 & 4 & 1\\ 0 & 2 & 5\\ 1 & -1 & 4\end{pmatrix}$, $A\cdot B=?$

2)Berilgan matritsaning rangini toping $A=\begin{pmatrix}2 & 1 & -2 & 3\\ -2 & 9 & -4 & 7\\ -4 & 3 & 1 & -1\end{pmatrix}$

3)Berilgan matritsaning teskari matritsasini toping. $\begin{pmatrix}2 & 1 & 1\\ 1 & 0 & 2\\ 3 & 1 & 2\end{pmatrix}$

7-variant

1)Matritsalar ustida amallarni bajaring $A=\begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}$, $A^2=?$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix} 0 & 2 & -4 \\ -1 & -4 & 5 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$A=\begin{pmatrix} 3 & 5 & 7 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$$

8-variant

1)Matritsalar ustida amallarni bajaring

$$A=\begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \\ 4 & 1 & 1 \end{pmatrix}, \quad 2A^2+3A+5E=?$$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix} 2 & 3 & 4 & 0 \\ 1 & 5 & 7 & 0 \\ 3 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$A=\begin{pmatrix} -1 & 0 & 8 \\ 5 & 9 & 0 \\ 0 & 4 & 3 \end{pmatrix}$$

9-variant

1)Matritsalar ustida amallarni bajaring

$$A=\begin{pmatrix} 3 & 4 & 2 \\ 1 & 0 & 5 \end{pmatrix}, B=\begin{pmatrix} 2 & 0 \\ 1 & 3 \\ 0 & 5 \end{pmatrix},$$

$$C=\begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}, \quad A\cdot B-C^2=?$$

2)Berilgan matritsaning rangini toping

$$A=\begin{pmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$\begin{pmatrix} 1 & -1 & 1 \\ -38 & 41 & -34 \\ 27 & -29 & 24 \end{pmatrix}$$

10-variant

1)Matritsalar ustida amallarni bajaring

$$A=\begin{pmatrix} 1 & 2 & -3 \\ 1 & 0 & 2 \\ 4 & 5 & 3 \end{pmatrix}, B=\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix},$$

$$C=\begin{pmatrix} 2 & 0 & 5 \end{pmatrix}, \quad A\cdot B\cdot C-3E=?$$

2)Berilgan matritsaning rangini toping

$$A=\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}.$$

3)Berilgan matritsaning teskari matritsasini toping.

$$\begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}$$

11-variant

1)Matritsalar ustida amallarni bajaring

$$A=\begin{pmatrix} 2 & 0 & 1 \\ -2 & 3 & 2 \\ 4 & -1 & 5 \end{pmatrix}, B=\begin{pmatrix} -3 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 3 \end{pmatrix},$$

$$A\cdot B=?$$

2)Berilgan matritsaning rangini toping

$$A=\begin{pmatrix} 1 & 2 & 1 & 3 \\ 4 & -1 & -5 & -6 \\ 1 & -3 & -4 & -7 \\ 2 & 1 & -1 & 0 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$\begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix}$$

12-variant

1)Matritsalar ustida amallarni bajaring $\begin{pmatrix}1 & -3 & 2\\ 3 & -4 & 1\\ 2 & -5 & 3\end{pmatrix}\cdot\begin{pmatrix}2 & 5 & 6\\ 1 & 2 & 5\\ 1 & 3 & 2\end{pmatrix}=?$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix}1 & 2 & 1 & 3 & 4\\ 3 & 4 & 2 & 6 & 8\\ 1 & 2 & 1 & 8 & 4\end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}X=\begin{pmatrix}3 & 5\\ 5 & 9\end{pmatrix}$$

13-variant

1)Matritsalar ustida amallarni bajaring

$$\begin{pmatrix}2 & -1 & 3 & -4\\ 3 & -2 & 4 & -3\\ 5 & -3 & -2 & 1\\ 3 & -3 & -1 & 2\end{pmatrix}\cdot\begin{pmatrix}7 & 8 & 6 & 9\\ 5 & 7 & 4 & 5\\ 3 & 4 & 5 & 6\\ 2 & 1 & 1 & 2\end{pmatrix}=?$$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix}1 & 2 & 1 & 3 & 4\\ 3 & 4 & 2 & 6 & 8\\ 1 & 2 & 1 & 8 & 4\end{pmatrix}$$

3)Berilgan tenglamani yeching.

$$\begin{pmatrix}1 & 2 & -3\\ 3 & 2 & -4\\ 2 & -1 & 0\end{pmatrix}X=\begin{pmatrix}1 & -3 & 0\\ 10 & 2 & 7\\ 10 & 7 & 8\end{pmatrix}.$$

14-variant

1)Matritsalar ustida amallarni bajaring

$$\begin{pmatrix}5 & 7 & -3 & -4\\ 7 & 6 & -4 & -5\\ 6 & 4 & -3 & -2\\ 8 & 5 & -6 & -1\end{pmatrix}\cdot\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 4 & 5\\ 1 & 3 & 5 & 7\\ 2 & 4 & 6 & 8\end{pmatrix}=?$$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix}4 & 3 & -5 & 2 & 3\\ 8 & 6 & -7 & 4 & 2\\ 4 & 3 & -8 & 2 & 7\\ 4 & 3 & 1 & 2 & -5\\ 8 & 6 & -1 & 4 & -6\end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$\begin{pmatrix}-1 & 1\\ 4 & 2\end{pmatrix}$$

15-variant

1)Matritsalar ustida amallarni bajaring

$A=\begin{pmatrix}3 & 5\\ 4 & 1\end{pmatrix}$, $B=\begin{pmatrix}2 & 3\\ 1 & -2\end{pmatrix}$,

$2A+5B=?$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix}17 & -28 & 45 & 11 & 39\\ 24 & -37 & 61 & 13 & 50\\ 25 & -7 & 32 & -18 & -11\\ 31 & 12 & 19 & -43 & -55\\ 42 & 13 & 29 & -55 & -68\end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$A=\begin{pmatrix}1 & 0 & 5\\ 4 & -2 & -1\\ 2 & 1 & 3\end{pmatrix}$$

16-variant

1)Matritsalar ustida amallarni bajaring

$A=\begin{pmatrix}3 & 5 & 7\\ 2 & -1 & 0\\ 4 & 3 & 2\end{pmatrix}$ $B=\begin{pmatrix}1 & 2 & 4\\ 2 & 3 & -2\\ -1 & 0 & 1\end{pmatrix}$,

$A+B=?$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix}24 & 19 & 36 & 72 & -38\\ 49 & 40 & 73 & 147 & -80\\ 73 & 59 & 98 & 219 & -118\\ 47 & 36 & 71 & 141 & -72\end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$A=\begin{pmatrix} 2 & 5 \\ -4 & 2 \end{pmatrix}.$$

17-variant

1)Matritsalar ustida amallarni bajaring

$$A=\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 5 \end{pmatrix},\ C=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},\ A\cdot C=?$$

2)Berilgan matritsaning rangini toping

$$A=\begin{pmatrix} 1 & 2 & 1 & 0 \\ 1 & 1 & 3 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 0 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$A=\begin{pmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$$

18-variant

1)Matritsalar ustida amallarni bajaring

$$A=\begin{pmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix},\ F=\begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 0 \end{pmatrix},\ A\cdot F=?$$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

19-variant

1)Matritsalar ustida amallarni bajaring

$$A=\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix},$$

$$B=\begin{pmatrix} 5 & 7 \\ -1 & 2 \end{pmatrix},\ A^2-A\cdot B+2B\cdot A=?$$

2)Berilgan matritsaning rangini toping

$$A=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$A=\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

20-variant

1)Matritsalar ustida amallarni bajaring

$$A=\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix},\ A^2+A+E=?$$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix} 47 & -67 & 35 & 201 & 155 \\ 26 & 98 & 23 & -294 & 6 \\ 16 & -428 & 1 & 1284 & 52 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$B=\begin{pmatrix} 0 & -1 & 3 \\ 3 & 5 & 2 \\ 4 & -2 & 1 \end{pmatrix}$$

21-variant

1)Matritsalar ustida amallarni bajaring

$$\begin{pmatrix} 5 & 2 & -2 & 3 \\ 6 & 4 & -3 & 5 \\ 9 & 2 & -3 & 4 \\ 7 & 6 & -4 & 7 \end{pmatrix}\cdot\begin{pmatrix} 2 & 2 & 2 & 2 \\ -1 & -5 & 3 & 11 \\ 16 & 24 & 8 & -8 \\ 8 & 16 & 0 & -16 \end{pmatrix}=$$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$\begin{pmatrix} 1 & 3 & 1 \\ 2 & 0 & 4 \\ 1 & 2 & 3 \end{pmatrix}$$

22-variant

1)Matritsalar ustida amallarni bajaring

$$\begin{pmatrix} 1 & 3 \\ 2 & 0 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & 3 \\ 5 & 4 & 0 \end{pmatrix} + \begin{pmatrix} -10 & -9 & 7 \\ 1 & 5 & 8 \\ -1 & -3 & 6 \end{pmatrix} =$$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix} 1 & 7 & 5 & 8 & 9 & 2 \\ 3 & 21 & 15 & 24 & 27 & 6 \\ 2 & 14 & 10 & 16 & 18 & 4 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$\begin{pmatrix} 0 & -1 & 3 \\ 3 & 5 & 2 \\ 4 & -2 & 1 \end{pmatrix}$$

23-variant

1)Matritsalar ustida amallarni bajaring

$$\begin{pmatrix} 1 & 1 & 1 & -1 \\ -5 & -3 & -4 & 4 \\ 5 & 1 & 4 & -3 \\ -16 & -11 & -15 & 14 \end{pmatrix} \cdot \begin{pmatrix} 7 & -2 & 3 & 4 \\ 11 & 0 & 3 & 4 \\ 5 & 4 & 3 & 0 \\ 22 & 2 & 9 & 8 \end{pmatrix} = ?$$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix} 1 & 7 & 5 & 8 & 9 & 2 \\ 3 & 21 & 15 & 24 & 27 & 6 \\ 2 & 14 & 10 & 16 & 18 & 4 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$A = \begin{pmatrix} 1 & 2 \\ -2 & 0 \end{pmatrix}$$

24-variant

1)Matritsalar ustida amallarni bajaring

$$A = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}, \ A^{20} = ?$$

2)Berilgan matritsaning rangini toping

$$\begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 2 & 0 & 4 & 0 & 0 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$\begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$$

25-variant

1)Matritsalar ustida amallarni bajaring

$$\begin{pmatrix} 4 & 3 \\ 7 & 5 \end{pmatrix} \cdot \begin{pmatrix} -28 & 93 \\ 38 & -126 \end{pmatrix} \cdot \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix} = ?$$

2)Berilgan matritsaning rangini toping

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 2 \end{pmatrix}$$

3)Berilgan matritsaning teskari matritsasini toping.

$$B = \begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

5-mavzu: Ikki va uch noma'lumli chiziqli tenglamalar sistemasi. Kroneker-kapelli teoremasi. Chiziqli tenglamalar sistemasini yechishning kramer usuli.

Bizga quyidagi tenglamalar sistemasi berilgan bo‘lsin.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1m}x_m = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2m}x_m = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nm}x_m = b_n \end{cases}.$$

Kroneker-Kapelli teoremasi berilgan chiziqli tenglamalar sistemasining birgalikda yoki birgalikda emasligini aniqlaydi.

Bunda chiziqli tenglamalar sistemasining asosiy va ozod hadlar hisobiga kengaytirilgan matritsasini tuzamiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1m} \\ a_{21} & a_{22} & \ldots & a_{2m} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nm} \end{pmatrix} \text{ va } B = \left(\begin{array}{cccc|c} a_{11} & a_{12} & \ldots & a_{1m} & b_1 \\ a_{21} & a_{22} & \ldots & a_{2m} & b_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nm} & b_n \end{array}\right).$$

Teorema. Agar A matritsa rangi B matritsa rangiga teng bo‘lib, noma'lumlar soniga ham teng bo‘lsa, ya'ni $r(A) = r(B) = m$ bo‘lsa, tenglamalar sistemasi aniqlangan bo‘ladi, sistema esa birgalikda bo‘lib yagona yechimga ega bo‘ladi.

Agar $r(A) = r(B) < m$ bo‘lsa, sistema birgalikda bo‘lib, cheksiz ko‘p yechimga ega bo‘ladi.

Agar $r(A) < r(B)$ bo‘lsa, sistema birgalikda bo‘lmaydi, sistema yechimga ega bo‘lmaydi.

17-misol. $\begin{cases} x_1 + 3x_2 + 5x_3 + 7x_4 + 9x_5 = 1 \\ x_1 - 2x_2 + 3x_3 - 4x_4 + 5x_5 = 2 \\ 2x_1 + 11x_2 + 12x_3 + 25x_4 + 22x_5 = 4 \end{cases}$ sistemani birgalikda yoki birgalikda emasligini tekshiring.

Yechish: Buning uchun asosiy va kengaytirilgan matritsalaning ranglarini hisoblaymiz:

$$A = \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 1 & -2 & 3 & -4 & 5 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 3 & 9 & 15 & 21 & 27 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 1 & 3 & 5 & 7 & 9 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix}$$

2-satr elementlaridan 1-satr elementlarini ayiramiz:

$$A \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix}. \text{ Demak, } r(A) = 2.$$

$$B = \left(\begin{array}{ccccc|c} 1 & 3 & 5 & 7 & 9 & 1 \\ 1 & -2 & 3 & -4 & 5 & 2 \\ 2 & 11 & 12 & 25 & 22 & 4 \end{array}\right).$$

Ushbu matritsa rangini topish uchun yana yuqoridagi ishni takrorlaymiz, natijada B matritsa quyidagi ko‘rinishni oladi.

$$B \sim \left(\begin{array}{ccccc|c} 1 & 3 & 5 & 7 & 9 & 1 \\ 1 & -2 & 3 & -4 & 5 & 2 \\ 2 & 11 & 12 & 25 & 22 & 4 \end{array}\right),, \quad B_1 = \begin{pmatrix} 7 & 9 & 1 \\ 0 & 0 & 1 \\ 25 & 22 & 4 \end{pmatrix}$$

matritsa rangini topamiz:

$$M = |B_1| = \begin{vmatrix} 7 & 9 & 1 \\ 0 & 0 & 1 \\ 25 & 22 & 4 \end{vmatrix} = 225 - 154 = 71, \text{ bundan } r(B) = 3.$$

Demak, $r(B) = 3$. bo‘lib, $r(A) \neq r(B)$ va sistema birgalikda emas.

18-misol. $\begin{cases} x_1 + 5x_2 + 4x_3 + 3x_4 = 1 \\ 2x_1 - x_2 + 2x_3 - x_4 = 0 \\ 5x_1 + 3x_2 + 8x_3 + x_4 = 1 \end{cases}$ sistemani birgalikda yoki birgalikda emasligini tekshiring.

Yechish: Sistema birgalikda yoki birgalikda emasligini tekshiring.

Ozod hadlar hisobiga kengaytirilgan matritsa tuzamiz:

$$B=\left(\begin{array}{cccc|c}1 & 5 & 4 & 3 & 1\\ 2 & -1 & 2 & -1 & 0\\ 5 & 3 & 8 & 1 & 1\end{array}\right)$$

3-satr elementlaridan 1-satr elementlarini ayiramiz:

$$B=\left(\begin{array}{cccc|c}1 & 5 & 4 & 3 & 1\\ 2 & -1 & 2 & -1 & 0\\ 4 & -2 & 4 & -2 & 0\end{array}\right)\sim\left(\begin{array}{cccc|c}1 & 5 & 4 & 3 & 1\\ 2 & -1 & 2 & -1 & 0\\ 2 & -1 & 2 & -1 & 0\end{array}\right)\sim\left(\begin{array}{cccc|c}1 & 5 & 4 & 3 & 1\\ 2 & -1 & 2 & -1 & 0\\ 0 & 0 & 0 & 0 & 0\end{array}\right)$$

A matritsa B matritsaning qismi bo'lgani uchun $r(A)=r(B)=2$ ekanini ko'rish mumkin. Demak, sistema birgalikda.

Chiziqli tenglamalar sistemasining yechimi Kramer formulalari deb atalgan quyidagi formulalar bo'yicha topiladi: $x_1=\frac{\Delta_1}{\Delta},\quad x_2=\frac{\Delta_2}{\Delta},\quad x_3=\frac{\Delta_3}{\Delta},\ \dots,\ x_n=\frac{\Delta_n}{\Delta}.$

Bu yerda Δ noma'lumlar oldidagi koeffitsiyentlardan tuzilgan kvadrat matritsa determinanti, $\Delta_1, \Delta_2, \Delta_3, \dots, \Delta_n$ lar asosiy matritsada mos ravishda $1, 2, 3, \dots, n$ -ustun elementlarini ozod hadlar bilan almashtirishdan hosil bo'lgan determinantlar. Shuni ta'kidlash kerakki, sistemada noma'lumlar va tenglamalar soni teng bo'lgan hollarda Kramer formulasini qo'llash maqsadga muvofiq.

Agar $\Delta \neq 0$ bo'lsa, sistema yagona yechimga ega bo'ladi.

Agar $\Delta = 0$ bo'lib, $\Delta_1, \Delta_2, \Delta_3, \dots, \Delta_n$ lardan kamida bittasi noldan farqli bo'lsa, sistema yechimga ega emas.

Agar $\Delta = 0$ bo'lib, $\Delta_1=\Delta_2=\Delta_3=\ldots=\Delta_n=0$ bo'lsa, sistema aniqmas, cheksiz ko'p yechimga ega bo'ladi. Formulani 3 noma'lumli 3 ta chiziqli tenglamalar sistemasi misolida keltiramiz:

$$\begin{cases}a_{11}x_1+a_{12}x_2+a_{13}x_3=b_1\\ a_{21}x_1+a_{22}x_2+a_{23}x_3=b_3\\ a_{31}x_1+a_{32}x_2+a_{33}x_3=b_3\end{cases}$$

Bu sistema uchun determinantlar quyidagicha bo'ladi:

$$\Delta=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix},\qquad \Delta_1=\begin{vmatrix}b_1 & a_{12} & a_{13}\\ b_2 & a_{22} & a_{23}\\ b_3 & a_{32} & a_{33}\end{vmatrix},$$

$$\Delta_2=\begin{vmatrix}a_{11} & b_1 & a_{13}\\ a_{21} & b_2 & a_{23}\\ a_{31} & b_3 & a_{33}\end{vmatrix},\qquad \Delta_3=\begin{vmatrix}a_{11} & a_{12} & b_1\\ a_{21} & a_{22} & b_2\\ a_{31} & a_{32} & b_3\end{vmatrix}.$$

Buni misollarda ko'ramiz:

19-misol. $\begin{cases}x_1+2x_2+x_3=8\\ 3x_1+2x_2+x_3=10\\ 4x_1+3x_2-2x_3=4\end{cases}$ sistemani Kramer formulasi bilan yeching.

Yechish: $\Delta=\begin{vmatrix}1 & 2 & 1\\ 3 & 2 & 1\\ 4 & 3 & -2\end{vmatrix}=-4+8+9-8-3+12=14.$

$\Delta\neq 0$ bo'lgani uchun sistema aniq, yagona yechim Kramer formulalari yordamida topiladi.

$$\Delta_1=\begin{vmatrix}8 & 2 & 1\\ 10 & 2 & 1\\ 4 & 3 & -2\end{vmatrix}=-32+8+30-8+40-24=14,$$

$$\Delta_2=\begin{vmatrix}1 & 8 & 1\\ 3 & 10 & 1\\ 4 & 4 & -2\end{vmatrix}=-20+32+12-40-4+48=28,$$

$$\Delta_3=\begin{vmatrix}1 & 2 & 8\\ 3 & 2 & 10\\ 4 & 3 & 4\end{vmatrix}=8+80+72-64-24-30=42,$$

$$x_1=\frac{14}{14}=1,\qquad x_2=\frac{28}{14}=2,\qquad x_3=\frac{42}{14}=3.\qquad (x_1;x_2;x_3)=(1;2;3).$$

20-misol. $\begin{cases} 4x_1 + 2x_2 + 3x_3 = -2 \\ 3x_1 + 8x_2 - x_3 = 8 \\ 9x_1 + x_2 + 8x_3 = 0 \end{cases}$ sistemani Kramer usulida yeching.

Yechish: $\Delta = \begin{vmatrix} 4 & 2 & 3 \\ 2 & 8 & -1 \\ 9 & 1 & 8 \end{vmatrix} = 256 + 6 - 18 - 216 - 32 + 4 = 266 - 266 = 0.$

$\Delta = 0$, demak, Kramer teoremasi tasdig'iga asosan sistema yoki aniqmas, yoki birgalikda emas. Δ_1 ni hisoblaymiz:

$$\Delta_1 = \begin{vmatrix} -2 & 2 & 3 \\ 8 & 8 & -1 \\ 0 & 1 & 8 \end{vmatrix} = -128 + 24 - 128 - 2 = -234 \neq 0.$$

$\Delta = 0$, $\Delta_1 \neq 0$ bo'lgani uchun Kramer teoremasiga ko'ra sistema aniqlanmagan.

21-misol. $\begin{cases} -2x_1 + x_2 - x_3 = 7 \\ 4x_1 + 5x_2 - 3x_3 = -5 \\ x_1 + 3x_2 - 2x_3 = 1 \end{cases}$ sistemani Kramer usulida yeching.

Yechish: $\Delta = \begin{vmatrix} -2 & 1 & -1 \\ 4 & 5 & -3 \\ 1 & 3 & -2 \end{vmatrix} = 20 - 3 - 12 + 5 + 8 - 18 = 33 - 33 = 0, \ \Delta = 0.$

Demak, sistema yoki aniqmas, yoki birgalikda emas. $\Delta_1, \Delta_2, \Delta_3$ larni hisoblaymiz:

$$\Delta_1 = \begin{vmatrix} 7 & 1 & -1 \\ -5 & 5 & -3 \\ 1 & 3 & -2 \end{vmatrix} = -70 + 15 - 3 + 5 - 10 + 63 = 83 - 83 = 0,$$

$$\Delta_2 = \begin{vmatrix} -2 & 7 & -1 \\ 4 & -5 & -3 \\ 1 & 1 & -2 \end{vmatrix} = -20 - 21 - 4 - 5 + 56 - 6 = 56 - 56 = 0,$$

$$\Delta_3 = \begin{vmatrix} -2 & 1 & 7 \\ 4 & 5 & -5 \\ 1 & 3 & 1 \end{vmatrix} = -10 - 5 + 84 - 35 - 4 - 30 = 84 - 84 = 0.$$

$\Delta = 0$, $\Delta_1 = \Delta_2 = \Delta_3 = 0$ bo'lgani uchun sistema aniqmas, cheksiz ko'p yechimga ega.

Sistemani Gauss algoritmi bilan yechamiz:

$$\left(\begin{array}{ccc|c} -2 & 1 & -1 & 7 \\ 4 & 5 & -3 & -5 \\ 1 & 3 & -2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} -2 & 1 & -1 & 7 \\ 0 & 7 & -5 & 9 \\ 0 & \frac{7}{2} & -\frac{5}{2} & \frac{9}{2} \end{array}\right) \sim \left(\begin{array}{ccc|c} -2 & -1 & -1 & 7 \\ 0 & 7 & -5 & 9 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

berilgan tenglama $\begin{cases} -2x_1 + x_2 = x_3 + 7 \\ 4x_1 + 5x_2 = 3x_3 - 5 \\ x_3 \in R \end{cases}$ sistemaga teng kuchli.

Bu tenglamani Kramer formulasi bilan yechish mumkin.

$$\Delta = \begin{vmatrix} -2 & 1 \\ 4 & 5 \end{vmatrix} = -10 - 4 = -14,$$

$$\Delta_1 = \begin{vmatrix} x_3 + 7 & 1 \\ 3x_3 - 5 & 5 \end{vmatrix} = 5(x_3 + 7) - 3x_3 + 5 = 5x_3 + 35 - 3x_3 + 5 = 2x_3 + 40 = 2(x_3 + 20),$$

$$\Delta_2 = \begin{vmatrix} -2 & x_3 + 7 \\ 4 & 3x_3 - 5 \end{vmatrix} = -2(3x_3 - 5) - 4(x_3 + 7) = -6x_3 + 10 - 4x_3 - 28 = -10x_3 - 18 = -2(5x_3 + 9),$$

$$x_1 = \frac{2(x_3 + 20)}{-14} = -\frac{x_3 + 20}{7}, \quad x_2 = \frac{-2(5x_3 + 9)}{-14} = \frac{5x_3 + 9}{7}.$$

Sistemaning yechimi $\left(-\frac{x_3 + 20}{7}; \frac{5x_3 + 9}{7}; x_3\right)$ bo'ladi.

Chiziqli tenglamalar sistemasini teskari matritsa, Gauss va Gauss-Jordan usullari bilan yechish.

Chiziqli tenglamalar sistemasini teskari matritsa usulida yechishda berilgan sistemani

$$AX=B$$

matritsaviy ko'rinishida yozib olamiz. Bunda

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \qquad X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad B=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$AX=B$ tenglamani ikki tomonini chapdan A^{-1} (A matritsaning teskari matritsasi)ga ko'paytiramiz:

$$A^{-1}\cdot AX=A^{-1}\cdot B, \quad A^{-1}\cdot A=E$$

ekanidan foydalansak

$$X=A^{-1}\cdot B$$

tenglik hosil bo'ladi.

Bu formula bilan topilgan X - ustun matritsa sistemaning yechimi bo'ladi.

Misol. Sistemani matritsaviy usulda yeching.

$$\begin{cases} x_1+2x_2+x_3=8 \\ 3x_1+2x_2+x_3=10 \\ 4x_1+3x_2-2x_3=4 \end{cases}.$$

Yechish: Misolni matritsaviy usulda yechamiz: $A=\begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 4 & 3 & -2 \end{pmatrix}$ matritsa uchun teskari matritsa mavjud, chunki $\Delta=|A|=14\neq 0$.

$$A^{-1}=\frac{1}{14}\begin{pmatrix} -7 & 7 & 0 \\ 10 & -6 & 2 \\ 1 & 5 & -4 \end{pmatrix},$$

$$X=A^{-1}\cdot B=\frac{1}{14}\begin{pmatrix} -7 & 7 & 0 \\ 10 & -6 & 2 \\ 1 & 5 & -4 \end{pmatrix}\cdot\begin{pmatrix} 8 \\ 10 \\ 4 \end{pmatrix}=\frac{1}{14}\begin{pmatrix} -56+70 \\ 80-60+8 \\ 8+50-16 \end{pmatrix}=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Javob: $(1;2;3)$.

Gaussning klassik usuli - bu berilgan sistemaning umumiy yechimini topishdan iborat bo'lib, bunda sistemaning tenglamalari ustida elementar almashtirishlar bajarib berilgan sistema trapetsiyali yoki uchburchakli ko'rinishga keltiriladi. So'ng oxirgi tenglamadan boshlab noma'lumlar ketma-ket topiladi.

Misol. $$\begin{cases} x_1+2x_2-4x_3=-4 \\ 3x_1-2x_2+x_3=11 \\ 4x_1-5x_2+x_3=9 \end{cases} \sim \begin{cases} x_1+2x_2-4x_3=-4 \\ -8x_2+13x_3=23 \\ -13x_2+17x_3=25 \end{cases} \sim \begin{cases} x_1+2x_2-4x_3=-4 \\ -8x_2+13x_3=23 \\ -\frac{33}{8}x_3=-\frac{99}{8} \end{cases}$$

$x_3=3$, $x_2=2$, $x_1=4$. Javob: $(4;2;3)$.

Gauss-Jordan usuli noma'lumlarni ketma-ket yo'qotish algoritmi Gauss usuli va teskari matritsa qurish Jordan algoritmiga asoslangan. Gauss-Jordan usuliga sxema ko'rinishida quyidagicha yoziladi: $(A|B)\sim(E|X)$.

$(A|B)$-asosiy matritsani ozod hadlar hisobiga kengaytirilgan matritsa.

E-birlik matritsa. X-tenglama yechimini ifodalovchi ustun matritsa.

Misol. $\begin{cases} x_1+x_2-6x_3-4x_4=6 \\ 3x_1-x_2-6x_3-4x_4=2 \\ 2x_1+3x_2+9x_3+2x_4=6 \\ 3x_1+2x_2+3x_3+8x_4=-7 \end{cases}$ sistemani Gauss-Jordan usuli bilan yeching.

Yechish:

$$\left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 3 & -1 & -6 & -4 & 2 \\ 2 & 3 & 9 & 2 & 6 \\ 3 & 2 & 3 & 8 & -7 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 0 & -4 & 12 & 8 & -16 \\ 0 & 1 & 21 & 10 & -6 \\ 0 & -1 & 21 & 20 & -25 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & -3 & -2 & 4 \\ 0 & 1 & 21 & 10 & -6 \\ 0 & -1 & 21 & 20 & -25 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & -3 & -2 & 4 \\ 0 & 0 & 24 & 12 & -10 \\ 0 & 0 & 18 & 18 & -21 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & -3 & -2 & 4 \\ 0 & 0 & 1 & 1/2 & -5/12 \\ 0 & 0 & 18 & 18 & -21 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -3 & -2 & 2 \\ 0 & 1 & -3 & -2 & 4 \\ 0 & 0 & 1 & 1/2 & -5/12 \\ 0 & 0 & 0 & 9 & -27/2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1/2 & 3/4 \\ 0 & 1 & 0 & -1/2 & 11/4 \\ 0 & 0 & 1 & 1/2 & -5/12 \\ 0 & 0 & 0 & 1 & -3/2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1/3 \\ 0 & 0 & 0 & 1 & -3/2 \end{array}\right)$$

Javob: (0; 2; 1/3; -3/2).

Misol. Berilgan $\begin{cases} x_1 - x_2 + x_3 = 1 \\ x_1 + x_2 - 2x_3 = 3 \\ 2x_1 + 2x_2 - 4x_3 = 6 \end{cases}$ sistema birgalikda, chunki

$$r\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -2 \\ 2 & 2 & -4 \end{pmatrix} = r\left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 1 & -2 & 3 \\ 2 & 2 & -4 & 6 \end{array}\right).$$

Yechish: Sistema cheksiz ko'p yechimga ega, umumiy yechimni Gauss-Jordan usuli yordamida topamiz:

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 1 & -2 & 3 \\ 2 & 2 & -4 & 6 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 2 & -3 & 2 \end{array}\right) \to$$

$$\to \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & -1,5 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -0,5 & 2 \\ 0 & 1 & -1,5 & 1 \end{array}\right)$$

$$\begin{cases} x_1 - 0,5x_3 = 2 \\ x_2 - 1,5x_3 = 1 \end{cases} \Rightarrow \begin{cases} x_1 = 0,5x_3 + 2 \\ x_2 = 1,5x_3 + 1 \end{cases}.$$

Javob: $\left(\frac{1}{2}x_3 + 2;\ \frac{3}{2}x_3 + 1;\ x_3\right) \quad x_3 \in R.$

Mustaqil yechish uchun mashqlar

1-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.
$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 + x_2 + 2x_3 = 1 \\ x_1 + x_2 + 3x_3 = 2 \end{cases}$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching
$$\begin{cases} 2x + y + 4z + 8t = -1 \\ x + 3y - 6z + 2t = 3 \\ 3x - 2y + 2z - 2t = 8 \\ 2x + y - 2z = 4 \end{cases}.$$

2-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.
$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 + x_2 + 2x_3 = 1 \\ 2x_1 + 2x_2 + 4x_3 = 1 \end{cases}$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching
$$\begin{cases} 3x + 2y + z = 5 \\ x + y - z = 0 \\ 4x - y + 5z = 3 \end{cases}.$$

3-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.
$$\begin{cases} 2x + 3y + 2z = 9 \\ x + 2y - 3z = 14 \\ 3x + 4y + z = 16 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} 2x_1 + x_2 - x_3 = 5 \\ 3x_1 - x_2 + 2x_3 = -5 \\ 7x_1 + x_2 - x_3 = 10 \end{cases}.$$

4-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ x_1 + 4x_2 - 3x_3 = 7 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} 2x - 3y + z = 2 \\ x + 5y - 4z = -5 \\ 4x + y - 3z = -4 \end{cases}.$$

5-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} 2x_1 - x_2 = 3 \\ 3x_1 - 5x_2 = 1 \\ 4x_1 - 7x_2 = 1 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} x_1 + x_2 + x_3 = 2 \\ 3x_1 + 2x_2 + 2x_3 = 1 \\ 4x_1 + 3x_2 + 3x_3 = 4 \end{cases}$$

6-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1 + 3x_2 + 5x_3 - 2x_4 = 3 \\ x_1 + 4x_2 - 2x_3 + 3x_4 = 2 \\ -x_1 - 2x_2 - 12x_3 - 7x_4 = -4 \\ 3x_1 + 11x_2 + x_3 - 4x_4 = 7 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} x_1 + x_2 - 2x_3 = -7 \\ 3x_1 - 3x_2 + x_3 = 12 \\ 5x_1 - x_2 - 4x_3 = -5 \end{cases}$$

7-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} 3x_1 + 2x_2 = 4 \\ x_1 - 4x_2 = -1 \\ 7x_1 + 10x_2 = 12 \\ 5x_1 + 6x_2 = 8 \\ 3x_1 - 16x_2 = -5 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} 2x_1 + 3x_2 + 11x_3 + 5x_4 = 2 \\ x_1 + x_2 + 5x_3 + 2x_4 = 1 \\ 2x_1 + x_2 + 3x_3 + 2x_4 = -3 \\ 3x_1 + x_2 + 3x_3 + 4x_4 = -7 \end{cases}.$$

8-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1 + 5x_2 + 4x_3 = 1 \\ 2x_1 + 10x_2 + 8x_3 = 3 \\ 3x_1 + 15x_2 + 12x_3 = 5 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} 3x_1 + 4x_2 + x_3 + 2x_4 + 3 = 0 \\ 3x_1 + 5x_2 + 3x_3 + 5x_4 + 6 = 0 \\ 6x_1 + 8x_2 + x_3 + 5x_4 + 8 = 0 \\ 3x_1 + 5x_2 + 3x_3 + 7x_4 + 8 = 0 \end{cases}.$$

9-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1 - 3x_2 + 2x_3 = -1 \\ x_1 + 9x_2 + 6x_3 = 3 \\ x_1 + 3x_2 + 4x_3 = 1 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} 2x - y + 3z = 9 \\ 3x - 5y + z = -4 \\ 4x - 7y + z = 5 \end{cases}.$$

10-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1 + 2x_2 - 4x_3 = 18 \\ 3x_1 - x_2 + 4x_3 = 4 \\ 2x_1 + x_2 + 5x_3 = 0 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} 2x - 5y + 3z + t = 5 \\ 3x - 7y + 3z - t = -1 \\ 5x - 9y + 6z + 2t = 7 \\ 4x - 6y + 3z + t = 8 \end{cases}.$$

11-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 5 \\ x_1 - 3x_2 + 5x_3 = 4 \\ 2x_1 + x_2 + 5x_3 = 0 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} x_1 - x_2 + 3x_3 = 3 \\ 2x_1 + 3x_2 - 4x_3 = -1 \\ 3x_1 + 2x_2 - x_3 = 2 \end{cases}.$$

12-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 + 4x_2 - 3x_3 = 2 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 0 \\ 2x_1 - 3x_2 - x_3 = 1 \\ 3x_1 + x_2 + 4x_3 = -1 \end{cases}.$$

13-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 + 4x_2 - 3x_3 = 5 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} x_1 - 3x_2 + 2x_3 + x_4 = 2 \\ 2x_1 + x_2 + 4x_3 + 3x_4 = 1 \\ x_1 + 5x_2 - x_3 + x_4 = -4 \\ 3x_1 - x_2 + 6x_3 + 5x_4 = 0 \end{cases}.$$

14-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1 - 3x_2 + 5x_3 + 7x_4 + 9x_5 = 1 \\ x_1 - 2x_2 + 3x_3 - 4x_4 + 5x_5 = 2 \\ 2x_1 + 11x_2 + 12x_3 + 25x_4 + 22x_5 = 4 \end{cases}$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} 2x_1 + x_2 + 3x_3 - 4x_4 = 3 \\ x_1 - 2x_2 + x_3 - 3x_4 = -1 \\ 3x_1 + 4x_2 - 5x_3 + x_4 = 4 \\ 2x_1 - 4x_2 + 2x_3 - 6x_4 = 5 \end{cases}$$

15-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 + 4x_2 - 3x_3 = 2 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 14 \\ 3x_1 + 2x_2 + x_3 = 10 \\ x_1 + x_2 + x_3 = 6 \\ 2x_1 + 3x_2 - x_3 = 5 \\ x_1 + x_2 = 3 \end{cases}.$$

16-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} 3x_1 + 2x_2 = 4 \\ x_1 - 4x_2 = -1 \\ 7x_1 + 10x_2 = 12 \\ 5x_1 + 6x_2 = 8 \\ 3x_1 - 16x_2 = -5 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} 3x_1-2x_2-5x_3+x_4=3 \\ 2x_1-3x_2+x_3+5x_4=-3 \\ x_1+2x_2-x_3-4x_4=-1 \\ x_1-x_2-4x_3+9x_4=22 \end{cases}.$$

17-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1-2x_2+x_3=4 \\ x_1+3x_2+x_3=0 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} x_1+x_2+x_3+x_4+x_5=15 \\ x_1+2x_2+3x_3+4x_4+5x_5=35 \\ x_1+3x_2+6x_3+10x_4+15x_5=70 \\ x_1+4x_2+10x_3+20x_4+35x_5=126 \\ x_1+5x_2+15x_3+35x_4+70x_5=210 \end{cases}$$

18-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} 3x_1+x_2=0 \\ -x_1+2x_2=5 \\ 2x_1-4x_2=1 \end{cases}$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} x+2y+3z=5 \\ 2x-y-z=1 \\ x+3y+4z=6 \end{cases}.$$

19-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} 2x_1-x_2+x_3-x_4=5 \\ x_1+2x_2-2x_3+3x_4=-6 \\ 3x_1+x_2-x_3+2x_4=-1 \end{cases}$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} x_1-2x_2-5x_3=1 \\ 4x_1+x_2-2x_3=-3 \\ -x_1+3x_2+7x_3=2 \end{cases}.$$

20-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1-3x_2=-5 \\ -x_1+x_2=1 \\ 4x_1-x_2=2 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} -x_1+2x_2-x_3=4 \\ 3x_1+x_2-2x_3=1 \\ 4x_1-x_2+x_3=-3 \end{cases}.$$

21-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1-x_2-3x_3=6 \\ -2x_1+2x_2+6x_3=-9 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} -x_1+x_2+x_3-x_4=-2 \\ x_1+2x_2-2x_3-x_4=-5 \\ 2x_1-x_2-3x_3+2x_4=-1 \\ x_1+2x_2+3x_3-6x_4=-10 \end{cases}.$$

22-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1+x_2-x_3=4 \\ 2x_1+4x_2+x_3=9 \\ x_1-x_2+x_3=-2 \\ 2x_1+5x_2-3x_3=15 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} 2x_1+2x_2-x_3+x_4=4 \\ 4x_1+3x_2-x_3+2x_4=6 \\ 8x_1+5x_2-3x_3+4x_4=12 \\ 3x_1+3x_2-2x_3+2x_4=6 \end{cases}.$$

23-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x_1 + 2x_2 + x_3 = 8 \\ x_2 + 3x_3 + x_4 = 15 \\ 4x_1 + x_3 + x_4 = 11 \\ x_1 + x_2 + 5x_4 = 23 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} x_1 + 3x_2 - 4x_3 = -1 \\ x_1 - 5x_2 + x_3 = 7 \\ 2x_1 + x_2 - 3x_3 = 3 \end{cases}.$$

24-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} 2x_1 - x_2 + x_3 - x_4 = 5 \\ x_1 + 2x_2 - 2x_3 + 3x_4 = -6 \\ 3x_1 + x_2 - x_3 + 2x_4 = -1 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} 2x - y - 6z + 3t + 1 = 0 \\ 7x - 4y + 2z - 15t + 32 = 0 \\ x - 2y - 4z + 9t - 5 = 0 \\ x - y + 2z - 6t + 8 = 0 \end{cases}.$$

25-variant

1) Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

$$\begin{cases} x + y - 3z = -1 \\ 2x - y + z = 2 \\ 3x + 2y - 4z = 1 \end{cases}.$$

2) Berilgan tenglamalar sistemasini a) teskari matritsa usuli, b) Kramer usuli, c) Gauss usullari bilan yeching

$$\begin{cases} 2x_1 + 5x_2 + 4x_3 + x_4 = 20 \\ x_1 + 3x_2 + 2x_3 + x_4 = 11 \\ 2x_1 + 10x_2 + 9x_3 + 7x_4 = 40 \\ 3x_1 + 8x_2 + 9x_3 + 2x_4 = 37 \end{cases}.$$

II BOB ANALITIK GEOMETRIYA ELEMENTLARI BILAN TANISHTIRISH METODIKASINI ISHLAB CHIQISH

6-mavzu: Koordinatalar sistemasini kiritish. Affin koordinatalar sistemasi. Qutb koordinatalar sistemasi.

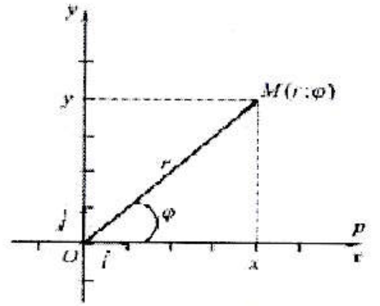
Dekart koordinatalari. Qutb koordinatalari. Koordinatalarni almashtirish

Umumiy boshlang'ich O nuqtaga va bir xil masshtab birligiga ega bo'lgan o'zaro perpendikular OX va OY o'qlar tekislikda dekart koordinatalar sistemasini hosil qiladi. Bu sistemaning OX o'qiga abssissalar o'qi, OY o'qiga ordinatalar o'qiva ular birgalikda *koordinata o'qlari deb ataladi.* Bunda OX va OY o'qlarning ortlari $\bar{i}$ va $\bar{j}$ bilan belgilanadi ($|\bar{i}| = |\bar{j}| = 1,\ \bar{i} \perp \bar{j}$) O *nuqtaga koordinatalar boshi deyiladi.* OX, OY o'qlar joylashgan tekislik *koordinata tekisligi deb ataladi* va Oxy bilan belgilanadi.

Oxy *tekislik* M *nuqtasining* OM *vektoriga* M *nuqtaning radius vektori deyiladi.*

OM *radius vektorning koordinatalariga* M *nuqtaning to'g'ri burchakli dekart koordinatalari deyiladi.* Agar $OM = \{x, y\}$ bo'lsa, u holda M nuqtaning koordinatalari $M(x, y)$ kabi belgilanadi, bu yerda x soni M nuqtaning abssissasi, y soni M nuqtaning ordinatasi deb ataladi.

Tekislikda sanoq boshiga, musbat yo'nalishga va masshtab birligiga ega bo'lgan OP *o'q qutb o'qi, uning* O *sanoq boshi qutb deb ataladi.* Tekislikning qutb bilan ustma-ust tushmaydigan ixtiyoriy M nuqtasining holati ikkita son, O qutbdan M nuqtagacha bo'lgan r masofa va OP qutb o'qi bilan $O\dot{M}$ yo'nalgan kesma orasidagi φ burchak bilan aniqlanadi r va φ sonlariga M nuqtaning qutb koordinatalari deyiladi va $M(r, \varphi)$ deb yoziladi. Bunda r masofa qutb radiusi, φ burchak qutb burchagi deb ataladi. Qutb koordinatalari $0 \le r < \infty$, $-\pi < \varphi \le \pi$ kabi o'zgaradi. *Nuqtaning qutb* *koordinatalaridan dekart koordinatalariga* $x = r\cos\varphi,\ y = r\sin\varphi$ *. tengliklar bilan o'tiladi*(1-shakl).



1-shakl.

Nuqtaning dekart koordinatalaridan qutb koordinatalariga o'tish $r = \sqrt{x^2 + y^2},\ tg\varphi = \frac{y}{x}$ *tengliklar orqali amalga oshiriladi. Bunda* φ *burchakning qiymati nuqtaning* joylashgan choragiga (x, y , larning ishoralari asosida) qarab, $-\pi < \varphi \le \pi$ oraliqda tanlanadi.

1-misol. $M(-3,3)$ nuqta berilgan. M nuqtaning qutb koordinatalarini toping.

$r = \sqrt{x^2 + y^2},\ tg\varphi = \frac{y}{x}$ formuladan topamiz: $r = \sqrt{(-3)^2 + 3^2} = \sqrt{18} = 3\sqrt{2},\ tg\varphi = \frac{-3}{3} = 1$,

$\varphi = arctg1 = \frac{\pi}{4} + \pi n$ M nuqtan III chorakda yotadi. U holda

$n = -1, \varphi = \frac{\pi}{4} - \pi = -\frac{3\pi}{4}$ bo'ladi. Demak, $M(3\sqrt{2}, -\frac{3\pi}{4})$

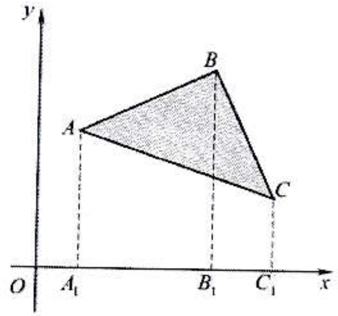
2-misol. Qutb koordinatalarida berilgan $M_1(r_1, \varphi_1)$ va $M_2(r_2, \varphi_2)$ nuqtalar orasidagi masofani toping.

Yechish: Ikki nuqta orasidagi masofa formulasida (1.1) bog'lanishni hisobga olib topamiz:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(r_1\cos\varphi_1 - r_2\cos\varphi_2)^2 + (r_1\sin\varphi_1 - r_2\sin\varphi_2)^2} =$$
$$\sqrt{r_1^2\cos^2\varphi - 2r_1r_2\cos\varphi_1\cos\varphi_2 + r_2^2\cos^2\varphi_2 + r_1^2\sin^2\varphi - 2r_1r_2\sin\varphi_1\sin\varphi_2 + r_2^2\sin^2\varphi_2} =$$
$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\varphi_1 - \varphi_2)}$$

Demak: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\varphi_1 - \varphi_2)}$

3-misol. ABC uchburchakning uchlari berilgan: $A(x_1, y_1),\ B(x_2, y_2),\ C(x_3, y_3)$. Uchburchakning yuzini koordinatalar usuli bilan toping.



A, B, C uchlardan OX o'qiga AA_1 BB_1 CC_1 perpendikularlar tushiramiz yuqoridagi shakldan topamiz: $S_{ABC} = S_{AA_1B_1B} + S_{B_1BCC_1} - S_{A_1ACC_1}$. Bundan

$$S_{ABC} = \frac{y_1 + y_2}{2}(x_2 - x_1) + \frac{y_2 + y_3}{2}(x_3 - x_2) - \frac{y_1 + y_3}{2}(x_3 - x_1) =$$
$$\frac{1}{2}(x_2y_1 - x_1y_1 + x_2y_2 - x_1y_2 + x_3y_2 - x_2y_2 + x_3y_3 - x_2y_3 - x_3y_1 + x_1y_1 - x_3y_3 + x_1y_3) =$$
$$\frac{1}{2}(x_3(y_2 - y_1) - x_1(y_2 - y_1) - x_2(y_3 - y_1) + x_1(y_3 - y_1)) =$$
$$\frac{1}{2}((y_2 - y_1)(x_3 - x_1) - (y_3 - y_1)(x_2 - x_1)) = \frac{1}{2}\begin{vmatrix} x_3 - x_1 & x_2 - x_1 \\ y_3 - y_1 & y_2 - y_1 \end{vmatrix}$$

Demak, $S_{ABC} = \frac{1}{2}\begin{vmatrix} x_3 - x_1 & x_2 - x_1 \\ y_3 - y_1 & y_2 - y_1 \end{vmatrix}$

Nuqtaning bir sistemadagi koordinatalarini uning boshqa sistemadagi koordinatalari bilan almashtirishga koordinatalarni almashtirish deyiladi.

Tekislikda Oxy to'g'ri burchakli koordinatalar sistemasi berilgan bo'lsin. Koordinata o'qlarini parallel ko'chirish –bu Oxy sistemadan uning o'qlari yo'nalishlarini va masshtablarini o'zgartirmasdan faqat koordinatalar boshining joylashishini o'zgartirish orqali yangi Ox_1y_1 sistemaga o'tishdir. Koordinata o'qlarini parallel ko'chirishda tekislik ixtiyoriy M nuqtasining Oxy sistemadagi (x, y) koordinatalari Ox_1y_1 sistemadagi (x', y') koordinatalari orqali $x = x_0 + x', y = y_0 + y'$ formulalar bilan bog'lanadi, bu yerda $x_0; y_0$ - Ox_1y_1 Sistema O_1 koordinatalar boshining Oxy sistemadagikoordinatalari. Koordinata o'qlarini burish –bu Oxy sistemadan uning koordinatalar boshini va o'qlari masshtablarini o'zgartirmasdan faqat koordinata o'qlarini biror burchakka burish orqali yangi Ox_1y_1 sistemaga o'tishdir.

Umumiy O nuqtaga va bir xil masshtabli o'qlarga ega bo'lgan Oxy va Ox_1y_1 koordinatalar sistemalarida M nuqtaning koordinatalari $x = x_0 + x'\cos\alpha - y'\sin\alpha,\ y = y_0 + x'\sin\alpha + y'\cos\alpha$.

4-misol. To'g'ri burchakli koordinatalar sistemasining o'qlari $A(12;-6)$ nuqtaga parallel ko'chirilgan va $\alpha = arctg\frac{3}{4}$ burchakka burilgan. Yangi sistemaga nisbatan A va $B(5;5)$ nuqtalarning koordinatalarini toping.

Yechish: (1.6) formulalardan topamiz:
$x - x_0 = x'\cos\alpha - y'\sin\alpha,\ y - y_0 = x'\sin\alpha + y'\cos\alpha$

Bundan

$x' = (x - x_0)\cos\alpha + (y - y_0)\sin\alpha,\ y' = (y - y_0)\cos\alpha - (x - x_0)\sin\alpha$. $\alpha = arctg\frac{3}{4}$ da

$$\cos\alpha = \frac{1}{\sqrt{1 + tg^2(arctg\frac{3}{4})}} = \frac{4}{5},\ \sin\alpha = \sqrt{1 - (\frac{4}{5})^2} = \frac{3}{5}$$

U holda

$$x' = \frac{4(x - x_0) + 3(y - y_0)}{5},\ y' = \frac{4(y - y_0) - 3(x - x_0)}{5}$$

Nuqtalarning yangi sistemadagi koordinatalarini oxirgi tengliklar bilan topamiz:

A nuqta uchun:

$$x' = \frac{4(12 - 12) + 3(-6 + 6)}{5} = 0,\ y' = \frac{4(-6 + 6) - 3(12 - 12)}{5} = 0 \text{ ya'ni } A(0;0);$$

B nuqta uchun:

$$x' = \frac{4(5 - 12) + 3(5 + 6)}{5} = 1,\ y' = \frac{4(5 + 6) - 3(5 - 12)}{5} = 13 \text{ ya'ni } B(1;13)$$

Mustaqil yechish uchun misollar;

1-variant

1) Berilgan $M_1(-2,3)$ va $M_2(5,4)$. Nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

2) $A(-2,-1), B(6,1), C(3,4)$ nuqtalardan o'tuvchi uchburchak, to'g'ri burchakli uchburchak ekanligini isbotlang.

3) $2x-3y+10=0, 5x-y+4=0$; ikki to'g'ri chiziq orasidagi burchaklarni toping

2-variant

1) $x-3y-9=0, 3x-y-6=0$; ikki to'g'ri chiziq orasidagi burchaklarni toping

2) α ning qanday qiymatlarida berilgan ikki to'g'ri chiziq kesishadi, parallel bo'ladi va perpindikulyar bo'ladi. 1) $2x - 3y + 4 = 0, \alpha x - 6y + 7 = 0$;

3) $A(3,1), M(5,-2), N(4,-1)$; A nuqtadan o'tuvchi va M, N nuqtadan o'tuvchi nuqtadan o'tuvchi to'g'ri chiziqqa parallel to'g'ri chiziqning tenglamasini tuzing.

3-variant

1) $x-y+6=0, -x-y-8=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping.

2) α ning qanday qiymatlarida berilgan ikki to'g'ri chiziq kesishadi, parallel bo'ladi va perpindikulyar bo'ladi 2) $\alpha x - 4y + 1 = 0, -2x + y + 2 = 0$;

3) $A(-1,-3), M(5,4), N(2,5)$; A nuqtadan o'tuvchi va M, N nuqtadan o'tuvchi nuqtadan o'tuvchi to'g'ri chiziqqa parallel to'g'ri chiziqning tenglamasini tuzing.

4-variant

1) $2x-3y-16=0$, $x+y-1=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping.

2) α ning qanday qiymatlarida berilgan ikki to'g'ri chiziq kesishadi, parallel bo'ladi va perpindikulyar bo'ladi 3) $4x+y-6=0, 3x+\alpha y-2=0$;

3) $A(3,-3), M(-3,-4), N(-6,-10)$; A nuqtadan o'tuvchi va M, N nuqtadan o'tuvchi nuqtadan o'tuvchi to'g'ri chiziqqa parallel to'g'ri chiziqning tenglamasini tuzing.

5-variant

1) $x+y-5=0, x+2y-13=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping.

2) α ning qanday qiymatlarida berilgan ikki to'g'ri chiziq kesishadi, parallel bo'ladi va perpindikulyar bo'ladi 4) $x-\alpha y+5=0, 2x+3y+3=0$?

3) $A(-1,1), M(-3,-2), N(0,2)$; A nuqtadan o'tuvchi va M, N nuqtadan o'tuvchi nuqtadan o'tuvchi to'g'ri chiziqqa parallel to'g'ri chiziqning tenglamasini tuzing.

6-variant

1) $2x+2y+7=0, x+y+2=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping

2) $A(2,1), M(5,-4), N(0,-2)$; A nuqtadan o'tuvchi va M, N nuqtadan o'tuvchi nuqtadan o'tuvchi to'g'ri chiziqqa parallel to'g'ri chiziqning tenglamasini tuzing.

3) $A(7,4), B(4,-8)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

7-variant

1) $A(7,-2), B(-3,0)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

2) $-2x-3y+3=0, -x-y+1=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping

3) $A(-1,-3), M(5,4), N(20,16)$; A nuqtadan o'tuvchi va M, N nuqtadan o'tuvchi nuqtadan o'tuvchi to'g'ri chiziqqa parallel to'g'ri chiziqning tenglamasini tuzing.

8-variant

1) $A(5,4), B(14,-8)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

2) $2x-3y+5=0, 3x+2y+21=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping

3) $A(-1,1), M(-3,-4), N(-4,-5)$; A nuqtadan o'tuvchi va M, N nuqtalardan o'tuvchi to'g'ri chiziqqa perpendikulyar to'g'ri chiziqning tenglamasini tuzing

9-variant

1) $-2x-5y+9=0, 5x-2y+19=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping

2) $A(2,-3), M(-3,4), N(-9,6)$; A nuqtadan o'tuvchi va M, N nuqtalardan o'tuvchi to'g'ri chiziqqa perpendikulyar to'g'ri chiziqning tenglamasini tuzing

3) $A(5,-4), B(0,-8)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

10-variant

1) $A(-3,-2), B(-6,-14)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

2) $x-5y-22=0$, $5x-2y+37=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping

3) $A(3,1)$, $M(7,-4)$, $N(6,-8)$; A nuqtadan o'tuvchi va M, N nuqtalardan o'tuvchi to'g'ri chiziqqa perpendikulyar to'g'ri chiziqning tenglamasini tuzing

11-variant

1) $A(7,-4), B(10,-8)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

2) $x-5y-22=0$, $-3x-2y-3=0$ ikki to'g'ri chiziqlar orasidagi burchakni toping

3) $A(-1,1)$, $M(5,-2)$, $N(8,0)$; A nuqtadan o'tuvchi va M, N nuqtalardan o'tuvchi to'g'ri chiziqqa perpendikulyar to'g'ri chiziqning tenglamasini tuzing

12-variant

1) $A(5,-4), B(-4,-16)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

2) $x-3y-14=0$, $-x+2y-17=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping

3) $A(-1,1)$, $M(5,-2)$, $N(8,0)$; A nuqtadan o'tuvchi va M, N nuqtalardan o'tuvchi to'g'ri chiziqqa perpendikulyar to'g'ri chiziqning tenglamasini tuzing

13-variant

1) $A(7,-4), B(-3,4)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

2) $x+3y+15=0$, $-3x-2y+15=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping

3) $A(3,2)$, $M(5,-2)$, $N(7,2)$; A nuqtadan o'tuvchi va M, N nuqtalardan o'tuvchi to'g'ri chiziqqa perpendikulyar to'g'ri chiziqning tenglamasini tuzing

14-variant

1) $A(-3,4), B(-2,8)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

2) $-2x-y+7=0$, $-3x-2y-3=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping

3) $A(-1,-3)$, $M(-3,-2)$, $N(-8,-1)$; A nuqtadan o'tuvchi va M, N nuqtalardan o'tuvchi to'g'ri chiziqqa perpendikulyar to'g'ri chiziqning tenglamasini tuzing

15-variant

1) $A(7,-2), B(5,-10)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

2) $-2x+5y+24=0$, $x-y+5=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping

3) $A(2,-3)$, $M(-3,-2)$, $N(7,2)$. A nuqtadan o'tuvchi va M, N nuqtalardan o'tuvchi to'g'ri chiziqqa perpendikulyar to'g'ri chiziqning tenglamasini tuzing

16-variant

1) $A(5,-4), B(2,-1)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

2) $(\alpha+1)x+(3-\alpha)y-8=0$. $(a-3)x+(2x-3)y=0$ α ning qanday qiymatlarida berilgan ikki to'g'ri chiziq kesishadi, parallel bo'ladi va perpindikulyar bo'ladi

3) $2x+3y+8=0$, $-x-2y-5=0$. ikki to'g'ri chiziqlar orasidagi burchakni toping

17-variant

1) $A(-3,-4), B(-6,-2)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

2) Kvadratning qarama-qarshi $A(3,5)$ va $C(1,-3)$ nuqtalari berilgan bo'lib, ushbu kvadraqtning yuzini toping.

3) Berilgan ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing A(1, —5), B(4, 3) va to'g'ri chiziq o'rtasini koordinatasini toping.

18-variant

1) Berilgan $A(2,4), B(-3,7)$ va $C(-6,6)$ — uchta nuqtalar parallelogramning ketma ket burchaklarining koordinatalari bo'lsa u holda parallelogramning to'rtinchi burchagi koordinatasini toping.

2) $A(7,4), B(17,12)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

3) a ning qanday qiymatlarida berilgan to'g'ri chiziq

$(\alpha^2-\alpha)x+(2+\alpha)y-3\alpha+1=0$: Koordinatalar o'qlariga parallel va perpendikulyar bo'ladi?

19-variant

1) $A(-3,-4), B(-8,-2)$; ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing

2) $A(3,2), B(3,8), C(6,2)$ nuqtalardan o'tuvchi uchburchakning barcha tomonlari tenglamalarini tuzing.

3) Berilgan nuqtalardan o'tuvchi uchburchakning $A(2,3), B(6,3), C(6,-5)$ BM bissektrissasining tenglamasi va uzunligini toping.

20-variant

1) α va β ning qanday qiymatlarida $(\alpha-\beta)x+(2\alpha+\beta)y-1=0$

to'g'ri chiziq OX o'qidan ajratgan kesmasining uzunligi 1/7 ga OY o'qidan ajratgan kesmasining uzunligi 1/2 ga teng bo'ladi?

2) $x-5y-22=0$, $5x-2y+37=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping

3) $2x-3y+4=0, \alpha x-6y+7=0$; α ning qanday qiymatlarida berilgan ikki to'g'ri chiziq kesishadi, parallel bo'ladi va perpindikulyar bo'ladi

21-variant

1) $A(-8,-5)$, $B(-2,5)$, $C(-5,2)$; nuqtalardan o'tuvchi uchburchakning barcha tomonlari tenglamalarini tuzing.

2) $(\alpha+1)x+(3-\alpha)y-8=0.$ $(a-3)x+(2x-3)y=0$ α ning qanday qiymatlarida berilgan ikki to'g'ri chiziq kesishadi, parallel bo'ladi va perpindikulyar bo'ladi

3) $2x+3y+8=0$, $-x-2y-5=0$. ikki to'g'ri chiziqlar orasidagi burchakni toping

22-variant

1) $A(4,20)$, $B(-2,5)$, $C(-6,1)$; nuqtalardan o'tuvchi uchburchakning barcha tomonlari tenglamalarini tuzing

2) $x+3y+15=0$, $-3x-2y+15=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping

3) $A(3,2)$, $M(5,-2)$, $N(7,2)$; A nuqtadan o'tuvchi va M, N nuqtalardan o'tuvchi to'g'ri chiziqqa perpendikulyar to'g'ri chiziqning tenglamasini tuzing

23-variant

1) $A(-3,15)$, $B(3,5)$, $C(-3,2)$; nuqtalardan o'tuvchi uchburchakning barcha tomonlari tenglamalarini tuzing

2) α ning qanday qiymatlarida berilgan ikki to'g'ri chiziq kesishadi,

parallel bo'ladi va perpindikulyar bo'ladi 4) $x - \alpha y + 5 = 0, 2x + 3y + 3 = 0$?

3) $A(-1,1), M(-3,-2), N(0,2)$; A nuqtadan o'tuvchi va M, N nuqtadan o'tuvchi nuqtadan o'tuvchi to'g'ri chiziqqa parallel to'g'ri chiziqning tenglamasini tuzing.

24-variant

1) $A(-11,-10)$, $B(-2,5)$, $C(-8,11)$; nuqtalardan o'tuvchi uchburchakning barcha tomonlari tenglamalarini tuzing

2) α ning qanday qiymatlarida berilgan ikki to'g'ri chiziq kesishadi, parallel bo'ladi va perpindikulyar bo'ladi. 1) $2x - 3y + 4 = 0, \alpha x - 6y + 7 = 0$;

3) $A(3,1), M(5,-2), N(4,-1)$; A nuqtadan o'tuvchi va M, N nuqtadan o'tuvchi nuqtadan o'tuvchi to'g'ri chiziqqa parallel to'g'ri chiziqning tenglamasini tuzing.

25-variant

1) $A(6,20)$, $B(3,5)$, $C(6,6)$ nuqtalardan o'tuvchi uchburchakning barcha tomonlari tenglamalarini tuzing

2) $-2x-5y+9=0$, $5x-2y+19=0$; ikki to'g'ri chiziqlar orasidagi burchakni toping

3) $A(2,-3)$, $M(-3,4)$, $N(-9,6)$; A nuqtadan o'tuvchi va M, N nuqtalardan o'tuvchi to'g'ri chiziqqa perpendikulyar to'g'ri chiziqning tenglamasini tuzing

7-mavzu: To'g'ri chiziq va uning tenglamalari. To'g'ri chiziqlar va ular orasidagi burchak. Berilgan nuqtadan to'g'ri chiziqqacha bo'lgan masofani topish.

1°. Tekislikdagi $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalar orasidagi masofa:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \qquad (1)$$

2°. Tekislikda yo'naltirilgan kesmaning, yoki boshi $A(x_1;y_1)$ va oxiri $B(x_2;y_2)$ bo'lgan $\overline{AB}$ vektorning koordinata o'qlaridagi proyeksiyalari:

$$\Pr_x \overline{AB} = X = x_2 - x_1, \quad \Pr_y \overline{AB} = Y = y_2 - y_1 \qquad (2)$$

3°. Kesmani berilgan nisbatda bo'lish: $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalar berilgan AB kesmani $AN:NB=\lambda$ nisbatda bo'luvchi $N(x;y)$ nuqtaning koordinatalari ushbu:

$$x=\frac{x_1+\lambda x_2}{1+\lambda}, \quad y=\frac{y_1+\lambda y_1}{1+\lambda} \qquad (3)$$

formulalar bilan aniqlanadi. Xususiy holda kesmani teng ikkiga, ya'ni $\lambda=1:1=1$ nisbatda bo'lganda quyidagicha bo'ladi:

$$x=\frac{x_1+x_2}{2}, \quad y=\frac{y_1+y_2}{2} \qquad (4)$$

4°. Uchlari $A(x_1;y_1)$, $B(x_2;y_2)$, $C(x_3;y_3)$, ..., $F(x_n;y_n)$ nuqtalarda bo'lgan ko'pburchak yuzi quyidagi formula bilan aniqlanadi:

$$S=\pm\frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}+\begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix}+\ldots+\begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix}\right] \qquad (5)$$

5°. To'g'ri chiziqning burchak koeffitsiyentli tenglamasi:

$$y=kx+b \qquad (6)$$

k parametr to'g'ri chiziqning Ox o'qining musbat yo'nalishiga og'ish burchagi α ning tangensiga teng bo'lib ($k=tg\,\alpha$), to'g'ri chiziqning burchak koeffitsenti, ba'zan qiyaligi deyiladi. b parametr boshlang'ich ordinata yoki Oy o'qdan ajratgan kesma kattaligi hisoblanadi.

6°. To'g'ri chiziqning umumiy tenglamasi:

$$Ax+By+C=0,\ \ (A^2+B^2\neq 0) \qquad (7)$$

Xususiy hollar:

a) $C=0$ bo'lsa, $y=-\frac{A}{B}x$ to'g'ri chiziq koordinatalar boshidan o'tadi;

b) $B=0$ bo'lsa, $x=-\frac{C}{A}=a$ to'g'ri chiziq Oy o'qqa parallel bo'ladi;

c) $A=0$ bo'lsa, $y=-\frac{C}{B}=b$ to'g'ri chiziq Ox o'qqa parallel bo'ladi;

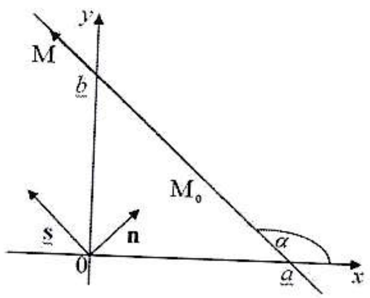
d) $B=C=0$ bo'lsa, $Ax=0$ yoki $x=0$ to'g'ri chiziq Oy o'qdan iborat;

e) $A=C=0$ bo'lsa, $By=0$ yoki $y=0$ to'g'ri chiziq Ox o'qdan o'tadi.

7°. To'g'ri chiziqning o'qlardan ajratgan kesmalari bo'yicha tenglamasi:

$$\frac{x}{a}+\frac{y}{b}=1 \qquad (8)$$

Bu yerda a va b to'g'ri chiziqning o'qlardan kesgan kesmalarining kattaliklari bo'ladi.



8°. To'g'ri chiziqning vektor parametrli tenglamasi:

$$\overline{M_0M}=t\cdot\overline{s} \qquad (9)$$

Bu yerda $M(x;y)$ to'g'ri chiziqning ixtiyoriy nuqtasi $\overline{M_0M}\,(x-x_0;y-y_0)$ vektor va $\overline{s}(m;n)$ yo'naltiruvchi vektori o'zaro kollinear, t-ixtiyoriy haqiqiy son yoki parametr.

$$\begin{cases} x-x_0=tm \\ y-y_0=tn \end{cases} \qquad (10)$$

9°. (9) tenglamani koordinatalarda

ifodalab, to'g'ri chiziqning parametrli tenglamasini hosil qilish mumkin.

10°. (10) tenglamalarda t parametr yo'qotilsa, to'g'ri chiziqning kanonik tenglamasi hosil bo'ladi:

$$\frac{x-x_0}{m}=\frac{y-y_0}{n} \qquad (11)$$

11°. Agar $|\bar a|=P$ $(P\geq 0)$, $v=\frac{\bar a}{P}=(\cos\alpha, \cos\beta)$ $\bar a$ normal radius vektorninig birlik vektori bo'lib, to'g'ri chiziqning ixtiyoriy $M(x;y)$ nuqtasining mos radius vektori $\bar r\,(x;y)$ bo'lsa, u holda $\bar r$ radius vektorning $\bar a$ yoki $\bar v$ vektordagi sonli proyektsiyasi P ga teng:

$$\Pr_v \bar r=P \ \text{ yoki } \ |v|\Pr_v \bar r=P \ \text{ yoki } \ (r\cdot v)=P\ \ (P\geq 0) \qquad (12)$$

Bu tenglama to'g'ri chiziqning vektor ko'rinishdagi tenglamasi deyiladi. (12) tenglama koordinatalarda

$$x\cdot\cos\alpha+y\cdot\cos\beta=P \ \text{ yoki } \ x\cdot\cos\alpha+y\cdot\sin\alpha=P\ \ (P\geq 0) \qquad (13)$$

ko'rinishni oladi. Bunda α - $\bar a$ yoki $\bar v$ vektorning Ox o'qining musbat yo'nalishi bilan hosil qilgan burchak kattaligi. (13) shakldagi tenglama to'g'ri chiziqning normal tenglamasi deyiladi.

12°. (7) shakldagi tenlamadan (13) shakldagi tenglamaga o'tish uchun umumiy ko'rinishdagi tenglama normallovchi ko'paytuvchi deb ataladigan

$\mu=\pm\frac{1}{\sqrt{A^2+B^2}}$ songa ko'paytiriladi, bunda "+" yoki "–" ishoradan C ozod had ishorasining qarama-qarshisi tanlanadi, aks holda $P=-\mu, C\geq 0$ munosabat bajarilmaydi.

1-misol. $3x+4y-8=0$ tenglamani normal ko'rinishga keltiring.

Yechish: Berilgan umumiy shakldagi tenglama uchun normallovchi ko'paytuvchi $\mu=\pm\frac{1}{\sqrt{3^2+4^2}}=\pm\frac{1}{5}$.

Tenglamani, $\mu=\frac{1}{5}$ ga ko'paytiramiz, natijada to'g'ri chiziq tenglamasi quyidagi ko'rinishda normal holga keltiriladi:

$\frac{3}{5}x+\frac{4}{5}y=\frac{8}{5}$.

13°. $y=k_1x+b_1$ to'g'ri chiziqdan $y=k_2x+b_2$ to'g'ri chiziqqacha soat strelkasiga qarshi yo'nalishda hisoblanuvchi φ burchak quyidagi formula bilan aniqlanadi:

$$tg\varphi=\frac{k_2-k_1}{1+k_1k_2}. \qquad (14)$$

14°. $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ tenglamalar bilan berilgan to'g'ri chiziqlar uchun (14) formula quyidagi ko'rinishga ega bo'ladi:

$$tg\,\varphi=\frac{A_1B_2-A_2B_1}{A_1A_2+B_1B_2} \qquad (15)$$

Yoki

$$\cos\varphi=\frac{(n_1\cdot n_2)}{|n_1|\cdot|n_2|}=\frac{A_1A_2+B_1B_2}{\sqrt{A_1^2+B_1^2}\cdot\sqrt{A_2^2+B_2^2}} \qquad (16)$$

15°. To'g'ri chiziqlarning parallellik sharti:

$$k_1=k_2 \text{ yoki } \frac{A_1}{A_2}=\frac{B_1}{B_2} \qquad (17)$$

16°. To'g'ri chiziqlarning *perpendikulyarlik* sharti:

$$k_1\cdot k_2=-1 \text{ yoki } A_1A_2+B_1B_2=0 \qquad (18)$$

17°. Berilgan $A(x_1;y_1)$ nuqtadan o'tuvchi to'g'ri chiziqlar dastasining tenglamasi:

$$y-y_1=k(x-x_1) \qquad (19)$$

18°. Berilgan ikki $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1} \qquad (20)$$

19°. Parallel bo'lmagan ikki $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ to'g'ri chiziqlarning kesishish nuqtasini topish uchun ularning tenglamalarini birgalikda yechish bilan x va y ni hosil qilamiz.

$$x=\frac{\begin{vmatrix}-C_1 & B_1\\ -C_2 & B_2\end{vmatrix}}{\begin{vmatrix}A_1 & B_1\\ A_2 & B_2\end{vmatrix}},\quad y=\frac{\begin{vmatrix}A_1 & -C_1\\ A_2 & -C_2\end{vmatrix}}{\begin{vmatrix}A_1 & B_1\\ A_2 & B_2\end{vmatrix}} \qquad (21)$$

20°. $(x_0;y_0)$ nuqtadan to'g'ri chiziqqacha bo'lgan d masofani topish uchun to'g'ri chiziq normal tenglamasining chap tomonidagi o'zgaruvchi koordinatalar o'rniga $(x_0;y_0)$ koordinatalarni qo'yib, hosil bo'lgan sonning absolyut qiymatini olamiz, ya'ni

$$d=|x_0\cos\beta+y_0\sin\beta-P| \qquad (22)$$

yoki

$$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}} \qquad (23)$$

21°. $Ax+By+C=0$ va $A_1x+B_1y+C_1=0$ to'g'ri chiziqlar orasidagi burchaklar bissektrissalarining tenglamalari:

$$\frac{Ax+By+C}{\sqrt{A^2+B^2}}=\pm\frac{A_1x+B_1y+C_1}{\sqrt{A_1^2+B_1^2}} \qquad (24)$$

22°. Berilgan ikki to'g'ri chiziqning kesishish nuqtasidan o'tuvchi to'g'ri chiziqlar dastasining tenglamasi:

$$\alpha(Ax+By+C)+\beta(A_1x+B_1y+C_1)=0 \qquad (25)$$

$\alpha=1$ deb olish mumkin, u holda biz (25) dastadan berilgan to'g'ri chiziqlardan ikkinchisini yo'qotgan bo'lamiz, ya'ni u vaqtda (25) dan ikkinchi to'g'ri chiziqning tenglamasini hosil qila olmaymiz.

Mustaqil yechish uchun mashqlar

1-variant

1) $\frac{x+2\sqrt{5}}{4}+\frac{y-2\sqrt{5}}{2}=0$ to'g'ri chiziq berilgan. To'g'ri chiziqning umumiy tenglamasi yozing.

2) Oy o'qidan $b=1$ birlikka teng kesma ajratuvchi Ox o'qining musbat yo'nalishi bilan $\alpha=\frac{2\pi}{3}$ burchak hosil qiluvchi to'g'ri chiziq tenglamasini yozing.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (3,-2,1) $\frac{x+3}{-3}=\frac{y-2}{1}=\frac{z-1}{4}$.

2-variant

1) $\frac{x+2\sqrt{5}}{4}+\frac{y-2\sqrt{5}}{2}=0$ to'g'ri chiziq berilgan. To'g'ri chiziqning burchak koeffitsientli tenglamasini yozing.

2) Oy o'qidan $b=1$ birlikka teng kesma ajratuvchi Ox o'qining

musbat yo'nalishi bilan $\alpha=\frac{2\pi}{3}$ burchak hosil qiluvchi to'g'ri chiziq tenglamasini yozing.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (4,5,-2) $\frac{x+1}{4}=\frac{y-5}{3}=\frac{z}{-2}$.

3-variant

1) $\frac{x+2\sqrt{5}}{4}+\frac{y-2\sqrt{5}}{2}=0$ to'g'ri chiziq berilgan. To'g'ri chiziqning kesmalarga nisbatan tenglamasini yozing.

2) Koordinata boshidan va $A(-2;-3)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini yozing.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (-3,1,2) $\frac{x-4}{2}=\frac{y}{-4}=\frac{z+1}{3}$.

4-variant

1) $4x+3y-36=0$ to'g'ri chiziq koordinata o'qlari bilan hosil qilgan uchburchakning yuzini toping.

2) Koordinata boshidan va $A(-2;-3)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini yozing.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (-1,2,1) $\frac{x+2}{4}=\frac{y}{-3}=\frac{z-5}{2}$.

5-variant

1) To'g'ri chiziq koordinata o'qlaridan teng kesmalar ajratadi. Agar to'g'ri chiziq koordinata o'qlari bilan hosil qilgan uchburchak yuzi 8 kv.birlik. bo'lsa, to'g'ri chiziq tenglamasini yozing

2) $M(-3;-4)$ nuqtadan o'tuvchi koordinata o'qlariga parallel to'g'ri chiziqlar tenglamasini yozing.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (2,1,2) $\frac{x+7}{4}=\frac{y-5}{-3}=\frac{z+2}{8}$.

6-variant

1) $A(2;5)$ nuqtadan o'tuvchi va ordinata o'qida $b=7$ kesma ajratuvchi to'g'ri chiziq tenglamasini yozing.

2) $M(-3;-4)$ nuqtadan o'tuvchi koordinata o'qlariga parallel to'g'ri chiziqlar tenglamasini yozing.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (-2, 3, 1) $\frac{x}{2}=\frac{y-1}{-3}=\frac{z+5}{4}$.

7-variant

1) Agar to'g'ri chiziq koordinata o'qlaridan teng kesmalar ajratsa va to'g'ri chiziqni koordinata o'qlari orasidagi kesmasi 5 $\sqrt{2}$ ga teng bo'lsa, to'g'ri chiziq tenglamasini yozing.

2) $O(0;0)$ va $A(-3;0)$ nuqtalar berilgan OA kesmada parallelogramm yasalgan, uning diogonallari $B(0;2)$ nuqtada kesishadi. Parallelogramm tomonlari va diogonallari tenglamasini yozing.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (-4,-1,2) $\frac{x-5}{1}=\frac{y+2}{3}=\frac{z-1}{-2}$

8-variant

1) $y=-2,\ y=4$ to'g'ri chiziqlar $3x-4y-5=0$ to'g'ri chiziqni A va B nuqtalarda kesib o'tadi. $\overline{AB}$ vektor uzunligi va uning koordinata o'qlaridagi proyektsiyalarini toping.

2) $O(0;0)$ va $A(-3;0)$ nuqtalar berilgan OA kesmada parallelogramm yasalgan, uning diogonallari $B(0;2)$ nuqtada kesishadi. Parallelogramm tomonlari va diogonallari tenglamasini yozing.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (-3,0,2) $\frac{x}{5}=\frac{y-6}{2}=\frac{z+4}{-3}$

9-variant

1) $\begin{cases} y=2x-3 \\ y=\frac{1}{2}x+1 \end{cases}$ to'g'ri chiziqlar orasidagi burchakni toping.

2) Asoslari 8 sm va 2 sm bo'lgan teng yonli trapetsiyaning o'tkir burchagi 45^0. Trapetsiyaning katta asosi Ox o'qida yotsa, Oy o'qi esa trapetsiyaning simmetriya o'qi bo'lsa, trapetsiyaning tomonlari tenglamasini yozing.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (1,2,3) $\frac{x+7}{3}=\frac{y-6}{2}=\frac{z+6}{-2}$.

10-variant

1) $\begin{cases} 5x-y+7=0 \\ 2x-3y+1=0 \end{cases}$ to'g'ri chiziqlar orasidagi burchakni toping.

2) Asoslari 8 sm va 2 sm bo'lgan teng yonli trapetsiyaning o'tkir burchagi 45^0. Trapetsiyaning katta asosi Ox o'qida yotsa, Oy o'qi esa trapetsiyaning simmetriya o'qi bo'lsa, trapetsiyaning tomonlari tenglamasini yozing.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (1,-1,-2) $\frac{x}{-4}=\frac{y-5}{2}=\frac{z+1}{5}$.

11-variant

1) $\begin{cases} 2x+y=0 \\ y=3x-4 \end{cases}$ to'g'ri chiziqlar orasidagi burchakni toping.

Agar to'g'ri chiziq koordinata o'qlari bilan hosil qilgan uchburchak yuzi 6 kvadrat birlik bo'lsa va to'g'ri chiziq $(-4;6)$ nuqtadan o'tsa, uning tenglamasini yozing

2) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (-3,2,4) $\frac{x-3}{2}=\frac{y+1}{4}=\frac{z}{-3}$.

12-variant

1) $3x-2y+7=0,\ 6x-4y-9=0,\ 6x+4y-5=0,\ 2x+3y-6=0$ to'g'ri chiziqlar orasidan parallel va perpendikulyar bo'lgan to'g'ri chiziqlarni aniqlang.

2) Agar to'g'ri chiziq koordinata o'qlari bilan hosil qilgan uchburchak yuzi 6 kvadrat birlik bo'lsa va to'g'ri chiziq $(-4;6)$ nuqtadan o'tsa, uning tenglamasini yozing

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (4,-3,1) $\frac{x-5}{3}=\frac{y+5}{-4}=\frac{z}{5}$.

13-variant

1) $A(2;3)$ nuqtadan o'tuvchi to'g'ri chiziqlar dastasini yozing. Bu dastadan Ox o'qi bilan 1) 45°, 2) 60°, 3) 135°, 4) 0° burchaklar tashkil etuvchi to'g'ri chiziqni toping.

2) Berilgan to'g'ri chiziqlar orasidagi burchakni toping:

$$\begin{cases}3x+2y=0\\6x+4y+9=0\end{cases},$$

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (4,5,1) $\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-2}{-3}$.

14-variant

1) $A(-2;5)$ nuqta va $2x-y=0$ to'g'ri chiziqni yasang. A nuqtadan o'tuvchi va berilgan to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini yozing.

2) Berilgan to'g'ri chiziqlar orasidagi burchakni toping:

$$\begin{cases}3x+2y=0\\6x+4y+9=0\end{cases},$$

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (4,2,-2) $\frac{x+4}{2}=\frac{y-2}{2}=\frac{z+1}{3}$.

15-variant

1) $A(-2;5)$ nuqta va $2x-y=0$ to'g'ri chiziqni yasang. A nuqtadan o'tuvchi va berilgan to'g'ri chiziqqa perpendikulyar to'g'ri chiziq tenglamasini yozing.

2) Uchlari $A(-2;0)$, $B(2;4)$ va $C(4;0)$ bo'lgan uchburchak berilgan. Uchburchak tomonlari, AE medianasi, BD balandlik tenglamalarini, AE mediana uzunligini toping.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (0,2,1) $\frac{x+2}{2}=\frac{y-1}{-1}=\frac{z}{3}$.

16-variant

1) $2x-5y-10=0$ to'g'ri chiziqni koordinata o'qlari bilan kesishish nuqtalariga perpendikulyar qo'yilgan. Ularning tenglamasini yozing.

2) Uchlari $A(-2;0)$, $B(2;4)$ va $C(4;0)$ bo'lgan uchburchak berilgan. Uchburchak tomonlari, AE medianasi, BD balandlik tenglamalarini, AE mediana uzunligini toping.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (5,-1,2) $\frac{x-3}{2}=\frac{y+1}{5}=\frac{z}{-4}$.

17-variant

1) $A(-1;3)$ va $B(4;-2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini yozing.

2) Tomonlari $x+y=4,\ 3x-y=0,\ x-3y-8=0$ tenglamalar bilan

berilgan uchburchakning burchaklari, uchlari va yuzini toping

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (4,2,-1) $\frac{x-3}{-5}=\frac{y-4}{2}=\frac{z+1}{3}$.

18-variant

1) Uchlari $A(-2;0), B(4;-2)$ va $C(4;2)$ nuqtalarda bo'lgan uchburchakka BD balandlik va BE mediana o'tkazilgan. AC tomon, BE mediana va BD balandlik tenglamalarini yozing.

2) Tomonlari $x+y=4,\ 3x-y=0,\ x-3y-8=0$ tenglamalar bilan berilgan uchburchakning burchaklari, uchlari va yuzini toping

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (-1,4,5) $\frac{x}{-3}=\frac{y-2}{3}=\frac{z+1}{4}$.

19-variant

1) Uchburchak tomonlari quyidagi tenglamalar bilan berilgan: $x+3y=0,\ x=3,\ x-2y+3=0$. Uchburchakning burchaklari va uchlarini toping.

2) Tomonlari $x+y=4,\ 3x-y=0,\ x-3y-8=0$ tenglamalar bilan berilgan uchburchakning burchaklari, uchlari va yuzini toping

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (-4,-1,2) $\frac{x-1}{6}=\frac{y+3}{4}=\frac{z}{-3}$.

20-variant

1) Kvadrat tomonlaridan birining tenglamasi $x+3y-7=0$ va diogonallari kesishgan nuqta $P(0;-1)$ berilgan. Kvadratning qolgan uchta tomon tenglamalarini yozing.

2) Koordinatalar boshidan $2x+y=a$ to'g'ri chiziq bilan teng yonli

uchburchak hosil qiluvchi ikki o'zaro perpendikulyar to'g'ri chiziq o'tkazilgan. Shu uchburchakning yuzini toping

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (2,5,-1) $\frac{x+3}{2}=\frac{y-5}{4}=\frac{z}{-1}$.

21-variant

1) Romb tomonlaridan birining tenglamasi $5x+2y-9=0$. Agar romb diogonallari $O(0;0)$ da kesishgan bo'lib, ulardan birining tenglamasi $y=2x$ bo'lsa, rombning qolgan uchta tomon tenglamasini yozing.

2) Koordinatalar boshidan $2x+y=a$ to'g'ri chiziq bilan teng yonli uchburchak hosil qiluvchi ikki o'zaro perpendikulyar to'g'ri chiziq o'tkazilgan. Shu uchburchakning yuzini toping

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (5,0,4) $\frac{x}{-3}=\frac{y-2}{2}=\frac{z-1}{1}$.

22-variant

1) Uchburchak tomonlarining o'rtasi berilgan $P(1;2)$ AB tomonining o'rtasi, $R(-4;3)$ BC tomonining o'rtasi, $Q(5;-1)$ AC tomonining o'rtasi, CF balandlik va AR mediana kesishgan nuqtani toping.

2) Uchburchak AB tomonining tenglamasi $x-3y+3=0$ va AC tomonining tenglamasi $x+3y+3=0$ hamda AD balandligining asosi $D(-1;3)$ berilgan bo'lsa, uchburchakning ichki burchaklarini toping.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (3,-2,1) $\frac{x+3}{-3}=\frac{y-2}{1}=\frac{z-1}{4}$.

23-variant

1) Rombning ikki qarama-qarshi uchlarining koordinatalari berilgan, $A(1;-4)$ $C(-1;3)$. Romb diogonallarining tenglamasini yozing.

2) Uchburchak AB tomonining tenglamasi $x-3y+3=0$ va AC

tomonining tenglamasi $x+3y+3=0$ hamda AD balandligining asosi $D(-1;3)$ berilgan bo'lsa, uchburchakning ichki burchaklarini toping.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (3,0,2) $\frac{x+2}{4}=\frac{y-1}{-3}=\frac{z-2}{5}$.

24-variant

1) Agar $A(-5;5)$ va $B(3;1)$ uchburchakning uchlari, $D(2;5)$ esa balandliklari kesishgan nuqta bo'lsa, uchburchak tomonlarining tenglamasini yozing.

2) Romb ikki tomonining tenglamalari $x+2y=4$ va $x+2y=10$ hamda diogonallaridan birining tenglamasi $y=x+2$ ma'lum bo'lsa, romb uchlarining koordinatalarini hisoblang.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (-5,3,-4) $\frac{x-3}{2}=\frac{y+3}{6}=\frac{z}{-3}$.

25-variant

1) $2x+2y-5=0$ to'g'ri chiziq Ox o'qining musbat yo'nalishi bilan qanday burchak hosil qiladi?

2) Romb ikki tomonining tenglamalari $x+2y=4$ va $x+2y=10$ hamda diogonallaridan birining tenglamasi $y=x+2$ ma'lum bo'lsa, romb uchlarining koordinatalarini hisoblang.

3) Berilgan nuqta va to'g'ri chiziq orqali o'tuvchi tekislik tenglamasini tuzing. A (4,3,1) $\frac{x-2}{4}=\frac{y+1}{3}=\frac{z+2}{-1}$

III BOB MATEMATIK ANALIZGA KIRISH, BIR O'ZGARUVCHI FUNKSIYASINING DIFFERENSIAL HISOBI, BIR O'ZGARUVCHI FUNKSIYASINING INTEGRAL HISOBI MAVZULARINI FANLARARO BOG'LIQLIKDA O'QITISHNI TASHKIL QILISH

8-mavzu: Funksiya tushunchasi. Funksiyaning asosiy xossalari. Ratsional va irratsional funksiyalar.

Ta'rif. Agar x miqdorning X sohadagi har bir qiymatiga biror f qonuniyatga ko'ra y miqdorning Y -sohadan aniq bir qiymati mos keltirilsa, y miqdor x miqdorning X -sohadagi funksiyasi deyiladi va $y=f(x)$ kabi yoziladi.

Bu holda x - argument yoki erkli o'zgaruvchi, y - esa funksiya yoki erksiz o'zgaruvchi deyiladi. Agar y, x ning funksiyasi bo'lsa, u holda x va y lar orasidagi bog'lanish funksiyali bog'lanish deyiladi va quyidagicha yoziladi: $y=f(x)$, $y=g(x)$, $y=\varphi(x)$,... . Agar yuqoridagi misollarga e'tibor qaratsak, doiraning yuzi radiusning funksiyasi, kvadratning yuzi tomonining funksiyasi ekanligi ma'lum bo'ladi.

Argument qabul qilishi mumkin bo'lgan qiymatlari to'plami funksiyaning aniqlanish sohasi, funksiyaning o'zi qabul qilishi mumkin bo'lgan qiymatlari to'plami funksiyaning o'zgarish sohasi yoki qiymatlari to'plami deyiladi.

Funksiyaning berilish usullari. Funksiya sharoitiga qarab jadval, analitik va grafik usullar bilan berilishi mumkin.

Funksiya jadval usulida berilganda, argumentning ma'lum tartibdagi $x_1, x_2, x_3, ..., x_n, ...$ qiymatlari va funksiyaning ularga mos keluvchi $y_1, y_2, y_3, ..., y_n, ...$ qiymatlari jadval holida beriladi:

X	x_1	x_2	x_3	...	x_n	...
Y	y_1	y_2	y_3	...	y_n	...

Funksiyalarning jadval usulida berilishiga misol qilib kvadratlar, kublar, kvadrat ildizlar jadvallarni ko'rsatish mumkin. Bu usuldan ko'pincha miqdorlar orasida tajribalar o'tkazishda foydalaniladi.

Funksiyaning grafik usulda berilishi. $y=f(x)$ funksiyaning grafigi deb koordinatalari $y=f(x)$ ni to'g'ri tenglikka aylantiruvchi tekislikdagi barcha nuqtalar to'plamiga aytiladi. Agar funksiyaning grafigi tasvirlangan bo'lsa, funksiya grafik usulda berilgan deyiladi.

Funksiyaning analitik usulda berilishi. Formula yordamida berilgan funksiyalarga analitik usulda berilgan deyiladi. Masalan, $y=x^2$, $y=kx+b$, $y=a^x$, $y=\lg x$, $y=\sin x$, $y=\cos x$,... funksiyalar analitik usulda berilgan. Agar analitik usulda berilgan funksiyaning aniqlanish sohasi to'g'risida alohida shart qo'yilmagan bo'lsa, u holda $y=f(x)$ da o'ng tomonda turuvchi ifoda ma'noga ega bo'ladigan x ning qiymatlari olinadi. Masalan, agar $y=x^2$ ni kvadratning tomoni bilan yuzi ifodalovchi bog'lanish sifatida olsak, u holda aniqlanish sohasi barcha musbat sonlardan iborat bo'ladi.

Funksiyaning aniqlanish sohasini topishga doir misollar keltiramiz. Quyidagi funksiyalarning aniqlanish sohasini toping:

1. $y=\lg(2x-1)$ funksiyaning aniqlanish sohasini toping.

Yechish: Logarifmik funksiya faqat musbat sonlar uchun aniqlangan. Demak, $2x-1>0$ bo'lishi kerak. Bundan, $x>\frac{1}{2}$. Demak, aniqlanish sohasi $(\frac{1}{2},+\infty)$ dan iborat.

2. $y=\frac{1}{\lg(2x-1)}$ funksiyaning aniqlanish sohasini toping.

Yechish: Agar yuqoridagilarni inobatga olsak, $2x-1>0$, $2x-1\neq 1$ bo'ladi. Bundan $x>\frac{1}{2}$, $x\neq 1$ kelib chiqadi. Demak, aniqlanish sohasi $(\frac{1}{2};+1)\cup(1;+\infty)$ dan iborat.

А) analitik usul funksiyaning o'rganish jarayonida juda ko'p uchraydigan usuldir, lekin ba'zi xollarda funksiyaning qiymatini topish murakkab hisoblashlarga olib keladi,

Б) $y=f(x)$ yozuv hali funksiyaning analitik usulda berilishi bo'lmasligi mumkin. Masalan, ushbu Dirixle funksiyasini olaylik,

$$y=\begin{cases}1, & \text{агар} \quad x-\text{рационал сон бўлса}\\ 0, & \text{агар} \quad x-\text{иррационал сон бўлса.}\end{cases}$$

Demak $y=f(x)$ funksiya berilgan, uning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iborat, ammo funksiyaning analitik ifodasi berilgan emas,

В) funksiyaning jadval usulida berilishi qulaydir, chunki bir necha qiymatlar topilgan bo'ladi, lekin funksiyaning sohasi cheksiz to'plam bo'lganda, uning barcha qiymatlarini ko'rsatib bo'lmaydi,

Г) funksiyaning grafik usulda berilishi uning o'zgartirishlarini ko'rgazmali qilish imkonini beradi.

Funksiyaning grafigi – egri chiziq (hususiy holda to'g'ri chiziq), ba'zi hollarda biror nuqtalar to'plami bo'ladi.

Funksiya grafigi. $y=f(x)$ funksiyaning grafigini hosil qilish uchun $M(x,f(x))$ nuqtalarni hosil qilib, ular bir-biriga juda yaqin bo'lganda, silliq chiziq bilan tutashtiriladi.

Misol. 1) $y=\frac{1}{x}$ funksiyaning grafigi chizilsin.

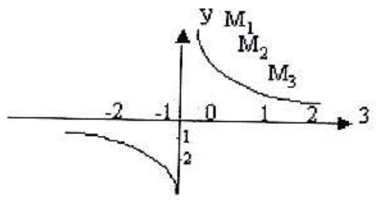
Bu funksiyaning aniqlanish sohasi $x\neq 0$ haqiqiy sonlar to'plami, ya'ni $(-\infty;0)\cup(0;+\infty)$ dan iborat.

Endi, aniqlanish sohasidan x ning bir necha qiymatlarini olib, uning ularga mos keladigan qiymatlarini topamiz.

x	1	2	3	−1	-2	-3	$\frac{1}{2}$	$-\frac{1}{2}$	...

y	1	$\frac{1}{2}$	$\frac{1}{3}$	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	2	-2	...

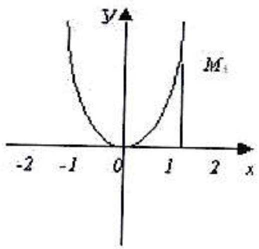
Koordinata tekisligida $M_1(1;1)$, $M_2(2;\frac{1}{2})$, $M_3(3;\frac{1}{3})$,... nuqtalarni hosil qilamiz. Bir biriga yaqin turga nuqtalarni uzluksiz chiziq yordamida tutashtirsak, funksiyaning grafigini ifoda qiladigan egri chiziq giperbola hosil bo'ladi. (2-chizma)



2-чизма.

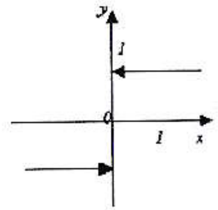
2) $y = x^2$ ning grafigi chizilsin.

$M_1(0;0)$, $M_2(1;1)$, $M_3(2;4)$ nuqtalarni hosil qilamiz. Ularni silliq chiziq bilan tutashtirsak, parabola egri yaizig'i hosil bo'ladi. (3-chizma)



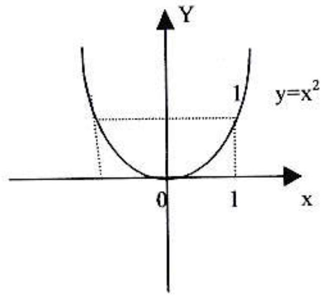
3-чизма.

3) $y = \begin{cases} 1, & \text{агар } x > 0 \text{ бўлса}, \\ 0, & \text{агар } x = 0 \text{ бўлса}, \\ -1, & \text{агар } x < 0 \text{ бўлса}. \end{cases}$ funksiyaning grafigi ko'rsatilgan. (4-чизма)



4-чизма.

Juft va toq funksiyalar. $y = f(x)$ funksiyaning aniqlanish sohasiga tegishli x o'zgaruvchining har bir qiymati bilan $-x$ qiymat ham shu funksiyaning aniqlanish sohasiga tegishli bo'lsa va bunda $f(-x) = f(x)$ tenglik bajarilsa, $y = f(x)$ funksiya juft funksiya deyiladi. Masalan, $y = x^2$ funksiya juft funksiyadir. Haqiqatdan, bu funksiya R to'plamda aniqlangan va aniqlanish sohasi har qanday x bilan $-x$ ni o'z ichiga oladi. Bundan tashqari, $f(-x) = (-x)^2 = x^2 = f(x)$ tenglik bajariladi. Juft funksiya grafigi ordinata o'qiga nisbatan simmetrik bo'ladi (5-chizma).



5-chizma

Umuman, har qanday juft funksiyaning grafigi ordinata o'qiga nisbatan simmetrikdir. $y = f(x)$ funksiyaning aniqlanish sohasiga tegishli x ning har bir qiymati bilan $-x$ qiymat ham shu funksiyaning aniqlanish sohasiga tegishli bo'lsa va bunda $f(-x)=-f(x)$ tenglik bajarilsa, $y=f(x)$ funksiya toq funksiya deyiladi. Toq funksiyaning grafigi koordinata boshiga nisbatan simmetrik joylashadi. Masalan, $f(x)=x^3$ funksiya toq funksiyadir. Haqiqatdan ham, $f(-x)=(-x)^3=-x^3=-f(x)$, ya'ni $f(-x)=-f(x)$ tenglik bajariladi. Bu funksiyaning grafigi koordinata boshiga nisbatan simmetrik bo'lib, kubik paraboladan iboratdir (9- chizma). Har qanday funksiya ham juft yoki toq bo'lishi shart emas.

Masalan, $y=2x+5$, $y=x^2+5x^3$, $y=\sin x+\cos x$ juft ham, toq ham emas. Demak funksiyalar har doim juft yoki toq bo'lishi shart emas ekan.

Misol. $y=\dfrac{x(x^2+1)(2x-3)}{3x^2-6x+8}$ funksiyaning juft yoki toq ekanligini aniqlang.

Yechish: yuqoridagi ta'rifga ko'ra argument o'rniga qarama-qarshi ishoradagi argumentni qo'yib soddalashtirilganda, funksiyaning o'zi kelib chiqsa funksiya juft bo'ladi. Agar soddalashtirilgach funksya ham teskari ishora bilan kelib chiqsa, funksiya tok funksiya bo'ladi. Shuni tekshiramiz.

$$y(-x)=\frac{-x((-x)^2+1)(-2x-3)}{3(-x^2)+6x+8}=\frac{x(x^2+1)(2x+3)}{3x^2+6x+8}\neq y(x)\neq -y(x)$$

bundan ko'rinadiki bu funksiya juft ham emas toq ham emas.

Davriy funksiyalar Ta'rif. Agar $f(x)$ funksiya uchun shunday $t>0$ son mavjud va funksiyaning aniqlanish sohasidan olingan har bir x uchun $x+t$ va $x-t$ lar aniqlanish sohasiga joylashgan bo'lib, $f(x+t)=f(x)$ tenglik o'rinli bo'lsa, u holda $f(x)$ *davriy funksiya deb ataladi.* t sonlarni eng kichigi funksiyaning davri deyiladi.

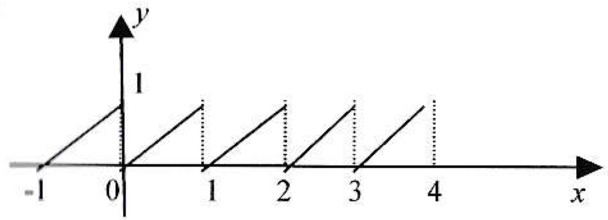
Misol. $y=sinx$, $y=cosx$, $y=tgx$, $y=x-[x]$ davriy funksiyalardir.

Davriy funksiyaning grafigini hosil qilish uchun uning bir davr ichidagi grafigini chizib, so'ngra uni chapga va o'ngga cheksiz ko'p marta ko'chirish kerak.

Misol. $f(x)=x-[x]=x-E(x)$ funksiya berilgan. Bunda $E(x)=[x]$ ifoda x ning butun qismini bildiradi. (E – fransuzcha Entier -ante-butun so'zining birinchi harfi). Masalan, $[x]=m\ (m\le x<m+1)$ m - butun son. $f(x)=x-E(x)=\{x\}$ bu funksiya x ning kasr qismini bildiradi, ya'ni $f(1)=0$; $f(1{,}05)=0{,}05$;... $f(x)$ funksiya davriydir va uning davri t=1 dir. Haqiqatdan,

$$f(x+1)=x+1-E(x+1)=x+1-E(x)-1=x-E(x)=f(x).$$

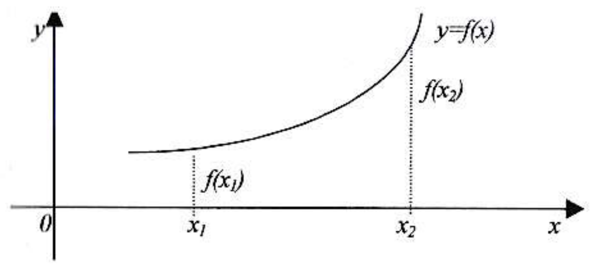
Demak, har qanday butun son ham davr bo'ladi. Funksiyaning grafigi ko'rsatilgan.



Monoton va teskari funksiyalar

1-ta'rif. $y=f(x)$ funksiyaning X sohadagi ihtiyoriy ikkita (x_1,x_2) qiymatlari uchun $x_1<x_2$ bo'lganda, $f(x_1)<f(x_2)$ tengsizlik o'rinli bo'lsa, u holda $y=f(x)$ funksiyasi X sohada o'suvchi funksiya deyiladi.

Ta'rifni geometrik nuqtai nazardan quyidagicha ko'rsatishimiz mumkin.

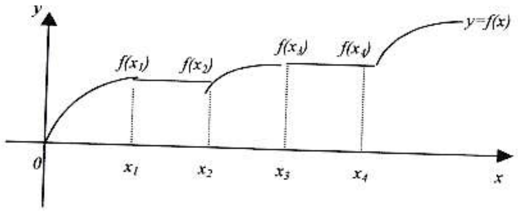


Yuqoridagi ta'rifdan ko'rinadiki, funksiya biror oraliqda o'suvchi bo'lishi uchun shu oraliqdagi argumentning kichik qiymatiga funksiyaning kichik qiymati, argumentning katta qiymatiga funksiyaning katta qiymati mos keladi.

1) $y=2^x$ funksiyasi butun sonlar o'qida o'suvchi.

2) $y=tgx$ funksiya ham o'suvchi funksiyadir.

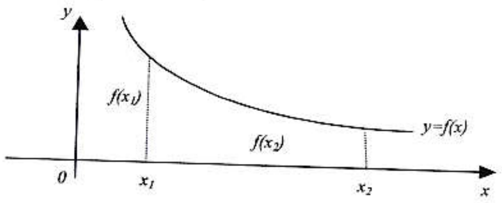
2-ta'rif. $y=f(x)$ funksiyaning X sohadagi ixtiyoriy ikkita (x_1,x_2) qiymatlari uchun $x_1 \le x_2$ bo'lganda $f(x_1)\le f(x_2)$ tengsizlik o'rinli bo'lsa, u holda $y=f(x)$ funksiya (x_1,x_2) oraliqda kamaymaydigan funksiya deyiladi.



3-ta'rif. $y=f(x)$ ning argumenti x ni $\forall(x_1,x_2)$ uchun $x_1<x_2$, bo'lganda $f(x_1)>f(x_2)$ tengsizlik o'rinli bo'lsa, $y=f(x)$ ni (x_1,x_2) oralig'ida kamayuvchi funksiya deyiladi.

Misol. $y=x^2$ funksiyaning olsak, bu funksiya $(-\infty,0)$ oralig'ida kamayuvchi, $(0,\infty)$ oralig'ida o'suvchi funksiyadir.

Misol. $y=sinx$ funksiya $(0,\frac{\pi}{2})$ oraliqda monoton o'suvchi bo'lib, $(\frac{\pi}{2},\frac{3\pi}{2})$ oraliqda monoton kamayuvchidir.



4-ta'rif. $y=f(x)$ ning argumentining ixtiyoriy (x_1,x_2) qiymatlari uchun $x_1\le x_2$ bo'lganda $f(x_1)\ge f(x_2)$ bo'lsa, u holda $y=f(x)$ funksiyasi (x_1,x_2) oralig'ida o'smaydigan funksiya deyiladi.

Agar berilgan oraliqda argumentning katta qiymatiga funksiyaning katta qiymati mos kelsa, ya'ni shu oraliqdagi ixtiyoriy x_1 va x_2 uchun $x_2>x_1$ shartdan $f(x_2)>f(x_1)$ kelib chiqsa, $y=f(x)$ funksiya shu oraliqda o'suvchi deyiladi.

5-ta'rif. Biror (x_1,x_2) oralig'ida o'suvchi va kamayuvchi funksiyalar monoton funksiyalar deyiladi.

Teskari funksiya tushunchasi. Teskari trigonometrik funksiyalarga o'tishdan avval umuman teskari funksiyalarga izoh berib o'tamiz.

Faraz qilaylik, $y=f(x)$ funksiya biror X sohada berilgan bo'lsin va x argument X sohada o'zgarganda, bu funksiya qabul qilgan barcha qiymatlar to'plami Y bilan ifodalansin. Odatda, X va Y lar oraliqlardan iborat bo'ladi.

Biz Y sohadan biror $y=y_0$ qiymatni tanlaylik; bu vaqtda X sohadan bizning funksiyamiz xuddi shu y_0 ga teng bo'ladigan $x=x_0$ qiymat, albatta, topiladi, demak, $f(x_0)=y_0$ bo'ladi.

x_0 ning bunday qiymatlari bir qancha bo'lishi ham mumkin. Shunday qilib, Y sohadagi y ning har bir qiymatiga xning bitta yoki bir qancha qiymati mos keladi; shu bilan Y sohada bir qiymatli yoki ko'p qiymatli $x=g(y)$ funksiya aniqlanib, buni $y=f(x)$ funksiyaning teskari funksiyasi deyiladi.

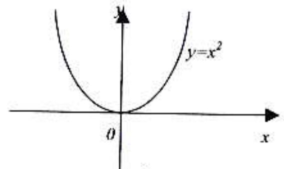
Misollar qaraymiz:

1) $y=a^x (a>1)$ funksiyani qaraylik, bu yerda x argument $X=(-\infty;+\infty)$ oraliqda o'zgaradi. Funksiya y ning qiymatlari $Y=(0;\ +\infty)$ oraliqni tashkil qiladi, shu bilan birga, bu oraliqdagi har bir y ga X dan bitta $x=log_a y$ qiymat mos keladi. Bu holda teskari funksiya bir qiymatli bo'ladi.

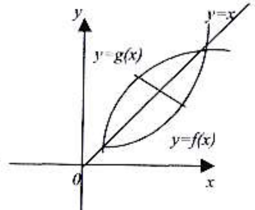
2) Aksincha, $y=x^2$ funksiya uchun x argument $X=(-\infty;+\infty)$ oraliqda o'zgarsa, teskari funksiya ikki qiymatli bo'ladi, chunki $Y=[0;\ +\infty)$ oraliqdagi y ning har bir qiymati uchun X da ikkita $x=\pm\sqrt{y}$ qiymat mos keladi. Odatda, bu ikki qiymatli funksiya o'rniga $x=\sqrt{y}$ va $x=-\sqrt{y}$ funksiya (ikki qiymatli funksiyaning "shoxchalari") tekshiriladi. Bularning har birini alohida $y=x^2$ ga teskari funksiya deb qarash ham mumkin, faqat bu vaqtda x ning o'zgarish sohasi $[0;\ +\infty)$ yoki $(-\infty;\ 0]$ oraliq bilan chegaralangan, deb faraz qilish kerak.

Berilgan $y=f(x)$ funksiyaning grafigiga qarab, bunga teskari $x=g(y)$ funksiyaning bir qiymatli bo'lish yoki bo'lmasligini sezish oson. Agar x o'qqa parallel bo'lgan har bir to'g'ri chiziq bu grafikni faqat bitta nuqtada kessa, u holda teskari funksiya bir qiymatli bo'ladi. Aksincha, bunday to'g'ri chiziqlardan ba'zilari grafikni bir nechta nuqtada kesib o'tsa, teskari funksiya ko'p qiymatli deyiladi. Bu holda grafikka qarab, har bir bo'lakka bu funksiyaning bir qiymatli "shoxchasi" mos keladigan qilib, x ning o'zgarish oralig'ini bo'laklarga bo'lish mumkin. Masalan, 1-chizmadagi $y=x^2$ funksiyaning grafigi bo'lgan parabolaga birinchi qarashimizdayoq, uning teskari funksiyasi ikki qiymatli ekanini aniq ko'ramiz va teskari funksiyaning bir qiymatli "shoxchalarini" olish uchun parabolaning o'ng va chap bo'laklarini, ya'ni x ning musbat va manfiy qiymatlarini alohida qarash yetarli.

Agar $x=g(y)$ funksiyasi $y=f(x)$ funksiyaga teskari bo'lsa, u vaqtda bu ikki funksiyaning grafigi bir xil bo'lishi ravshan. Teskari funksiyaning argumentini ham x bilan belgilashni, ya'ni $x=g(y)$ funksiya o'rniga $y=g(x)$ deb yozishni talab etish mumkin. U vaqtda gorizontal o'qni y o'q deb va vertikal o'qni esa x o'q (yangi) gorizontal, y o'q (yangi) vertikal bo'lsin desak, u vaqtda bu o'qlarning o'rinlarini almashtirib, birining o'rniga ikkinchisini qo'yish kerak, bu esa grafikni ham o'zgartiradi. Buni amalga oshirish uchun xOy chizma tekisligini birinchi koordinata burchak bissektritsasi atrofida 180^0 ga aylantirish maqsadga muvofiq.



1-чизма.



2-чизма.

Shunday qilib, $y=g(x)$ ning grafigi $y=f(x)$ ning grafigini shu bissektrisaga nisbatan ko'zgudagi aksi deb olish mumkin.

Mustaqil yechish uchun misollar.

1-variant

$y=\dfrac{6x^3}{(x+3)^2}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=\sin 5x$ funksiyaning eng kichik davrini toping.

$y=\dfrac{1}{x-2}+1$ funksiyaga teskari funksiyani toping.

$y=\lg\left(x^2-4x+3\right)$. funksiyaning aniqlanish sohasini toping

2-variant

$y=\dfrac{x(x-2)(x+2)}{x^2+4x-8}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=\lg\cos 2x$ funksiyaning eng kichik davrini toping.

$y=x^2+4x-7$ funksiyaga teskari funksiyani toping.

$y=\arcsin(3x-4)$. funksiyaning aniqlanish sohasini toping

3-variant

$y=\frac{3x^2}{x^4+4x^2}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=tg3x+\cos 4x$ funksiyaning eng kichik davrini toping.

$y=\frac{2x-1}{4-3x}$ funksiyaga teskari funksiyani toping.

$y=\frac{1}{\lg(1-x)}+\sqrt{x+2}$. funksiyaning aniqlanish sohasini toping

4-variant

$y=7x^6+5x^2+2|x|+7$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=tgx+3\sin\frac{x}{2}-3\cos\frac{x}{3}$ funksiyaning eng kichik davrini toping.

$y=3x^2+4$ funksiyaga teskari funksiyani toping.

$y=\sqrt{\sin x}-\sqrt{9-x^2}$. funksiyaning aniqlanish sohasini toping

5-variant

$y=\frac{x^4-2x^2}{x}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=5x^2+2\sin x-7\cos kx,\quad k\in Z$ funksiyaning eng kichik davrini toping.

$y=3x^2-9$ funksiyaga teskari funksiyani toping.

$y=\arcsin\frac{1+x^2}{2x}$. funksiyaning aniqlanish sohasini toping

6-variant

$y=\frac{(x-8)^2+5}{4x}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=4ctg\frac{x}{2}+3tg\frac{x}{3}$ funksiyaning eng kichik davrini toping.

$y=x^2-4x+1$ funksiyaga teskari funksiyani toping.

$y=x^{\frac{1}{\lg x}}$. funksiyaning aniqlanish sohasini toping

7-variant

$y=|x+13|+x^2$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=\cos(8x-7)$, $y=\sin(4x+13)$ funksiyaning eng kichik davrini toping.

$y=2^{4x+5}$ funksiyaga teskari funksiyani toping.

$y=tg\sqrt{16-x^2}$. funksiyaning aniqlanish sohasini toping

8-variant

$y=\frac{x^3+x-\sin x}{x^5}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=ctg(8x+7)$, $y=tg(4x-7)$ funksiyaning eng kichik davrini toping.

$y=3^x+7$ funksiyaga teskari funksiyani toping.

$y=\frac{1}{\sqrt{|x|-2|x-1|}}$. funksiyaning aniqlanish sohasini toping

9-variant

$y = 2x|x| + x^3 - 4x$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y = tg\frac{5\pi}{6}x$, $y = ctg\frac{5\pi}{3}x$ funksiyaning eng kichik davrini toping.

$y = 15 \cdot 5^{6x-7} - 13$ funksiyaga teskari funksiyani toping.

$f(x) = \ln\cos x$. funksiyaning aniqlanish sohasini toping

10-variant

$y = \frac{2x}{x^2 - 7}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y = tg3x$, $y = \sin(6x + 5)$ funksiyaning eng kichik davrini toping.

$y = 3 \cdot 4^{x+2} + 1$ funksiyaga teskari funksiyani toping.

$y = \sqrt{x^2 + x - 2}$.

11-variant

$y = \sqrt{x^2 - 2x + 2} + \sqrt{x^2 + 2x + 2}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$ctg6x$, $y = \cos(3x - 1)$ funksiyaning eng kichik davrini toping.

$y = 4 \cdot 8^{3x-5} + 11$ funksiyaga teskari funksiyani toping.

$y = \log_2(-x)$. funksiyaning aniqlanish sohasini toping

12-variant

$y = \log_5 4x^2 + x^4$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y = 2\sin\frac{\pi x}{3} + 3\cos\frac{\pi x}{4} + tg\frac{\pi x}{2}$ funksiyaning eng kichik davrini toping.

$y = 2 \cdot 3^{x-12} - 25$ funksiyaga teskari funksiyani toping.

$y = \arccos\left(\frac{x}{2} - 1\right)$. funksiyaning aniqlanish sohasini toping

13-variant

$y = x + 4\log_2 3^x$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y = (2 + \sin\frac{x}{2}) \cdot (1 + \cos\frac{x}{6}) \cdot tg(\frac{x}{3} - 7)$ funksiyaning eng kichik davrini toping.

$y = \log_2 x + 4$ funksiyaga teskari funksiyani toping.

$y = \sqrt{3^x - 5^x}$. funksiyaning aniqlanish sohasini toping

14-variant

$y = \arccos x$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y = x^4 \sin 3x$ funksiyaning eng kichik davrini toping.

$y = \log_3(2x + 3) - 4$ funksiyaga teskari funksiyani toping.

$y = \frac{1}{xe^x}$. funksiyaning aniqlanish sohasini toping

15-variant

$y = \frac{3^x - 3^{-x}}{4}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y = x^4 - x^2 + x$ funksiyaning eng kichik davrini toping.

$y = \log_3(6x + 12) - 15$ funksiyaga teskari funksiyani toping.

$y=\sin\sqrt{x-3}$. funksiyaning aniqlanish sohasini toping

16-variant

$y=\dfrac{2^x+5^x}{2^x-5^x}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=\lg\cos x$ funksiyaning eng kichik davrini toping.

$y=\log_5 7x$ funksiyaga teskari funksiyani toping.

$y=\lg\cos x$. funksiyaning aniqlanish sohasini toping

17-variant

$y=x^2+\cos x$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=tgx+3\sin\dfrac{x}{2}-3\cos\dfrac{x}{3}$ funksiyaning eng kichik davrini toping.

$y=\lg 10x+2$ funksiyaga teskari funksiyani toping.

$y=\sin\sqrt{x}$. funksiyaning aniqlanish sohasini toping

18-variant

$y=x^2+\cos x$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=5x^2+2\sin x-7\cos kx,\quad k\in Z$ funksiyaning eng kichik davrini toping.

$y=\lg 100x^2-4$ funksiyaga teskari funksiyani toping.

$y=\dfrac{|\sin x|}{1-\cos x}$. funksiyaning aniqlanish sohasini toping

19-variant

$y=\sin 2x+tg4x$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=4ctg\dfrac{x}{2}+3tg\dfrac{x}{3}$ funksiyaning eng kichik davrini toping.

$y=\ln x^3+4$ funksiyaga teskari funksiyani toping.

$y=\sin(\arccos x)$. funksiyaning aniqlanish sohasini toping

20-variant

$y=\sqrt[3]{(1-x)^2}+\sqrt[3]{(1+x)^2}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=\cos(8x-7)$, $y=\sin(4x+13)$ funksiyaning eng kichik davrini toping.

$y=\ln 10x^2+11$ funksiyaga teskari funksiyani toping.

$y=\left|\dfrac{10^x+1}{10^x-1}\right|$. funksiyaning aniqlanish sohasini toping

21-variant

$y=\ln\dfrac{1-x}{1+x}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=ctg(8x+7)$, $y=tg(4x-7)$ funksiyaning eng kichik davrini toping.

$y=\log_3(2x+3)-4$ funksiyaga teskari funksiyani toping.

$y=\lg\dfrac{x+3}{x-3}$. funksiyaning aniqlanish sohasini toping

22-variant

$y=\ln(x+\sqrt{1+x^2})$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=tg\frac{5\pi}{6}x$, $y=ctg\frac{5\pi}{3}x$ funksiyaning eng kichik davrini toping.

$y=\log_3(6x+12)-15$ funksiyaga teskari funksiyani toping.

$y=\sqrt{-x^2-x+2}$. funksiyaning aniqlanish sohasini toping

23-variant

$y=2^x+2^{-x}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=tg3x$, $y=\sin(6x+5)$ funksiyaning eng kichik davrini toping.

$y=\lg 100x^2-4$ funksiyaga teskari funksiyani toping.

$y=\frac{1}{\arcsin(1-x)}$. funksiyaning aniqlanish sohasini toping

24-variant

$y=\sin 2x+tg4x$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$ctg6x$, $y=\cos(3x-1)$ funksiyaning eng kichik davrini toping.

$y=\ln x^3+4$ funksiyaga teskari funksiyani toping.

$y=\frac{x+1}{x-2}$. funksiyaning aniqlanish sohasini toping

25-variant

$y=2^x+2^{-x}$ funksiyaning juft yoki toq ekanligini ikki hil usulda tekshiring.

$y=2\sin\frac{\pi x}{3}+3\cos\frac{\pi x}{4}+tg\frac{\pi x}{2}$ funksiyaning eng kichik davrini toping.

$y=\ln 10x^2+11$ funksiyaga teskari funksiyani toping.

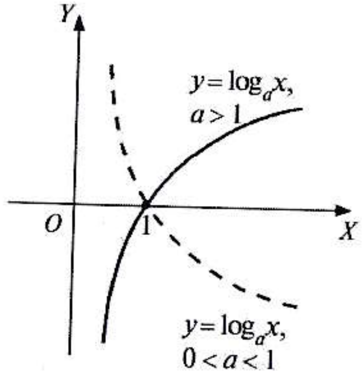
$y=\frac{1}{x^2+1}+1$. funksiyaning aniqlanish sohasini toping

9-mavzu: Ko'rsatkichli va logorifmik funksiyalar, xossalari va ularning grafiklari.

Logarifmlar. Logarifmik funksiya.

> $a>0, a\neq 1$ bo'lsin. N sonining a asos bo'yicha *logarifmi* deb, N sonini hosil qilish uchun a sonini ko'tarish kerak bo'lgan daraja ko'rsatkichiga aytiladi va $\log_a N$ bilan belgilanadi

Ta'rifga ko'ra, $a^x=N$ $(a>0, a\neq 1)$ tenglamaning x yechimi $x=\log_a N$ sonidan iborat. Ifodaning logarifmini topish amali shu ifodani logarifmlash, berilgan logarifmiga ko'ra shu ifodaning o'zini topish esa potensirlash deyiladi. $x=\log_a N$ ifoda potensirlansa, qaytadan $a^x=N$ $(a>0, a\neq 1)$ hosil bo'ladi. Bundan esa $x=\log_a N$ va $a^x=N$ tengliklar o'zaro teng kuchli ekanligi kelib chiqadi. Shu tariqa biz o'zining aniqlanish sohasida uzluksiz va monoton bo'lgan $y=\log_a x$ $(a>0, a\neq 1)$ funksiyaga ega bo'lamiz. Bu funksiya a asosli logarifmik funksiya deyiladi. $y=\log_a x$ funksiya $y=a^x$ funksiyaga teskari funksiyadir. Uning grafigi $y=a^x$ funksiya grafigini $y=x$ to'g'ri chiziqqa nisbatan simmitrik alamashtirish bilan hosil qilanadi.



Logarifmik funksiya ko'rsatkichli funksiyaga teskari funksiya bo'lganligi sababli, uning xossalarini ko'rsatkichli funksiya xossalaridan foydalanib hosil qilish mumkin. Jumladan $y=a^x$ funksiyaning aniqlanish sohasi

$D(f)=\{-\infty<x<\infty\}$, qiymatlar sohasi $E(f)=\{0<x<\infty\}$ edi. Shunga ko'ra $f(x)=\log_a x$ funksiya uchun $D(f)=\{0<x<\infty\}$, $E(f)=\{-\infty<x<\infty\}$ bo'ladi.

Logorifmik funksiyalarning xossalari.

1) $y=\log_a x$ funksiyada $a>1$ bo'lsa funksiya o'suvchi. $0<a<1$ funksiya kamayuvchi.

2) Funksiyaning aniqlanish sohasi $D(f)=\{0<x<\infty\}$ va qiymatlar sohasi $E(f)=\{-\infty<x<\infty\}$ dan iborat.

3) Funksiya davriy emas.

4) $y=\log_a x$ funksiya $a>1$ bo'lsa, argumentning $0<x<1$ qiymatlarida funksiya manfiy va argumentning $x>1$ qiymatlarida funksiya musbat qiymatlar qabul qiladi.

5) $y=\log_a x$ funksiya $a<1$ bo'lsa, argumentning $0<x<1$ qiymatlarida funksiya musbat va argumentning $x>1$ qiymatlarida funksiya manfiy qiymatlar qabul qiladi.

6) Ordinatalar o'qi $y=\log_a x$ funksiya uchun vertikal asimptota.

Asosiy logorifmik ayniyatlar:

1) $\log_a 1=0,\ \log_a a=1$

2) $\log_a b=\dfrac{\log_c b}{\log_c a}$

3) $\log_a(bc)=\log_a b+\log_a c$

4) $\log_a\dfrac{b}{c}=\log_a b-\log_a c$

5) $\log_a\dfrac{1}{b}=-\log_a b$

6) $\log_a b^c=c\log_a b$

7) $\log_{a^b} c=\dfrac{1}{b}\log_a c$

8) $\log_a b<\log_a c,\ va\ a>1\ bo'lsa\ b<c\ \ 0<a<1\ bo'lsa\ b>c\ bo'ladi$

9) $a^{\log_b c}=c^{\log_b a}$

10) $a^{\log_a b}=b$

Amaliyotda asosi 10 bo'lgan (o'nli logarifmlar) va asosi e =2,7182818... ga teng bo'lgan (natural logarifmlar) logarifmlar keng qo'llaniladi. Ularni mos ravishda lg$a$ va ln$a$ ko'rinishda belgilash qabul qilingan.

Ko'rsatkichli va logarifmik ifodalarni ayniy almashtirishlar.

Oldingi bandlarda logarifmning va logarifmik funksiyaning, shuningdek, darajaning va ko'rsatkichli funksiyaning xossalari bilan tanishgan edik. Bu xossalardan logarifmik va ko'rsatkichli ifodalarni shakl almashtirishlarda foydalaniladi.

Misol: $36^{\log_6 5}+10^{1-\lg 2}-3^{\log_9 36}$ ifodani hisoblang.

Yechish: yuqoridagi 6- va 10- xossalardan foydalanib, $36^{\log_6 5}=6^{2\log_6 5}=6^{\log_6 5^2}=6^{\log_6 25}=25$, $10^{1-\lg 2}=\dfrac{10}{10^{\lg 2}}=\dfrac{10}{2}=5$ va $3^{\log_9 36}=3^{\log_{3^2} 6^2}=3^{2\cdot\frac{1}{2}\log_3 6}=3^{\log_3 6}=6$ ekanligini topamiz va o'rniga keltirib quyidagiga ega bo'lamiz. $25+5-6=24$ demak ifodaning qiymati 24 ga teng ekan.

Misol: $(81^{\frac{1}{4}-\frac{1}{2}\log_9 4}+25^{\log_{125} 8})\cdot 49^{\log_7 2}$ ifodaning qiymatini toping.

Yechish:

$$(81^{\frac{1}{4}-\frac{1}{2}\log_9 4}+25^{\log_{125} 8})\cdot 49^{\log_7 2}=(\frac{81^{\frac{1}{4}}}{81^{\frac{1}{2}\log_9 4}}+25^{\log_{5^3}(2^3)})\cdot 7^{2\log_7 2}=$$

$$(\frac{3^{4\cdot\frac{1}{4}}}{9^{2\cdot\frac{1}{2}\log_9 4}}+25^{\frac{3}{3}\log_5 2})7^{\log_7 4}=(\frac{3}{9^{\log_9 4}}+5^{2\log_5 2})\cdot 4=(\frac{3}{4}+9)\cdot 4=$$

$$\frac{39}{4}\cdot 4=39$$

Misol: $\left[\left({}^{\log_{49}25}\sqrt[25]{5}+2\log_2\log_2\log_2 a^{2\log_a 4}\right)\cdot{}^{-\frac{1}{2}\log_3 4}\sqrt[4]{4}-a^2\right]:(1-a)$ ifodani soddalshtiring.

Yechish:

$$\left[\left(\sqrt[\log_{49}25]{5}+2\log_2\log_2\log_2 a^{2\log_a 4}\right)\cdot\sqrt[-\frac{1}{2}\log_3 4]{4}-a^2\right]:(1-a)=$$

$$\left[\left(5^{\frac{1}{\log_{49}25}}+2\log_2\log_2\log_2 a^{\log_a 16}\right)\cdot 4^{-\frac{1}{\frac{1}{2}\log_3 4}}-a^2\right]:(1-a)=$$

$$\left[\left(5^{\log_5 7}+2\log_2\log_2\log_2 16\right)\cdot 4^{-\log_2 3}-a^2\right]:(1-a)=$$

$$\left[\left(7+2\log_2\log_2 4\right)\cdot\frac{1}{2^{\log_2 9}}-a^2\right]:(1-a)=\left[\left(7+2\log_2 2\right)\cdot\frac{1}{9}-a^2\right]:(1-a)=$$

$$(1-a^2):(1-a)=1+a$$

Ko'rsatkichli tenglamalar va tengsizliklar. $a^x=b\ (a,b\in R)$ tenglama eng sodda ko'rsatkichli tenglamadir, bu yerda $a>0, a\neq 1$. Ko'rsatkichli funksiyaning qiymatlar to'plami $(0;\infty)$ oraliqdan iborat bo'lgani uchun $b\leq 0$ bo'lganda qaralayotgan tenglama yechimga ega bo'lmaydi. Agar $b>0$ bo'lsa, tenglama yagona yechimga ega va bu yechim $x=\log_a b$ sonidan iborat bo'ladi.

Teorema: agar $a>0, a\neq 1$ bo'lsa u holda $a^{f(x)}=a^{g(x)}$ tenglamaning yechimi $f(x)=g(x)$ tenglamaning yechimi kabi bo'ladi.

Misol: $8^{5x^2-46}=8^{2(x^2+1)}$ tenglamani yeching.

Yechish: yuqoridagi teoremadan foydalanamiz va

$$8^{5x^2-46}=8^{2(x^2+1)}$$
$$5x^2-46=2(x^2+1)$$
$$5x^2-46=2x^2+2$$
$$3x^2=48$$
$$x^2=16$$
$$x=\pm4$$

Bo'lib tenglamaning yechimi $x=\pm4$ ekanligi kelib chiqadi.

Ko'rsatkichli tengsizliklarni yechishda $y=a^x$ funksiyaning monotonligidan foydalaniladi. $a^{f(x)}>a^{g(x)}$ tengsizlikning yechimi, $a>1$ bo'lsa, $f(x)>g(x)$ tengsizlik yechimi bilan, $a<1$ bo'lsa $f(x)<g(x)$ tengsizlik yechimi bilan bir hil bo'ladi.

Misol: $0{,}3^{x^2+2x-5}<0{,}3^{x^2+x-7}$ tengsizlikni yeching.

$$0{,}3^{x^2+2x-5}<0{,}3^{x^2+x-7}$$
$$x^2+2x-5>x^2+x-7$$

Yechish: $2x-x>-7+5$

$$x>-2$$
$$x\in(-2;\infty)$$

Logarifmik tenglamalar va tengsizliklar. $\log_a x=b\ (a>0, a\neq 1)$ tenglamani qaraymiz. Bu tenglama eng sodda logarifmik tenglama deyiladi. $x=a^b$ son qaralayotgan tenglamaning ildizi bo'lishini ko'rish qiyin emas.

Berilgan tenglama $x=a^b$ dan boshqa ildizga ega emasligini $y=\log_a x$ logarifmik funksiyaning monotonligidan foydalanib isbotlash mumkin.

Mustaqil yechish uchun misollar:

1-variant

1) $\sqrt{\sqrt[\log_6^3]{25}+\sqrt[\log_8^7]{49}}$. ifodani soddalshtiring va qiymatini toping.

2) $5^{2x-1}+2^{2x}-5^{2x}+2^{2x+2}=0$ tenglamani yeching.

3) $\dfrac{\lg 8-\lg(x-5)}{\lg\sqrt{x+7}-\lg 2}=-1$. tenglamani yeching.

2-variant

1) $\sqrt[\log_3^1]{81}+\sqrt[\log_6^3]{27}+\sqrt[\log_7^3]{9}$. ifodani soddalashtiring va qiymatini toping.

2) $3^2 \cdot 3^5 \cdot 3^8 \cdot \ldots \cdot 3^{3n-1} = 27^5$. tenglikdan natural son n ni toping.

3) $0,5\left[\lg\left(x^2-55x+90\right)-\lg\left(x-36\right)\right]=\lg\sqrt{2}$. tenglamani yeching.

3-variant

1) $-\log_2 \log_2 \sqrt{\sqrt[4]{2}}$. ifodani soddalashtiring va qiymatini toping.

2) $\sqrt{3}\cdot 3^{\frac{x}{1+\sqrt{x}}}\cdot\left(\frac{1}{3}\right)^{\frac{2+\sqrt{x}+x}{2(1+\sqrt{x})}}=81$. tenglamani yeching.

3) $27^{\lg x}-7\cdot 9^{\lg x}-21\cdot 3^{\lg x}+27=0$ tenglamani yeching.

4-variant

1) $\left(1+\frac{1}{2dx}\right)\lg 3+\lg 2=\lg\left(27-\sqrt[x]{3}\right)$ ifodani soddalashtiring va qiymatini toping.

2) $\sqrt{2^x\sqrt[3]{4^x\sqrt[x]{0,125}}}=4\sqrt[3]{2}$. tenglamani yeching.

3) $\log_2\left(4\cdot 3^x-6\right)-\log_2\left(9x-6\right)=1$ tenglamani yeching.

5-variant

1) $-\log_3 \log_3 \sqrt[3]{\sqrt[3]{3}}$. ifodani soddalashtiring va qiymqtini toping.

2) $(\sqrt[3]{3})^x+(\sqrt[10]{3})^{x-10}=84$. tenglamani yeching.

3) $2\log_3\left(x-2\right)+\log_3\left(x-4\right)^2=0$ tenglamani yeching.

6-variant

1) $\dfrac{\left(\sqrt[\log_{[illegible]}]{27}+\sqrt[\log_{49}^{25}]{5}\right)\cdot\left(\sqrt[\log_4^9]{81}-\sqrt[\log_9^4]{8}\right)}{3+5^{\frac{1}{\log_{16}^{25}}}\cdot 5^{\log_5^3}}$. ifodani soddalashtiring.

2) $5^{3(\log_5 2+x)}-2=5^{x+\log_5 2}$. tenglamani yeching.

3) $5^{-2\log_{0,04}(3-4x^2)}+1,5\log_{\frac{1}{8}}4^x=0$. tenglamani yeching.

7-variant

1) $36^{\log_6^5}+10^{1-\lg 2}-3^{\log_9^{36}}$. ifodani soddiring va qiymatini toping.

2) $\sqrt{2}\cdot 0,5^{\frac{5}{4\sqrt{x}+10}}-\sqrt[\sqrt{x}+1]{4}=0$. tenglamani yeching.

3) $\log_3 x\cdot\log_9 x\cdot\log_{27} x\cdot\log_{81} x=\frac{2}{3}$. tenglamani yeching.

8-variant

1) $\left(81^{\frac{1}{4}-\frac{1}{2}\log_9^4}+25^{\log_{125}^8}\right)\cdot 49^{\log_7^2}$. ifodani soddalshtiring va qiymatini toping.

2) $8^{\frac{x-3}{3x-7}}\cdot\sqrt[3]{\sqrt[x-1]{0,5^{3x-1}}}=1$. tenglamani yeching.

3) $\log_5(3x-11)+\log_5(x-27)=3+\log_5 8$. tenglamani yeching.

9-variant

1) $\frac{\sqrt[\log_5^9]{81}+\sqrt[\log_6^3]{27}}{409}\cdot\left(\sqrt[\log_{25}^7]{7}-\sqrt[\log_{\sqrt{6}}^5]{125}\right).$ ifodani soddalashtiring va qiymatini toping.

2) $0,6^x\left(\frac{25}{9}\right)^{x^2-12}=\left(\frac{27}{125}\right)^3$. tenglamani yeching.

$\lg(5-x)+2\lg\sqrt{3-x}=1$. tenglamani yeching.

10-variant

1) $\left(2^{\log^{-1}{}_a\sqrt[4]{2}}-3^{\log^{-1}{}_{(a^2+1)}{}^3}-2a\right):\left(7^{4\log_{49}{}^a}-5^{\frac{1}{2}\log_{\sqrt{5}}a}-1\right).$ ifodani soddalashtiring va qiymatini toping.

2) $\sqrt[x]{100}+\sqrt[x]{25}=4,25\sqrt[x]{50}$. tenglamani yeching.

3) $9^{\log_{\frac{1}{8}}(x+1)}=5^{\log_{\frac{1}{5}}(2x^2+1)}$. tenglamani yeching.

11-variant

1) $\frac{\log_a\sqrt{a^2-1}\cdot\log^2{}_{1/2}\sqrt{a^2-1}}{\log_{a^2}(a^2-1)\cdot\log_{\sqrt[3]{a}}\sqrt[6]{a^2-1}}$. ifodani soddalashtiring.

2) $3\cdot5^{2x-1}-2\cdot5^{x-1}=0,2$. tenglamani yeching.

3) $4^{\log_5 x^2}-4^{\log_5 x+1}+4^{\log_5 x-1}-1=0$ tenglamani yeching.

12-variant

1) $\sqrt[\log_b a]{a^2}-2\cdot\sqrt[\log_b a]{a}\cdot\sqrt[\log_a b]{b}+\sqrt[\frac{1}{2}\log_a b]{b}.$ ifodani soddalshtiring.

2) $9^{x^2-1}-36\cdot3^{x^2-3}+3=0$. tenglamani yeching.

3) $x^{1-\frac{1}{3}\lg x^2}-\frac{1}{\sqrt[3]{100}}=0$. tenglamani yeching.

13-variant

1) $\left[\left(\sqrt[\log_{49}25]{5}+2\log_2\log_2\log_2 a^{2\log_a 4}\right)\cdot\sqrt[-\frac{1}{2}\log_3 4]{4}-a^2\right].$ ifodani soddalashtiring.

2) $4^x-10\cdot2^{x-1}-24=0$. tenglamani yeching.

3) $3\log_5 2+2-x=\log_5\left(3^x-5^{2-x}\right)$. tenglamani yeching.

14-variant

1) $(\log_a b+\log_b a+2)\cdot(\log_a b-\log_{ab} b)\log_b a-1$. ifodani soddalashtiring.

2) $\sqrt[x]{64}-\sqrt[x]{2^{3x+3}}+12=0$. tenglamani yeching.

3) $\sqrt{\log_3 x^9}-4\log_9\sqrt{3x}=1$. tenglamani yeching.

15-variant

1) $(\log_a b+\log_b a+2)\cdot(\log_a b-\log_{ab} b)\log_b a-1$. ifodani soddalashtiring.

2) $10^{1+x^2}-10^{1-x^2}=99$. tenglamani yeching.

3) $\log_{1-x}3-\log_{1-x}2-0,5=0$. tenglamani yeching.

16-variant

1) $\dfrac{1-\log^3{}_a b}{(\log_a b+\log_b a+1)\cdot\log_a \dfrac{a}{b}}$. ifodani soddalashtiring.

2) $3\cdot4^x+\frac{1}{3}\cdot9^{x+2}=6\cdot4^{x+1}-\frac{1}{2}\cdot9^{x+1}$. tenglamani yeching.

3) $\log_5(x-2)+\log_{\sqrt5}(x^3-2)+\log_{0,2}(x-2)=4$. tenglamani yeching.

17-variant

1) $\left(\sqrt[\lg a]{b^{\log_{100}a}}\cdot\sqrt[\lg b]{a^{\log_{100}b}}\right)^{2\log_{ab}(a+b)}$ ifodani soddalashtiring.

2) $\dfrac{2^x+10}{4}=\dfrac{9}{2^{x-2}}$. tenglamani yeching.

3) $\lg5+\lg(x+10)=1-\lg(2x-1)+\lg(21x-20)$. tenglamani yeching.

18-variant

1) $\left[\left(\log_b^4 a+\log_a^4 b+2\right)^{\frac12}+2\right]^{\frac12}-\log_b a-\log_a b$. ifodani soddalashtiring.

2) $4^{\log_5 x^2}-4^{\log_5 x+1}+4^{\log_5 x-1}-1=0$ tenglamani yeching.

3) $\log_2 182-2\log_2\sqrt{5-x}=\log_2(11-x)+1$. tenglamani yeching.

19-variant

1) $\log_2 2x^2+\log_2 x\cdot x^{\log_x(\log_2 x+1)}+\frac{1}{2}\log_4^2 x^4+2$. ifodani soddalashtiring.

2) $\log_5\sqrt{x-9}-\log_5 10+\log_5\sqrt{2x-1}=0$. tenglamani yeching.

3) $\log_x 9x^2\cdot\log^2{}_3 x=4$. tenglamani yeching.

20-variant

1) $\left[x\cdot\sqrt[\log_4 x^2]{x}+\sqrt[\log_{x^2}2]{2}+1\right]^{\frac12}$. ifodani soddalashtiring.

2) $\lg(3^x-2^{4-x})=2+\frac{1}{4}\log16-\frac{x\lg4}{2}$. tenglamani yeching.

3) $\lg(5-x)+2\lg\sqrt{3-x}=1$. tenglamani yeching.

21-variant

1) $\left[6\left(\log_b a\cdot\log_{a^2}b+1\right)+\log_a b^{-6}+\log_a^2 b\right]^{\frac12}-\log_a b, a>1$. ifodani soddalashtiring.

2) $\lg(x+1,5)=-\lg x$. tenglamani yeching.

3) $\lg\left[10^{\lg(x^2-21)}\right]-2=\lg x-\lg25$. tenglamani yeching.

22-variant

1) $\dfrac{\log_a b+\log_a\left[b^{\frac12\log_b a^2}\right]}{\log_a b-\log_{ab}b}\cdot\dfrac{\log_{ab}b\cdot\log_a b}{b^{2\log_b\log_a b}-1}$. ifodani soddalashtiring.

2) $x\lg\sqrt[5]{5^{2x-8}}-\lg25=0$. tenglamani yeching.

3) $\lg(5-x)-\frac{1}{3}\lg(35-x^{8})=0$. tenglamani yeching.

23-variant

1) $\log_a x=\alpha, \log_b x=\beta, \log_c x=\gamma, \log_d x=\delta$ va $x\neq 1$ ekanligi ma'lum. $\log_{abcd} x$ ni toping.

2) $\lg 81\sqrt[3]{3^{x^2-8x}}=0$. tenglamani yeching.

3) $\frac{\log_3 x-1}{\log_3 \frac{x}{3}}-2\log_3\sqrt{x}+\log^2{}_3 x=3$. tenglamani yeching.

24-variant

1) $\beta=10^{\frac{1}{1-lg\alpha}}$ va $\gamma=10^{\frac{1}{1-lg\beta}}$ ekanligi ma'lum. α ning γ ga bog'lanishini toping.

2) $\frac{2-\lg 4+\lg 0,12}{\lg(\sqrt{3x+1}+4)-\lg 2x}=1$. tenglamani yeching.

3) $\lg(x^2+1)=2\lg^{-1}(x^2+1)-1$. tenglamani yeching.

25-variant

1) $\log_{ab} c=\frac{\log_a c\cdot\log_b c}{\log_a c+\log_b c}$ ekanligini isbot qiling.

2) Agar $m^2=a^2-b^2$ ekanligi ma'lum bo'lsa,

$\log_{a+b} m+\log_{a-b} m-2\log_{a+b} m\cdot\log_{a-b} m$ ifodani soddalashtiring.

3) $x^{\lg^3 x}-5\lg x=0,0001$. tenglamani yeching.

10-mavzu: Trigonometriya elementlari. Trigonometrik funksiyalar. Teskari trigonometric funksiyalar.

Yoy va burchaklarning radian o'lchovi. Hammamizga ma'lumki burchaklar tushunchasi kiritilishi bilan burchak o'lchovi sifatida "Gradus" o'lchov birligi kiritiladi va odatda burchaklar gradusda o'lchanadi. Ammo trigonometriyaga kirishda burchaklarning gradus o'lchovidan ko'ra radian o'lchovi kiritiladi. Shuning uchun hozir biz burchak o'lchovlarida gradus o'lchovi va radian o'lchov birliklari orasidagi bog'lanishni kiritib o'tamiz.

Gradus o'lchovidan radion o'lchoviga o'tish formulasi quyidagi ko'rinishda yoziladi. $a=\frac{\pi}{180^0}\cdot\alpha$ bu yerda α burchakning gradus o'lchovi.

Masalan: 10^0 li burchakni radian o'lchovda ifodalasak, $a=\frac{\pi}{180^0}\cdot 10^0=\frac{\pi}{18}$ bo'lib hamma burchak o'lchovlari huddi shu kabi radian o'lchov birliklariga o'tkaziladi.

1^0 ning radian o'lchovi $0,0175$ radian;ga teng.

Radian o'lchovidan gradus o'lchoviga o'tish formulasi quyidagi ko'rinishda yoziladi. $\alpha=\frac{180^0}{\pi}\cdot a$ bo'lib bu yerda a burchakning radian o'lchovidir.

Masalan: $\frac{5\pi}{8}$ ga teng bo'lgan radian o'lchovdagi burchakning gradus o'lchovini toping. $\alpha=\frac{180^0}{\pi}\cdot\frac{5\pi}{8}=\frac{45\cdot 5}{2}=\frac{225}{2}=112,5^0$ ga teng bo'ladi va barcha radian o'lchovdagi burchaklar huddi shu kabi gradus o'lchoviga o'tkaziladi.

1 radianning gradus o'lchovi $57^0 17'44'',8\approx 57,3^0$ ga teng.

Aylana yoyining uzunligi yoyning radian o'lchovi bu yoyning radiusga ko'paytirilganiga teng: $l=aR$.

Doiraviy sektorning yuzi sector yoyining radian o'lchovini doira radiusining kvadrati ko'paytmasining yarmiga teng. $S_{sek}=a\cdot\frac{R^2}{2}$.

Endi ba'zi bir kelajakda ko'p ishlatiladigan ba'zi bir burchaklarning gradus o'lchovidan radian o'lchovga o'tkazilgandagi qiymatlarini ko'rib chiqamiz.

Masalan: $30^0, 45^0, 60^0, 90^0, 120^0, 135^0, 180^0, 270^0, 360^0$ ga teng bo'lgan burchaklarni radian o'lchovlarini topamiz.

$30^0 = \frac{\pi}{180^0} \cdot 30^0 = \frac{\pi}{6}, 45^0 = \frac{\pi}{180^0} \cdot 45^0 = \frac{\pi}{4}, 60^0 = \frac{\pi}{180^0} \cdot 60^0 = \frac{\pi}{3}, 90^0 = \frac{\pi}{180^0} \cdot 90^0 = \frac{\pi}{2},$

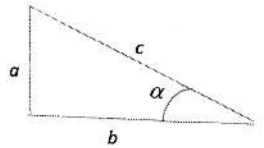
$120^0 = \frac{\pi}{180^0} \cdot 120^0 = \frac{2\pi}{3}, 135^0 = \frac{\pi}{180^0} \cdot 135^0 = \frac{3\pi}{4}, 180^0 = \frac{\pi}{180^0} \cdot 180^0 = \pi, 270^0 = \frac{\pi}{180^0} \cdot 270^0 = \frac{3\pi}{2}, 360^0 = \frac{\pi}{180^0} \cdot 360^0 = 2\pi$

bo'lib,

$30^0 = \frac{\pi}{6}, 45^0 = \frac{\pi}{4}, 60^0 = \frac{\pi}{3}, 90^0 = \frac{\pi}{2}, 120^0 = \frac{2\pi}{3}, 135^0 = \frac{3\pi}{4}, 180^0 = \pi, 270^0 = \frac{3\pi}{2}, 360^0 = 2\pi$

ekanligi kelib chiqadi.

Sonli argumentning trigonometric funksiyalari Hammamizga 9- sinf geometriya kursidan ma'lumki burchak sinusi, kosinusi, tangensi va kotengensi tushunchalari ixtiyoriy to'g'ri burchakli uchburchakning o'tkir burchaklari uchun kiritilgan bo'lib, quyidagicha ta'riflanar edi.

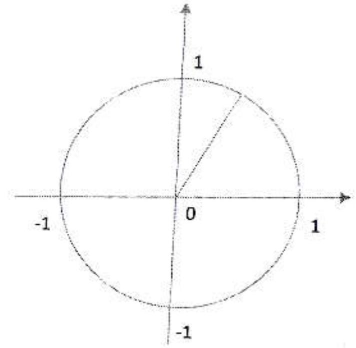


berilgan uchburchak uchun burchak sinusi, kosinusi, tangensi va katangen-
sining ta'riflarini keltirib o'tamiz. α burchak sinusi deb, berilgan burchak qarshisidagi katetni gipotenuzaga nisbatiga aytiladi ya'ni quyidagicha yoziladi. $\sin\alpha = \frac{a}{c}$.

Berilgan burchak kosinusi deb, berilgan burchakka yopishgan katetni gepotenuzaga nisbatiga aytiladi, ya'ni quyidagicha yoziladi. $\cos\alpha = \frac{b}{c}$.

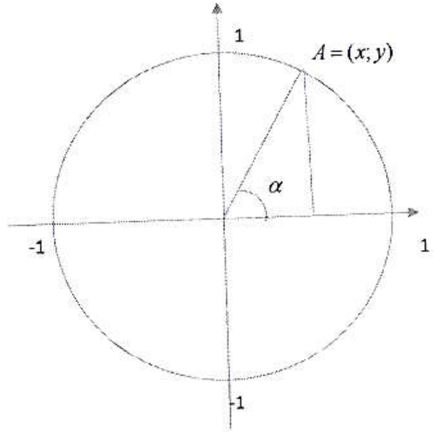
Berilgan burchak tangengensi deb, berilgan burchak qarshisidagi katetni yopishgan katetga nisbatiga aytiladi va quyidagicha yoziladi. $tg\alpha = \frac{a}{b}$.

Berilgan burchak katangensi deb, berilgan burchakka yopishgan katetni qarshisidagi katetga nisbatiga aytiladi va quyidagicha yoziladi. $ctg\alpha = \frac{b}{a}$. Shu o'rinda quyidagilarni ta'kidlab o'tish lozim. Ya'ni $tg\alpha = \frac{\frac{a}{c}}{\frac{b}{c}}$ ko'rinishida yozish mumkinligini inobatga oladigan bo'lsak u holda $tg\alpha = \frac{\sin\alpha}{\cos\alpha}$ ga teng bo'lish kelib chiqadi va $ctg\alpha = \frac{\frac{b}{c}}{\frac{a}{c}}$ ekanligidan esa $ctg\alpha = \frac{\cos\alpha}{\sin\alpha}$ ga tengligi kelib chiqadi. Bundan tashqari esa burchak tangensi va katangensi quyidagicha bog'liqligi mavjudligi kelib chiqadi ya'ni $tg\alpha = \frac{1}{ctg\alpha}$.



Endi triganometriya bo'limining asosiy xossalarini to'liq o'rganishda qulaylik uchun trigonametrik funksiyalarni Dekart koordinatalar sistemasida berilgan birlik aylana yordamida ko'rib chiqamiz. Ya'ni markazi $(0;0)$ nuqtada ya'ni koordinata tekisligining boshida bo'lgan radiusi 1 ga teng bo'lgan aylana kiritamiz va $(1;0)$ nuqtani burchaklarning boshlanish qismi sifatida qabul qilamiz va soat strelkasiga qarama qarshi yo'nalishni musbat yo'nalish sifatida qabul qilamiz.

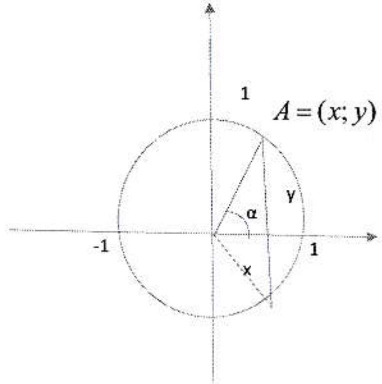
Endi ushbu birlik aylanada berilgan burchak sinusi, kosinusi, tangensi va kotangensi tariflarini kiritsak u holda quyidagiga kelamiz.



Ushbu burchak uchun yuqorida berilgan ta'riflarni kiritadigan bo'lsak u holda quyidagiga ega bo'lamiz. $\sin\alpha = \frac{y}{1} = y$, $\cos\alpha = \frac{x}{1} = x$, $tg\alpha = \frac{y}{x}$, $ctg\alpha = \frac{y}{x}$. Bundan esa burchak sinusi sonli birlik aylana nuqtasining ardinatasiga teng bo'ladi va shu burchakning kosinusi esa nuqtaning absissasiga teng bo'ladi. Burchak tangensi va katangensi esa berilgan burchakning mos sinus va kosinuslarining nisbatlariga teng bo'ladi.

Triganometrik funksiyalarning ishoralari, son qiymatlari va juft toqlik xossalari.

Endi triganometrik funksiyalarning juft va toqligini ko'rib chiqadigan bo'lsak yana yuqoridagi kabi birlik aylanadan foydalanamiz, ya'ni berilgan funksiyaning juft yoki toq ekanligini korsatish uchun quyidagidan foydalanardik. $f(-x) = f(x)$ shart bajarilsa u holda funksiya juft funksiya deyiladi va $f(-x) = -f(x)$ shrt bajarilsa u holda funksiya toq funksiya deyiladi va agar $f(-x) = g(x) \neq f(x)$ bo'lsa u holda funksiya juft ham emas toq ham emas deyiladi. Ushbu shrtlardan qaysi biri bajarilishini yuqorida berilgan birlik aylana yordamida tekshiramiz.



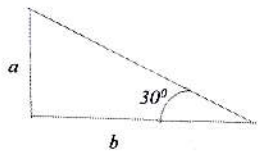
Ya'ni α burchakni koordinatalar sistemasining birinchi choragidan olingan deb hisoblasak u holda $-\alpha$ burchak esa soat strelkasi bo'ylab suriladi va to'rtinchi chorakda joylashadi. Endi bu burchaklarning sinusi va kosinuslarini ko'rib chiqamiz. Ya'ni burchklarning sinusi berilgan nuqtaning ardinatasiga teng bo'lganligi uchun, $\sin\alpha = y, \sin(-\alpha) = -y$ bo'lib bundan esa $\sin\alpha = -\sin(-\alpha)$ ga tengligi kelib chiqadi va sinus funksiya toq funksiya ekanligi kelib chiqadi. Burchakning kosinusi esa nuqtaning absissasiga teng ekanligini yuqorida ta'kidlab o'tgandik. Shuning uchun $\cos\alpha = x, \cos(-\alpha) = x$ bo'lib bundan $\cos\alpha = \cos(-\alpha)$ ekanligi kelib chiqadi va bu esa kosinus funksiyaning juft funksiya ekanligini anglatadi. Tangens va katangens funksiyalar ikki funksiya nisbati ya'ni juft va toq funksiyalarning nisbati yana toq funksiya bo'lishidan, tangens va katangens funksiyalar ham toq ekanligi kelib chiqadi.

$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$
$$tg(-\alpha) = -tg\alpha$$
$$ctg(-\alpha) = -ctg\alpha$$

Endi esa trigonometric funksiyalarning choraklar bo'yicha ishoralarini ko'rib chiqamiz. Buning uchun ham burchakning sinusi va kosinusi nuqtaning mos ravishta ardinatasi va absissasiga teng ekanligidan foydalanamiz, ya'ni koordinatalar tekisligining birinchi va ikkinchi choraklaridagi burchklarning sinusi ishorasi musbat va uchinchi va to'rtinchi chorakdagi burchaklarning sinusi ishorasi manfiy bo'ladi. Burchak kosinusi ishorasi esa birinchi va to'rtinchi choraklarda musbat, ikkinchi va uchunchu choraklardagi ishoralari esa manfiy bo'ladi. Bundan tashqari tangens va katangens ishoratari esa sinus va kosinus ishoralarining nisbati kabi topiladi, ya'ni tangens va katangens funksiyalarning ishoralari birinchi va uchinchi choraklarda musbat, ikkinchi va to'rtinchi choraklarda esa manfiy bo'ladi va ushbu ishoralarning barchasi jadval ko'rinishida quyida berilgandir.

	I chorak $0<\alpha<\frac{\pi}{2}$	IIchorak $\frac{\pi}{2}<\alpha<\pi$	III chorak $\pi<\alpha<\frac{3\pi}{2}$	IV chorak $\frac{3\pi}{2}<\alpha<2\pi$
$\sin\alpha$	+	+	-	-
$\cos\alpha$	+	-	-	+
$tg\alpha$	+	-	+	-
$ctg\alpha$	+	-	+	-

Endi esa trigonometric funksiyalarning ba'zi bir burchakdagi qiymatlarini hisoblab chiqamiz. Eng avvalo $\frac{\pi}{6}=30^0$ ga teng burchakning sinusi va kosinusini topamiz. Buning uchun bizga ma'lum bo'lgan 30^0 qarshisidagi katet gepotenuzaning yarmiga teng xossasidan foydalansak u holda



$c=2a$ va Pifagor teoremasidan $b=\sqrt{3}a$ ga teng ekanligi kelib chiqadi. Endi burchak sinusi va kosinusi ta'riflaridan quyidagiga ega bo'lamiz. $\sin 30^0=\frac{a}{c}=\frac{a}{2a}=\frac{1}{2}$, $\cos 30^0=\frac{\sqrt{3}a}{2a}=\frac{\sqrt{3}}{2}$. 45^0 ga teng bo'lgan burchakning sinusi va kosinusini ko'rib chiqamiz. Buning uchun yana to'g'ri burchakli uchburchakdan foydalanamiz ya'ni bir burchagi 45^0 ga teng bo'lgan to'g'ri burchakli uchburchak teng yonli ekanligidan foydalanamiz va $b=a$, $c=\sqrt{2}a$ ekanligidan $\sin 45^0=\frac{a}{c}=\frac{a}{\sqrt{2}a}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$, $\cos 45^0=\frac{a}{\sqrt{2}a}=\frac{\sqrt{2}}{2}$ ekanligi kelib chiqadi. Bundan tashqari 60^0 ga teng bo'lgan burchakning sinusi va kosinusi ham huddi yuqoridagi kabi hisoblanadi. 90^0, 180^0, 270^0 ga teng o'lchovga ega bo'lgan

burchak sinusi va kosinusini topish uchun birlik aylanadan foydalanamiz, ya'ni nuqtaning absissasi kosinusni ardinatasi esa sinusni berishni bilgan holda $\cos 90^0=0$, $\sin 90^0=1$ ekanligi kelib chiqadi. Burchakning ;tangens va kotangensini esa shu burchaklarning sinus va kosinus qiymatlarini mos ravishta nisbatini hisoblash orqali topamiz.

Ba'zi burchaklar trigonometric funksiyalarining qiymatlarini jadval shaklida berib o'tamiz.

	0 0^0	$30^0=\frac{\pi}{6}$	$45^0=\frac{\pi}{4}$	$60^0=\frac{\pi}{3}$	$90^0=\frac{\pi}{2}$	$180^0=\pi$	$270^0=\frac{3\pi}{2}$	$360^0=2\pi$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	−1	0	1
$tg\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∅	0	∅	0
$ctg\alpha$	∅	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∅	0	∅

Asosiy trigonometrik ayniyatlar Asosiy trigonometrik ayniyatlarni o'rganishdan oldin, ayniyatning o'zi nima ekanligi haqida qisqacha to'xtalib o'tamiz.

Birgina argumentning ikkita funksiyasi bir hil aniqlanish sohasiga ega bo'lib, argumentlarning barcha mumkin bo'lgan qiymatlarida bir hil qiymatlar qabul qilsa, bunday funksiyalar aynan teng deyiladi.

Argumentning qabul qilishi mumkin bo'lgan barcha qiymatlarida o'rinli bo'lgan tenglik ayniyat deyiladi.

Agar ayniyat tarkibiga trigonometric funksiyalar kirgan bo'lsa u holda ayniyat trigonometric ayniyat deyiladi.

Berilgan funksiyadan unga aynan teng funksiyaga o'tish funksiyani aynan shakl almashtirish deyiladi.

Trigonometric ayniyatlarni isbotlashda odatda quyidagi usullar qo'llaniladi: 1) tenglikning istilgan qismi (odatda qiyinroq ifoda qatnashgan qismi) ustida shunday shakl almashtirishlar bajariladiki, natijada tenglikning boshqa qismida turgan ifoda hosil bo'ladi; 2) isbotlanayotgan ayniyatning har ikkala qismini har ikkala qismda bir vaqtda aynan teng ifodalar hosil bo'lganligi ravshan bo'lgunga qadar shakl almashtiriladi; 3) proporsiyaning chetki va o'rta hadlari ko'paytmalarining tengligi xossasidan foydalanib, bu ko'paytmalarning tengligiga ishonch hosil qilinadi.

Asosiy trigonometric ayniyatlar.

1) $\sin^2\alpha+\cos^2\alpha=1$.

2) $tg\alpha\cdot ctg\alpha=1,\ \alpha\neq\frac{\pi k}{2}\ k\in Z$

3) $1+tg^2\alpha=\frac{1}{\cos^2\alpha},\ \alpha\neq\frac{\pi}{2}+\pi k\ k\in Z$

4) $1+ctg^2\alpha=\frac{1}{\sin^2\alpha},\ \alpha\neq\pi k\ k\in Z$.

Endi ushbu asosiy ayniyatlarni isbotlasak. Birinchi ayniyatni isbotlash uchun Pifagor teoremasi hamda sinus va kosinus funksiyalarning ta'riflaridan foydalanamiz. $\sin\alpha=\frac{a}{c}$ va $\cos\alpha=\frac{b}{c}$, bularni yuqoridagi ifodaga qo'yamiz. $\sin^2\alpha+\cos^2\alpha=(\frac{a}{c})^2+(\frac{b}{c})^2=\frac{a^2+b^2}{c^2}=\frac{c^2}{c^2}=1$ bo'lib bu esa birinchi ayniyatning to'g'ri ekanligini isbotlaydi.

Ikkinchi ayniyatning isboti ham huddi yuqoridagi kabi ta'rifdan foydalanib isbotlanadi. $tg\alpha=\frac{a}{b}$ va $ctg\alpha=\frac{b}{a}$ bo'lib, bularni ikkinchi ifodaga qo'yib tekshiramiz. $tg\alpha\cdot ctg\alpha=\frac{a}{b}\cdot\frac{b}{a}=1$ ekanligi kelib chiqadi bu esa tenglikning to'g'ri ekanligini ko'rsatadi.

Uchunchi ayniyatning to'g'ri ekanligini ko'rsatish uchun, $tg\alpha=\frac{\sin\alpha}{\cos\alpha}$ ekanligidan foydalanamiz. $1+tg^2\alpha=1+\frac{\sin^2\alpha}{\cos^2\alpha}=\frac{\sin^2\alpha+\cos^2\alpha}{\cos^2\alpha}=\frac{1}{\cos^2\alpha}$ bo'lib bu esa uchunchi ayniyat to'g'ri ekanligi kelib chiqadi.

To'rtinchi ayniyat ham huddi yuqoridagi uchinchi ayniyat kabi isbotlanadi.

Yuqoridagi ayniyatlardan foydalangan holda yana bir nechta ayniyatlarni keltirib chiqarish mumkin, masalan, $\sin\alpha=\pm\sqrt{1-\cos^2\alpha}$, $\cos\alpha=\pm\sqrt{1-\sin^2\alpha}$, $\cos\alpha=\pm\frac{1}{\sqrt{1+tg^2\alpha}}$ ... kabi bir qator ayniyatlarni.

Trigonometrik funksiyalarning davriyligi $f(x)$ funksiya $X\subset R$ to'plamda berilgan bo'lsin.

Ta'rif. Agar shunday o'zgarmas $T\ (T\neq0)$ son mavjud bo'lsaki, $\forall x\in X$ uchun

1) $x-T\in X$, $x+T\in X$

2) $f(x+T)=f(x)$

bo'lsa, $f(x)$ **davriy funksiya** deyiladi, T son esa $f(x)$ **funksiyaning davri** deyiladi.

Masalan, $f(x)=\sin x$, $f(x)=\cos x$ funksiyalar davriy funksiyalar bo'lib, ularning davri 2π ra, $f(x)=tgx$, $f(x)=ctgx$ funksiyalarning davri esa π ga teng.

Davriy funksiyalar quyidagi xossalarga ega:

а) Agar $f(x)$ davriy funksiya bo'lib, uning davri $T\ (T\neq0)$ bo'lsa, u holda

$$T_n=nT\ \ (n=\pm1,\pm2,)$$

sonlar ham shu funksiyaning davri bo'ladi.

б) Agar T_1 va T_2 sonlar $f(x)$ funksiyaning davri bo'lsa, u holda $T_1+T_2\neq0$ hamda $T_1-T_2\ (T_1\neq T_2)$ sonlar ham $f(x)$ funksiya-ning davri bo'ladi.

в) Agar $f(x)$ hamda $g(x)$ lar davriy funksiyalar bo'lib, ularning har birining davri $T\ (T\neq0)$ bo'lsa, u holda

$$f(x)+g(x),\ \ f(x)-g(x),\ \ f(x)\cdot g(x),\ \ \frac{f(x)}{g(x)}\ \ (g(x)\neq 0)$$

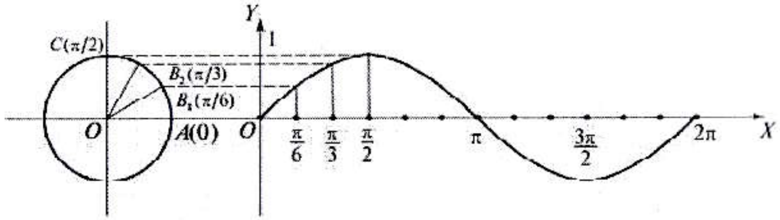
funksiyalar ham davriy funksiyalar bo'lib, T son ularning ham davri bo'ladi.

Davriy funksiya to'g'risida umumiy tushunchaga ega bo'ldik. Yuqorida ta'kidlaganimizdek trigonometric funksiyalar davriy bo'ladi va trigonometric funksiyalarning davriyligini trigonametrik tenglamalar mavzusini to'liq o'rganilgach isbotiga to'xtalamiz.

Bundan tashqari trigonometric funksiyalarning davriylik xossasini quyidagi ayniyatlar yordamida ifodalash mumkin.

$$\sin\alpha=\sin(\alpha+2\pi k)\ \ k\in Z;$$
$$\cos\alpha=\cos(\alpha+2\pi k)\ \ k\in Z;$$
$$tg\alpha=tg(\alpha+\pi k)\ \ k\in Z;$$
$$ctg\alpha=ctg(\alpha+\pi k)\ \ k\in Z;$$

Trigonometrik funksiyalarning grafiklari Hozir biz trigonometric funksiyalarning bizga ma'lum oraliqdagi, ya'ni biz biladigan qiymatlariga asoslangan holda grafigini chizamiz. Sinus va kosinus funksiyani $[0;2\pi]$ va tangens va kotangens funksiyalarning $[0;\pi]$ oraliqdagi grafigini yasaymiz.



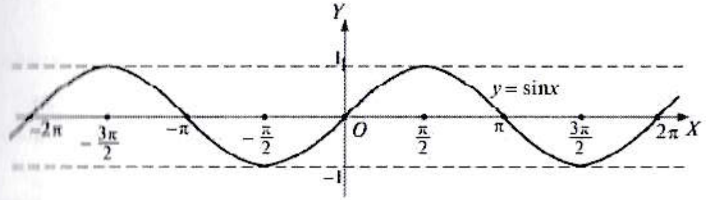
Bu $y=\sin x$ funksiyaning yuqorida aytilganidek $[0;2\pi]$ oraliqdagi grafigi bo'lib, ushbu grafik asosida funksiyaning juda ko'p xossalarini o'rganish mumkin. Ya'ni:

1) $y=\sin x$ funksiya argumentning barcha qiymatlarida aniqlangan, ya'ni funksiyaning aniqlanish sohasi $(-\infty;\infty)$ dan iborat.

2) $y=\sin x$ funksiyaning qiymatlar sohasi $[-1;1]$ oraliqdan iborat.

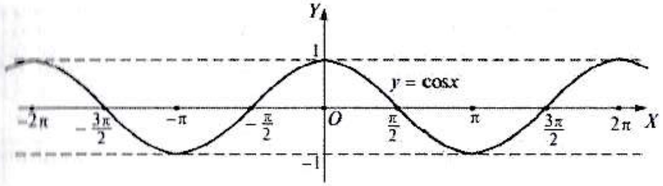
3) $y=\sin x$ funksiya berilgan oraliqning $[0;\frac{\pi}{2}]$ $[\frac{3\pi}{2};2\pi]$ sohalarida o'suvchi va $[\frac{\pi}{2};\frac{3\pi}{2}]$ oralig'ida kamayuvchidir.

Endi funksiyaning davriyligi va 2π davri ekanligidan kelib chiqqan holda funksiyaning umumiy grafigini yasaymiz.



Ushbu grafik $y=\sin x$ funksiyaning grafigi bo'lib, sinusoida deb ataladi.

Huddi yuqoridagi kabi cosinus funksiyaning grafigi ham shu tartibda chiziladi.



Bu esa cosinus funksiyaning grafigi bo'lib, cosinusoida deyiladi va grafik yordamida kosinus funksiyaning xossalarini o'rganib chiqshimiz mumkin.

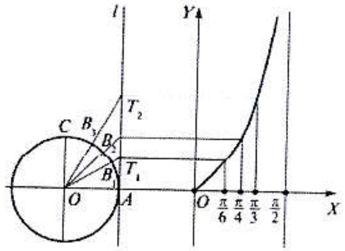
1) $y=\cos x$ funksiya argumentning barcha qiymatlarida aniqlangan, ya'ni funksiyaning aniqlanish sohasi $(-\infty;\infty)$ dan iborat.

2) $y=\cos x$ funksiyaning qiymatlar sohasi $[-1;1]$ oraliqdan iborat.

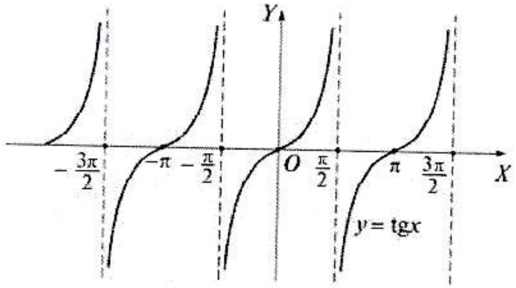
3) $y=\cos x$ funksiya berilgan oraliqning $[0;\pi]$ sohalarida kamayuvchi va $[\pi;2\pi]$ oralig'ida o'suvchidir.

4) Bulardan tashqari juft va toq funksiyalarning xossalariga ko'ra $y=\sin x$ funksiya toq bo'lganligi uchun koordinatalar boshiga nisbatan va $y=\cos x$ funksiya esa juft bo'lganligi uchun ardinatalar o'qiga nisbatan simmetrik bo'ladi.

Endi tangens va katangens funksiyalarning grafiklarini chizamiz. Ushbu funksiyalarning grafiklarini chizish uchun ha huddi yuqoradagi kabi chizamiz. Va bu grafikdan foydalanib funksiyalarni xossalarini o'rganishimiz mumkin, ya'ni yuqoridagi kabi o'sush va kamayish oraliqlarini, aniqlanish va qiymatlar sohasini, juft va toqligini topishimiz mumkin.



Bu $y = tgx$ funksiyaning $[0;\frac{\pi}{2})$ oraliqdagi grafigidir. Funksiyaning davriyligini inobatga olib umumiy ko'rinishdagi grafigi



Kabi bo'ladi. Garikdan ma'lumki;

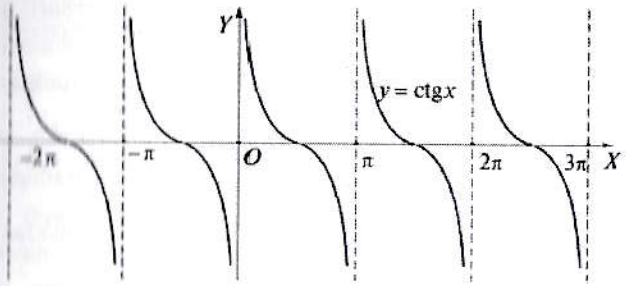
1) $y = tgx$ funksiya $\frac{\pi}{2} + \pi k,\ k \in Z$ nuqtalardan tashqari barcha haqiqiy sonlarda aniqlangandir.

2) $y = tgx$ funksiyaning qiymatlar to'plami esa barcha haqiqiy sonlar to'plamidan iborat.

3) $y = tgx$ funksiya o'zining aniqlanish sohasida doim o'suvchi bo'ladi.

4) $y = tgx$ funksiya toq bo'lganligi uchun koordinatalar boshiga nisbatan simmetrik bo'ladi.

$y = ctgx$ funksiyaning grafigini ham huddi yuqoridagi kabi chizamiz va quyidagi grafikka ega bo'lamiz.



Ushbu grafikdan quyidagilarni bilishimiz mumkin.

1) $y = ctgx$ funksiya $\pi k,\ k \in Z$ nuqtalardan tashqari barcha haqiqiy sonlarda aniqlangandir.

2) $y = ctgx$ funksiyaning qiymatlar to'plami esa barcha haqiqiy sonlar to'plamidan iborat.

3) $y = ctgx$ funksiya o'zining aniqlanish sohasida doim kamayuvchi bo'ladi.

4) $y = tgx$ funksiya toq bo'lganligi uchun koordinatalar boshiga nisbatan simmetrik bo'ladi.

Teskari trigonometric funksiyalar va ularning asosiy xossalari

Ushbu $y = \sin x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ funksiyani qaraymiz.

x argumentning qiymatlari $-\frac{\pi}{2}$ *dan* $\frac{\pi}{2}$ gacha o'sib borganda y nging qiymatlari -1 *dan* 1 gacha o'sib borishi va $[-1;1]$ kesmani to'ldirishi bizga ma'lum. Shundan kelib chiqib, y ning $[-1;1]$ kesmadagi har bir qiymatiga $\sin x = y,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ shartlarni qanoatlantiruvchi birgina x sonni, ya'ni $x = \arcsin y$ sonni mos qo'yish mumkinligi kelib chiqadi.

Har bir $y \in [-1;1]$ songa uning arksinusini mos qo'yib, quyidagi funksiyaga ega bo'lamiz.

$x = \arcsin y,\ -1 \le y \le 1$

x *va* y o'zgaruvchilarning $y=\sin x,\ -\frac{\pi}{2}\le x\le\frac{\pi}{2}$ shartni qanoatlantiruvchi har qanday qiymatlar jufti $x=\arcsin y,\ -1\le y\le 1$ shartni ham qanoatlantiradiva aksincha. Bundan esa yuqoridagi funksiyalar o'zaro teskari funksiyalar ekanligi kelib chiqadi. Odatda funksiyaning argumenti x va funksiyaning o'zi esa y bilan belgilangani uchun, ushbu teskari funksiyani ham $y=\arcsin x,\ -1\le x\le 1$ kabi yozish bir muncha qulaydir.

Endi $y=\arcsin x,\ -1\le x\le 1$ funksiyaning ba'zi xossalarini ko'rib chiqamiz.

1^0 $y=\arcsin x$ funksiyaning aniqlanish sohasi $-1\le x\le 1$ kesmadan iborat.

2^0 $y=\arcsin x$ funksiyaning qiymatlar sohasi $[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmadan iborat.

3^0 $y=\arcsin x$ funksiya $-1\le x\le 1$ kesmada o'sadi.

4^0 $y=\arcsin x$ funksiya toq funksiya, ya'ni $\arcsin(-x)=-\arcsin x$ tenglik barcha $-1\le x\le 1$ lar uchun o'rinli bo'ladi.

5^0 $y=\arcsin x$ funksiya davriy funksiya emas.

Ushbu xossalarning asosiy qismi $y=\arcsin x$ funksiya $y=\sin x$ funksiya teskari funksiya ekanligidan kelib chiqadi.

Shu o'rinda quyidagi takidlab o'tish lozim, ya'ni $y=\sin x$ funksiya $(-\infty;\infty)$ oraliqda teskarilanuvchi emas, chunki har qanday $y\in[-1;1]$ songa $\sin x=y$ shartni qanoatlantiruvchi cheksiz ko'p sonlar mos keladi. $y=\sin x$ funksiyaning teskarilanuvchi bo'lishligini ta'minlash uchun uning aniqlanish sohasini toraytiramiz. Ya'ni aniqlanish sohasi sifatida $[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmani olamiz.

Huddi yuqoridagi kabi mulohazalar yuritish orqali $y=\cos x$ teskaruvchanligi va unga teskari funksiyani kiritamiz.

Ushbu $y=\cos x,\ 0\le x\le\pi$ funksiyani qaraymiz.

x argumentning qiymatlari 0 *dan* π gacha o'sib borganda y ning qiymatlari -1 *dan* 1 gacha kamayib borishi va $[-1;1]$ kesmani to'ldirishi bizga ma'lum. Shundan kelib chiqib, y ning $[-1;1]$ kesmadagi har bir qiymatiga

$\cos x=y,\ 0\le x\le\pi$ shartlarni qanoatlantiruvchi birgina x sonni, ya'ni $x=\arccos y$ sonni mos qo'yish mumkinligi kelib chiqadi.

Har bir $y\in[-1;1]$ songa uning arccosinusni mos qo'yib, quyidagi funksiyaga ega bo'lamiz.

$$x=\arccos y,\ -1\le y\le 1$$

x *va* y o'zgaruvchilarning $y=\cos x,\ 0\le x\le\pi$ shartni qanoatlantiruvchi har qanday qiymatlar jufti $x=\arccos y,\ -1\le y\le 1$ shartni ham qanoatlantiradiva aksincha. Bundan esa yuqoridagi funksiyalar o'zaro teskari funksiyalar ekanligi kelib chiqadi. Odatda funksiyaning argumenti x va funksiyaning o'zi esa y bilan belgilangani uchun, ushbu teskari funksiyani ham $y=\arccos x,\ -1\le x\le 1$ kabi yozish bir muncha qulaydir.

Endi ushbu funksiyaning ham xossalarini ko'rib chiqamiz.

1^0 $y=\arccos x$ funksiyaning aniqlanish sohasi $-1\le x\le 1$ kesmadan iborat.

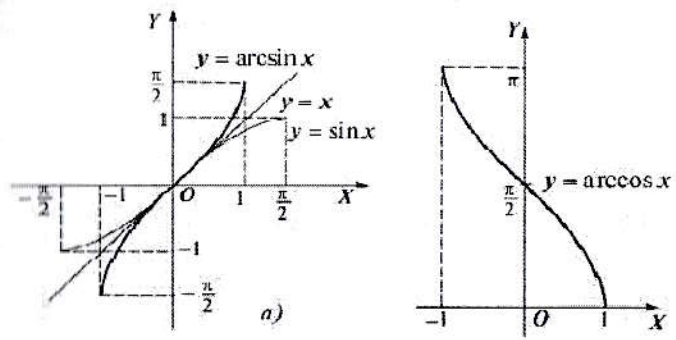
2^0 $y=\arccos x$ funksiyaning qiymatlar sohasi $[0;\pi]$ kesmadan iborat.

3^0 $y=\arccos x$ funksiya $-1\le x\le 1$ kesmada kamayadi.

4^0 $y=\arccos x$ funksiya juft ham emas toq ham emas, sababi $\arccos(-x)=\pi-\arccos x$ tenglik barcha $-1\le x\le 1$ lar uchun o'rinli bo'ladi.

5^0 $y=\arccos x$ funksiya davriy funksiya emas.

Endi $y=\arcsin x$ *va* $y=\arccos x$ funksiyalarning grafigi va asosiy xossalarining jadvval shaklini keltiramiz.



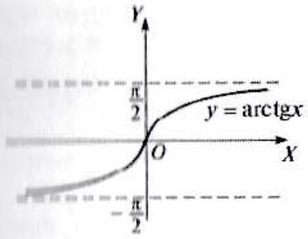
Yuqorida arcsinus va arkkosinusning grafiklari keltirilgan, endi asosiy xossalarini jadval ko'rinishida keltirb o'tamiz.

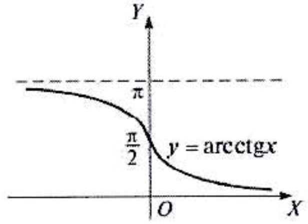
	$y = \arcsin x$	$y = \arccos x$
Aniqlanish sohasi	$[-1;1]$	$[-1;1]$
Qiymatlar sohasi	$[-\frac{\pi}{2};\frac{\pi}{2}]$	$[0;\pi]$
Monotonligi	$[-1;1]$ oraliqda o'suvchi	$[-1;1]$ oraliqda kamayuvchi
Juft- toqligi	Toq funksiya	Juft ham emas toq ham emas
Davriyligi	Davriy emas	Davriy emas

Huddi yuqoridagi kabi $y = tgx$ va $y = ctgx$ funksiyalar uchun ham teskari funksiyalar keltirilib chiqariladi va $y = tgx$ funksiyaga $y = arctgx$ funksiya va $y = ctgx$ funksiyaga esa $y = arcctgx$ funksiya teskari funksiya deyiladi. Ushbu teskari trigonometric funksiyalarning asosiy xossalarini huddi yuqoridagi kabi jadval shaklida berib ketamiz.

	$y = arctgx$	$y = arcctgx$
Aniqlanish sohasi	$(-\infty;\infty)$	$(-\infty;\infty)$
Qiymatlar sohasi	$(-\frac{\pi}{2};\frac{\pi}{2})$	$(0;\pi)$
Monotonligi	$(-\infty;\infty)$ oraliqda o'suvchi	$(-\infty;\infty)$ oraliqda kamayuvchi
Juft- toqligi	Toq funksiya	Juft ham emas toq ham emas
Davriyligi	Davriy emas	Davriy emas

Mos ravishta $y = arctgx$ va $y = arcctgx$ funksiyalarning grafiklari quyida keltirilgan.





Teskari trigonometrik funksiyalar qatnashgan ayrim ayniyatlar

Teskari trigonometric funksiyalarga berilgan ta'rifga ko'ra, masalan, $y = \sin x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ va $x = \arcsin y$, bir ma'noli munosabatlardir. Agar $y = \sin x$ tenglikda x o'rniga $x = \arcsin y$ qo'yilsa, ushbu $\sin(\arcsin y) = y,\ -1 \le y \le 1$ ayniyat paydo bo'ladi.

Shu ta'riqa quyidagi ayniyatlarni ham keltirib chiqarishimiz mumkin.

$$\cos(\arccos y) = y,\ -1 \le y \le 1$$
$$tg(arctgy) = y,\ -\infty < y < \infty$$
$$ctg(arcctgy) = y,\ -\infty < y < \infty$$

Agar $x = \arcsin y$ tenglikda y o'rniga $\sin x$ qo'yilsa:

$$\arcsin(\sin x) = x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$
$$\arccos(\cos x) = x,\ 0 \le x \le \pi$$
$$arctg(tgx) = x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$$
$$arcctg(ctgx) = x,\ 0 < x < \pi$$

Bo'lib ushbu ayniyatlardan foydalangan holda yana bir nechta ayniyatlar va tengliklarni keltirib chiqarishimiz mumkin.

Misollar:

1) $\arcsin(\cos x)$ ifodaning qiymatini toping.

Javob: ushbu ifoda qiymatini topish uchun quyidagicha almashtirish bajaramiz. $\arcsin(\cos x) = \arcsin(\sin(\frac{\pi}{2} - x)) = \frac{\pi}{2} - x$ ga teng bo'ladi.

2) $\sin(\arccos x)$ ifoda qiymatini toping.

Javob: ushbu ifoda qiymatini topish uchun ham almashtirish bajaramiz, almashtirish quyidagicha bajariladi.

$\sin(\arccos x)=\sqrt{1-\cos^2(\arccos x)}=\sqrt{1-(\cos(\arccos x))^2}=\sqrt{1-(x)^2}=\sqrt{1-x^2}$ ga teng bo'ladi.

3) $\arcsin x+\arccos x=\frac{\pi}{2}$ ekanligini isbotlang.

Javob: ayniyatni isbotlash uchun tenglikni har ikki qismini siniuslab yuboramiz va ikki burchak yig'indisi uchun formuladan quyidagiga ega bo'lamiz.

$\sin(\arcsin x+\arccos x)=\sin\frac{\pi}{2}\Rightarrow \sin(\arcsin x)\cos(\arccos x)+\cos(\arcsin x)\sin(\arccos x)=1$

ushbu ifodani soddalshtirish uchun yuqorida ikkinchi misolda isbotlagan tenglikdan foydalanamiz.

$\sin(\arcsin x)\cos(\arccos x)+\cos(\arcsin x)\sin(\operatorname{arc}\cos x)=x\cdot x+\sqrt{1-x^2}\cdot\sqrt{1-x^2}=x^2+1-x^2=1$

bo'lib bu esa tenglikning to'g'riligini isbotlaydi.

4) $\sin^2(arctg3-arctg(-\frac{1}{2}))$ ni xisoblang.

Javob: ushbu ifoda qiymatini topish ikki burchak yig'indisi uchun formulalardan foydalanamiz, ya'ni $\sin^2(arctg3-arctg(-\frac{1}{2}))=\sin^2(arctg3+arctg\frac{1}{2})=$ $(\sin(arctg3)\cos(arctg\frac{1}{2})+\sin(arctg\frac{1}{2})\cos(arctg3))^2$ bo'lib, sinus va tangens funksiyalarning, kosinus va tangens funksiyalarning o'zaro bog'liqliklari formulasidan ifodani soddalashtirib olamiz. $\sin x=\frac{tgx}{\sqrt{1+tg^2x}}$ va $\cos x=\frac{1}{\sqrt{1+tg^2x}}$,

$$(\sin(arctg3)\cos(arctg\frac{1}{2})+\sin(arctg\frac{1}{2})\cos(arctg3))^2=(\frac{tg(arctg3)}{\sqrt{1+tg^2(arctg3)}}\cdot\frac{1}{\sqrt{1+tg^2(arctg\frac{1}{2})}}+$$

$$+\frac{tg(arctg\frac{1}{2})}{\sqrt{1+tg^2(arctg\frac{1}{2})}}\cdot\frac{1}{\sqrt{1+tg^2(arctg3)}})^2=(\frac{3}{\sqrt{10}}\cdot\frac{2}{\sqrt{5}}+\frac{1}{\sqrt{5}}\cdot\frac{1}{\sqrt{10}})^2=(\frac{7}{\sqrt{50}})^2=\frac{49}{50}$$

Teskari trigonometrik tenglama va tengsizliklar Teskari trigonometric tenglamalarni yechishning turli usullari va berilgan misol

turiga qarab yechilish yo'llarining farqlanishini inobatga olgan holda, ushbu mavzuni yoritishda bir nechta misollar yordamida tushuntirib o'tamiz.

Misollar:

1) $\arccos\frac{3}{5}x+\arccos\frac{4}{5}x=\arccos x$ tenglamani yeching.

Javob: ushbu tenglamani yechish uchun tenglikning har ikki qismini kosinuslaymiz va soddalshtirib oddiy tenglama yechimini topamiz. $\cos(\arccos\frac{3}{5}x+\arccos\frac{4}{5}x)=\cos(\arccos x)$ tenglikning chap qismini ikki burchk yig'indisi formulasidan foydalanib yoyamiz va soddalashtiramiz. $\cos(\arccos\frac{3}{5}x+\arccos\frac{4}{5}x)=\cos(\arccos\frac{3}{5}x)\cos(\arccos\frac{4}{5}x)-\sin(\arccos\frac{3}{5}x)\sin(\arccos\frac{4}{5}x)$ ohirgi tenglikni soddalashtirish uchun yuqoridagi teskari trigonometric funksiyalar uchun formulalardan foydalanamiz.

$\cos(\arccos\frac{3}{5}x)\cos(\arccos\frac{4}{5}x)-\sin(\arccos\frac{3}{5}x)\sin(\arccos\frac{4}{5}x)=$

$=\frac{3}{5}x\cdot\frac{4}{5}x-\sqrt{1-\frac{9}{25}x}\cdot\sqrt{1-\frac{16}{25}x}=\frac{12}{25}x-\frac{4}{5}x\cdot\frac{3}{5}x=0$ bo'lib bu esa tenglamaning yechimi $x=0$ ekanligini bildiradi.

2) $4\arcsin^2x-5\arcsin x+1=0$ tenglama yechimini toping.

Javob: ushbu tenglamani yechish uchun belgilash kiritib ishlanadigan tenglamalar yoki (bikvadrat tenglamalar) dan foydalanib quyidagicha belgilash kiritamiz. $\arcsin x=t$ bundan esa quyidagiga kelamiz. $4\arcsin^2x-5\arcsin x+1=0\Rightarrow 4t^2-5t+1=0\Rightarrow t_1=\frac{1}{4}$ va $t_2=1$ bo'lib bundan esa tenglamaning ildizlari, $\arcsin x=\frac{1}{4}$ va $\arcsin x=1$ tenglamaning yechimlaridan iborat bo'ladi. Bu yerda $\frac{1}{4}$ va 1 sonlari $[-\frac{\pi}{2};\frac{\pi}{2}]$ oraliqqa tegishli bo'lganligi uchun, $x_1=\sin\frac{1}{4}$ va $x_2=\sin 1$ ga teng bo'ladi.

3) $\frac{6}{\pi^2}\arccos^2x-\frac{5}{\pi}\arccos x+1=0$ tenglamani yeching.

Javob: ushbu tenglamani yechimini topish uchun ham yuqoridagi kabi belgilash kiritamiz va $\frac{1}{\pi}\arccos x=t$, $6t^2-5t+1=0$ kabi tenglamaga kelamiz.

Kvadrat tenglamadan yechimini topsak, u holda $t_1=\frac{1}{3},\ t_2=\frac{1}{2}$ yechimlarga kelamiz. $\frac{1}{\pi}\arccos x=\frac{1}{3},\ \frac{1}{\pi}\arccos x=\frac{1}{3}\Rightarrow \arccos x=\frac{\pi}{3}$ *va* $\arccos x=\frac{\pi}{2}$ bo'lib bundan esa $x_1=\cos\frac{\pi}{3}=\frac{1}{2},\ x_2=\cos\frac{\pi}{2}=0$ bo'ladi.

Mustaqil yechish uchun misollar:

1-variant

1) Quyidagi gradus o'lchovi bilan berilgan burchaklarni radian o'lchoviga almashtiring. $150^0, 210^0, 225^0$

2) quyidagi funksiyalarning aniqlanish sohalarini toping. $y=\arcsin\frac{x+1}{2x-4}$

3) Quyidagi ifodalarni qiymatini toping. $\arcsin 0+\arccos 1$

4) quyidagi tenglamalrni yeching $\arcsin(\frac{x}{12}+36)=0$

2-variant

1) Quyidagi gradus o'lchovi bilan berilgan burchaklarni radian o'lchoviga almashtiring. $330^0, 240^0, 300^0$

2) quyidagi funksiyalarning aniqlanish sohalarini toping $y=\arccos\frac{x}{7x-3}$

3) Quyidagi ifodalarni qiymatini toping. $\arcsin\frac{1}{2}-\arccos\frac{\sqrt{3}}{2}$

4) quyidagi tenglamalrni yeching $\arccos(\frac{x}{2}-3)=1$

3-variant

1) Quyidagi gradus o'lchovi bilan berilgan burchaklarni radian o'lchoviga almashtiring. $315^0, 450^0, 480^0$

2) quyidagi funksiyalarning aniqlanish sohalarini toping $y=\arcsin x+\arccos\frac{x+1}{x-1}$

3) Quyidagi ifodalarni qiymatini toping. $\arccos 0+arctg1$

4) quyidagi tenglamalrni yeching $arctg(5x-1)=\frac{\pi}{2}$

4-variant

1) Quyidagi gradus o'lchovi bilan berilgan burchaklarni radian o'lchoviga almashtiring. $510^0, 540^0, 570^0$

2) quyidagi funksiyalarning aniqlanish sohalarini toping. $y=\arcsin(x^2+2x+2)+arctgx$

3) Quyidagi ifodalarni qiymatini toping. $2\arcsin\frac{\sqrt{2}}{2}-\arccos 0$

4) quyidagi tenglamalrni yeching $arcctg(\frac{2x+16}{3})=\frac{\pi}{2}$

5-variant

1) Quyidagi gradus o'lchovi bilan berilgan burchaklarni radian o'lchoviga almashtiring. $600^0, 720^0, 900^0$

2) quyidagi funksiyalarning aniqlanish sohalarini toping. $y=\arccos(5x^2+6x)$

3) Quyidagi ifodalarni qiymatini toping. $5\arccos\frac{\sqrt{3}}{2}-3\arccos(-\frac{\sqrt{2}}{2})$

4) quyidagi tenglamalrni yeching $\frac{2}{\pi^2}\arccos^2 x+\frac{5}{\pi}\arccos x+2=0$

6-variant

1) Quyidagi gradus o'lchovi bilan berilgan burchaklarni radian o'lchoviga almashtiring. $900^0, 990^0, 1080^0$

2) quyidagi funksiyalarning aniqlanish sohalarini toping $y = arcctg\sqrt{x^2+3x+2}$

3) Quyidagi ifodalarni qiymatini toping. $\arccos\frac{1}{3}+\arccos(-\frac{1}{3})$

4) quyidagi tenglamalrni yeching $arctgx - arcctgx = 0$

7-variant

1) Quyidagi gradus o'lchovi bilan berilgan burchaklarni radian o'lchoviga almashtiring. $1500^0, 2190^0, 2250^0$

2) quyidagi funksiyalarning aniqlanish sohalarini toping. $y = arctg\sqrt{x^2+7x+6} + arctg\frac{1}{x+6}$

3) Quyidagi ifodalarni qiymatini toping. $\arcsin\frac{1}{5}+\arccos\frac{4}{5}$

4) quyidagi tenglamalrni yeching $\arcsin x + \arccos x = 0$

8-variant

1) Quyidagi gradus o'lchovi bilan berilgan burchaklarni radian o'lchoviga almashtiring. $3360^0, 2400^0, 3030^0$

2) quyidagi funksiyalarning aniqlanish sohalarini toping $y = \frac{\arcsin(5x+12)}{\sqrt{x^2-5x+4}}$

3) Quyidagi ifodalarni qiymatini toping. $\arcsin 0 + \arccos 1$

4) quyidagi tenglamalrni yeching $\arcsin x - \arccos x = 0$

9-variant

1) Quyidagi gradus o'lchovi bilan berilgan burchaklarni radian o'lchoviga almashtiring. $3150^0, 4545^0, 4845^0$

2) quyidagi funksiyalarning aniqlanish sohalarini toping. $y = \arcsin(x^2-4x+8)$

3) Quyidagi ifodalarni qiymatini toping. $\arcsin(-\frac{3}{4}) + \arcsin\frac{\sqrt{7}}{4}$

4) quyidagi tenglamalrni yeching $\pi^2\arccos^2 x + 2\pi\arccos x + 1 = 0$

10-variant

1) Quyidagi gradus o'lchovi bilan berilgan burchaklarni radian o'lchoviga almashtiring. $5190^0, 5400^0, 5790^0$

2) quyidagi funksiyalarning aniqlanish sohalarini toping. $y = arctg(\frac{1}{2\sin x})$

3) Quyidagi ifodalarni qiymatini toping. $4\arccos\frac{\sqrt{3}}{2} + 3\arcsin\frac{1}{2}$

4) quyidagi tenglamalrni yeching $\arcsin^2 x + 7\arcsin x + 12 = 0$

11-variant

1) Quyidagi gradus o'lchovi bilan berilgan burchaklarni radian o'lchoviga almashtiring. $7230^0, 9000^0, 9900^0$

2) quyidagi funksiyalarning aniqlanish sohalarini toping. $y = \cos\frac{2x+1}{x^2+3x+2}$

3) Quyidagi ifodalarni qiymatini toping. $arctg0 + arcctg1$

4) quyidagi funksiyalarning qiymatlar sohasini toping $y = 1 + \sin x$

12-variant

1) Quyidagi gradus o'lchovi bilan berilgan burchaklarni radian o'lchoviga almashtiring. $6045^0, 10830^0$

2) quyidagi funksiyalarning aniqlanish sohalarini toping. $y = \sqrt{\sin x - 2}$

3) quyuda berilgan funksiyaning juft yoki toq $y = x^2 + \cos x$

ekanligini aniqlang.

4) quyidagi funksiyalarning qiymatlar sohasini toping $y = 1 + |\cos 2x|$

13-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. $\frac{11\pi}{6}, \frac{3\pi}{4}$

2) quyidagi funksiyalarning aniqlanish sohalarini toping $y = \sin\sqrt{2x-4}$

3) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang. $y = 5 - \cos 3x$

4) quyidagi funksiyalarning qiymatlar sohasini toping $y = 1 - 5\cos 3x$

14-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. $\frac{11\pi}{6}, \frac{3\pi}{4},$

2) quyidagi funksiyalarning aniqlanish sohalarini toping $y = \frac{1}{2\sin x - 1}$

3) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang. $y = \cos 2x - \sin^2 x$

4) quyidagi funksiyalarning qiymatlar sohasini toping $y = \cos^2 x + \cos x + 1$

15-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. $\frac{5\pi}{6}, \frac{7\pi}{6}$

2) quyidagi funksiyalarning aniqlanish sohalarini toping $y = \cos\sqrt{\frac{2x+3}{x-1}}$

3) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang. $y = tg 2x - ctg\frac{2}{3}x + \sin 7x$

4) quyidagi funksiyalarning qiymatlar sohasini toping $y = 3 - 2\sin^2 x$

16-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. $\frac{4\pi}{3}, \frac{5\pi}{3}$

2) quyidagi funksiyalarning aniqlanish sohalarini toping $y = \frac{2}{\cos^2 x - \sin^2 x}$

3) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang $y = tg^2 x + \cos 2x$

4) quyidagi funksiyalarning qiymatlar sohasini toping $y = 2 + 4\cos^2 x$

17-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. $\frac{5\pi}{4}, \frac{7\pi}{4}$

2) quyidagi funksiyalarning aniqlanish sohalarini toping $y = \sqrt{\sin 2x}$

3) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang $y = \sin(x^4 + 3x^2) - ctg^2 x$

4) quyidagi funksiyalarning qiymatlar sohasini toping $y = \frac{1 + 2\sin x}{3}$

18-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. $\frac{7\pi}{3}, \frac{7\pi}{12}$

2) quyidagi funksiyalarning aniqlanish sohalarini toping $y=\frac{1}{\cos^3 x \sin x}$

3) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang. $y=\sin\left(x^3+3x\right)-tg2x$

4) quyidagi funksiyalarning qiymatlar sohasini toping $y=1+2|\sin 3x|$

19-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. $\frac{5\pi}{54}, \frac{11\pi}{36}$

2) quyidagi funksiyalarning aniqlanish sohalarini toping $y=tg\frac{2x-1}{3}$

3) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang. $y=\sin x+tg(3x-2)+\cos 5x$

4) quyidagi funksiyalarning qiymatlar sohasini toping $y=4tgx$

20-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. $\frac{5\pi}{18}, \frac{5\pi}{9}$

2) quyidagi funksiyalarning aniqlanish sohalarini toping $y=tg\sqrt{x}+\sin x$

3) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang. $y=\sin x-\cos x$

4) quyidagi funksiyalarning qiymatlar sohasini toping $y=3^{\sin x}+2$

21-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. $\frac{5\pi}{18}, \frac{5\pi}{9}$

2) quyida berilgan funksiyalarning grafiklarini, $y=\sin x,\ y=\cos x,\ y=tgx\ va\ y=ctgx$ funksiyalarning garfiklaridan foydalanib chizing va monotonlik oraliqlarini, juft yoki toqligini, aniqlanish va qiymatlar sohasini toping. $y=2\sin x$

3) Quyida keltirilgan ifodalarning ishorasini aniqlang. $\cos 225^0+\sin(-145^0)+tg480^0$

4) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang. $y=tg2x-ctg\frac{2}{3}x+\sin 7x$

22-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. $\frac{13\pi}{20}, \frac{14\pi}{36}$

2) quyida berilgan funksiyalarning grafiklarini, $y=\sin x,\ y=\cos x,\ y=tgx\ va\ y=ctgx$ funksiyalarning garfiklaridan foydalanib chizing va monotonlik oraliqlarini, juft yoki toqligini, aniqlanish va qiymatlar sohasini toping $\sin 5\frac{1}{3}\pi+\cos 4\frac{2}{3}\pi$

3) Quyida keltirilgan ifodalarning ishorasini aniqlang.. $\arcsin 0+\arccos 1$

4) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang $y=tg^2x+\cos 2x$

23-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. $\frac{13\pi}{20}, \frac{14\pi}{36}$

2) quyida berilgan funksiyalarning grafiklarini, $y=\sin x,\ y=\cos x,\ y=tgx\ va\ y=ctgx$ funksiyalarning garfiklaridan foydalanib chizing va monotonlik oraliqlarini, juft yoki toqligini, aniqlanish va qiymatlar sohasini toping — $y=\sin\frac{1}{2}x$

3) Quyida keltirilgan ifodalarning ishorasini aniqlang. — $\sin 2\pi+\cos 3\pi-tg\pi$

4) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang — $y=\sin(x^4+3x^2)-ctg^2x$

24-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. — $\frac{17\pi}{30}, \frac{\pi}{90}$

2) quyida berilgan funksiyalarning grafiklarini, $y=\sin x,\ y=\cos x,\ y=tgx\ va\ y=ctgx$ funksiyalarning garfiklaridan foydalanib chizing va monotonlik oraliqlarini, juft yoki toqligini, aniqlanish va qiymatlar sohasini toping — $y=\cos x+2$

3) Quyida keltirilgan ifodalarning ishorasini aniqlang. — $\cos 3630^0+\sin 5460^0+tg1850^0$

4) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang. — $y=\sin\left(x^3+3x\right)-tg2x$

25-variant

1) Quyidagi radian o'lchovi bilan berilgan burchaklarni gradus o'lchovini toping. — $\frac{17\pi}{30}, \frac{\pi}{90}$

2) quyida berilgan funksiyalarning grafiklarini, $y=\sin x,\ y=\cos x,\ y=tgx\ va\ y=ctgx$ funksiyalarning garfiklaridan foydalanib — $y=tg2x$

chizing va monotonlik oraliqlarini, juft yoki toqligini, aniqlanish va qiymatlar sohasini toping

3) Quyida keltirilgan ifodalarning ishorasini aniqlang. — $\sin 120^0+\cos(-60^0)$

4) quyuda berilgan funksiyaning juft yoki toq ekanligini aniqlang. — $y=\sin x-\cos x$

11-mavzu: Funksiya limiti. Aniqmas ifodalar va ularni elementar usullarda ochish. Ajoyib limitlar.

Funksiya limiti

> ***Ta'rif.*** Ixtiyoriy $\forall\varepsilon>0$ son uchun shunday $\delta(\varepsilon)$ son topilib, argument x ning $|x-a|<\delta(\varepsilon)$ tengsizlikni qanoatlantiruvchi a dan farqli barcha qiymatlarida $f(x)$ funksiya $|f(x)-A|<\varepsilon$ tengsizlikni qanoatlantirsa, u holda x argument a ga intilganda $f(x)$ *funksiya A ga teng limitga ega deyiladi.*

Misollar:

1. $\lim\limits_{x\to3}(x^3+x-5)=25$ ekanligini limit ta'rifi yordamida isbotlang.

$\forall\varepsilon>0$ son uchun $\exists\delta(\varepsilon)$ son topilib, $|x-3|<\delta$ tengsizlik bajarilganda, $|x^3+x-30|<\varepsilon$ tengsizlik bajarilishi kerak.

$$|x^3-27+x-3|=|(x-3)(x^2+3x+9)+(x-3)|=|(x-3)(x^2+3x+10)|=$$

$$|(x-3)(x^2-6x+9+9x+1)|=|(x-3)((x-3)^2+9(x-3)+28)|<|\delta^3+9\delta^2+28\delta|<$$

$$|\delta^3+9\delta^2+27\delta+27|=|(\delta+3)^3|=\varepsilon\Rightarrow\delta+3=\sqrt[3]{\varepsilon}\Rightarrow\delta=\sqrt[3]{\varepsilon}-3$$

bu esa tenglik to'g'ri ekanligini isbotlaydi. Chunki funksiya limiti ta'rifida $\forall\varepsilon>0$ son uchun $\exists\delta(\varepsilon)$ topilib ya'ni δ son ε soniga bog'liq bo'lishi lozim. Ushbu misolda ham δ son ε songa bog'liq ravishta kelib chiqadi. Demak, haqiqatda ham yuqoridagi limit o'rinli.

Limitlar haqida teoremalar

> **Teorema 1.** Agar argument $x\to a$ da $f(x)$ va $g(x)$ funksiyalar limitga ega bo'lsa, u holda bu funksiyalarning yig'indisi va ayirmasi ham limitga ega bo'lib, bu limit $f(x)$ va $g(x)$ funksiyalar limitlarining yig'indisi va ayirmasiga teng bo'ladi
>
> $\lim\limits_{x\to a}(f(x)\pm g(x))=\lim\limits_{x\to a}f(x)\pm\lim\limits_{x\to a}g(x).$

> **Teorema 2.** Agar argument $x\to a$ da $f(x)$ va $g(x)$ funksiyalar limitga ega bo'lsa, u holda bu funksiyalarning ko'paytmasi ham limitga ega bo'lib, bu limit $f(x)$ va $g(x)$ funksiyalar limitlarining ko'paytmasiga teng bo'ladi.
>
> $$\lim_{x\to a}f(x)\cdot g(x)=\lim_{x\to a}f(x)\cdot\lim_{x\to a}g(x).$$
>
> **Teorema 3.** Agar argument $x\to a$ da $f(x)$ va $g(x)$ funksiyalar limitga ega bo'lib, $g(x)$ funksiya limiti noldan farqli bo'lsa $\dfrac{f(x)}{g(x)}$ nisbatning ham limiti mavjud bo'lib, uning limiti $f(x)$ va $g(x)$ funksiyalar limitlarining nisbatiga teng
>
> $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{\lim\limits_{x\to a}f(x)}{\lim\limits_{x\to a}g(x)}.$

Teoremalardan kelib chiqadigan natijalar:

1. O'zgarmas ko'paytuvchini limit belgisi oldiga chiqarish mumkin.

$$\lim_{x\to a}k\cdot f(x)=k\cdot\lim_{x\to a}f(x)$$

2. Agar n natural son bo'lsa, u holda

$$\lim_{x\to a}x^n=a^n,\ \lim_{x\to a}\sqrt[n]{x}=\sqrt[n]{a}$$

bo'ladi.

3. Ushbu $P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_n$ ko'phadning $x\to a$ dagi limiti bu ko'phadning $x=a$ dagi qiymatiga teng, ya'ni $\lim\limits_{x\to a}P(x)=P(a)$ ga teng.

4. Ushbu $R(x)=\dfrac{P(x)}{Q(x)}=\dfrac{a_0x^n+a_1x^{n-1}+\ldots a_n}{b_0x^m+b_1x^{m-1}+\ldots b_m}$ kasr ratsional funksiyaning $x\to a$ dagi limiti,agar a bu funksiyaning aniqlanish sohasiga tegishli bo'lsa, bu funksiyaning $x=a$ dagi qiymatiga teng, ya'ni $\lim\limits_{x\to a}R(x)=R(a)$ ga teng.

Misol. $\lim\limits_{x\to\infty}x(\sqrt{4x^2-1}-2x)$ limitni hisoblang.

Yechish: yuqoridagi teoremadan foydalanib argument o'rniga cheksizlikni qo'ysak u holda $\infty - \infty$ ko'rinishidagi aniqmaslik hosil bo'ladi. Bu aniqmaslikni yechish uchun irratsionallikdan qutqarish ishini qilimiz, ya'ni ifodani qo'shmasiga ko'paytirib bo'lamiz.

$$\lim_{x\to\infty} x(\sqrt{4x^2-1}-2x) = \lim_{x\to\infty}\frac{x(\sqrt{4x^2-1}-2x)(\sqrt{4x^2-1}+2x)}{(\sqrt{4x^2-1}+2x)} = \lim_{x\to\infty}\frac{x(4x^2-1-4x^2)}{(\sqrt{4x^2-1}+2x)}$$

$$\lim_{x\to\infty}\frac{-x}{(\sqrt{4x^2-1}+2x)}$$

bundan ohirgi tenglikdan argumentni cheksizlikka intiltirib, limit hisoblanganda $\frac{\infty}{\infty}$ ko'rinishidagi aniqmaslik hosil bo'ladi. Bu aniqmaslikdan qutilish uchun quyidagicha amal bajariladi: $\lim_{x\to\infty}\frac{-x}{(\sqrt{4x^2-1}+2x)} = -\lim_{x\to\infty}\frac{x}{x(\sqrt{4-\frac{1}{x^2}}+2)} = -\lim_{x\to\infty}\frac{1}{\sqrt{4-\frac{1}{x^2}}+2} = -\frac{1}{4}.$

Misol. $\lim_{x\to\infty}\frac{3x^5-2x^2}{x^3+3x-x^5}$ limitni hisoblang.

Yechish: yuqoridagi teoremaga asosan kasr ratsional ko'phadning surat va mahrajiga cheksizlikni qo'yib hisoblasak, u holda $\frac{\infty}{\infty}$ ko'rinishdagi aniqmaslik hosil bo'ladi. Bunday misollarni ishlashda quyidagicha yo'l tutiladi, ya'ni kasrning surat va maxrajidan argumentning eng katta darajasini chiqaramiz.

$$\lim_{x\to\infty}\frac{3x^5-2x^2}{x^3+3x-x^5} = \lim_{x\to\infty}\frac{x^5(3-\frac{2}{x^3})}{x^5(-1+\frac{1}{x^2}+\frac{3}{x^4})} = \lim_{x\to\infty}\frac{(3-\frac{2}{x^3})}{(-1+\frac{1}{x^2}+\frac{3}{x^4})} = \frac{3}{-1} = -3$$

ga teng bo'ladi.

Misol. $\lim_{x\to-2}\frac{\sqrt{2x+9}-\sqrt{x+7}}{x^2+2x+6}$ limitni hisoblang.

Yechish: ushbu limitni hisoblash uchun argument intilayotgan sonni funksiya o'rniga qo'yib tekshiramiz.

$\lim_{x\to-2}\frac{\sqrt{2x+9}-\sqrt{x+7}}{x^2+2x+6} = \frac{\sqrt{2\cdot(-2)+9}-\sqrt{-2+7}}{(-2)^2+2\cdot(-2)+6} = \frac{\sqrt{5}-\sqrt{5}}{6} = \frac{0}{6} = 0$ ekanligi kelib chiqadi va limit nolga teng ekanligi kelib chiqadi.

Misol. $\lim_{x\to3}\frac{\sqrt{2x+3}-3}{\sqrt{x-2}-1}$ limitni hisoblang.

Yechish: Ushbu misolni ishlashda ham huddi yuqoridagi kabi argument intilayotgan sonni funksiya o'rniga qo'yib tekshiramiz. $\frac{\sqrt{2x+3}-3}{\sqrt{x-2}-1} = \frac{\sqrt{6+3}-3}{\sqrt{3-2}-1} = \frac{0}{0}$ ekanligi kelib chiqadi va bu esa aniqmaslikka keladi.

$$\lim_{x\to3}\frac{\sqrt{2x+3}-3}{\sqrt{x-2}-1} = \lim_{x\to3}\frac{(\sqrt{2x+3}-3)(\sqrt{2x+3}+3)(\sqrt{x-2}+1)}{(\sqrt{x-2}-1)(\sqrt{x-2}+1)(\sqrt{2x+3}+3)} =$$

$$\lim_{x\to3}\frac{(2x+3-9)(\sqrt{x-2}+1)}{(x-2-1)(\sqrt{2x+3}+3)} = \lim_{x\to3}\frac{(2x-6)(\sqrt{x-2}+1)}{(x-3)(\sqrt{2x+3}+3)} = \lim_{x\to3}\frac{2(x-3)(\sqrt{x-2}+1)}{(x-3)(\sqrt{2x+3}+3)} =$$

$\lim_{x\to3}\frac{2(\sqrt{x-2}+1)}{\sqrt{2x+3}+3}$ ko'rinishga keladi. Endi argumentning intilayotgan nuqtasini funksiya o'rniga qo'yib hisoblansa, $\lim_{x\to3}\frac{2(\sqrt{x-2}+1)}{\sqrt{2x+3}+3} = \frac{2\cdot2}{6} = \frac{2}{3}$ ga teng bo'ladi.

Demak limit $\frac{2}{3}$ ga teng.

Misol. $\lim_{x\to\infty}\frac{x+\sqrt{9x^2+1}}{2x+\sqrt{x^2-1}}$ limitni hisoblang.

Yechish: Ushbu limitni hisoblash uchun limit ostidagi funksiyaning surat va maxrajidan argumentni qavsdan tashqariga chiqaramiz. Sababi bu funksiya $\frac{\infty}{\infty}$ aniqmaslikka teng bo'lganligi uchun.

$$\lim_{x\to\infty}\frac{x+\sqrt{9x^2+1}}{2x+\sqrt{x^2-1}} = \lim_{x\to\infty}\frac{x(1+\frac{\sqrt{9x^2+1}}{x})}{x(2+\frac{\sqrt{x^2-1}}{x})} = \lim_{x\to\infty}\frac{1+\sqrt{\frac{9x^2+1}{x^2}}}{2+\sqrt{\frac{x^2-1}{x^2}}} = \lim_{x\to\infty}\frac{1+\sqrt{9+\frac{1}{x^2}}}{2+\sqrt{1-\frac{1}{x^2}}} =$$

$\frac{1+3}{2+1}=\frac{4}{3}$. Bundan limitning qiymati $\frac{4}{3}$ ga teng bo'ladi.

Ajoyib limitlar haqida qisqacha tushunchalar. Biz quyida ketma-ketlik va funksiyalarning limitlarida uchraydigan ba'zi bir aniqmasliklarni ko'rib chiqamiz. Hozirga qadar $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty-\infty$ ko'rinishidagi aniqmasliklarni ko'rib, ularni ochish o'rganildi. Endilikda esa $\frac{0}{0}$ aniqmaslikni va shu bilan birga 1^{∞} ko'rinishidagi aniqmasliklarni ham bartaraf qilishni o'rganamiz, bularni maktab o'quvchilariga o'rgatishning oson, qulay, tushunarli va misollardagi tadbiqlarini o'rganib, shu bilan birga bu ketma ketliklarning maktab matematikasida tutgan o'rnini tahlil qilamiz.

Bularni o'rganishni ketma-ketliklar misolida qaraydigan bo'lsak, u holda quyidagi ajoyib limit formulasini keltirish mumkin. $\lim_{n\to\infty}(1+\frac{1}{n})^n=e$ formula ketma-ketliklarning limiti uchun ajoyib limit formulasidir. Endi ushbu formulaning isbotiga to'xtalsak, bu formulani isbotlash uchun $\lim_{n\to\infty}(1+\frac{1}{n})^n$ ko'phadnig darajasini Nyuton binomidan foydalanib ochib chiqamiz.

$$(1+\frac{1}{n})^n=1^n+C_n^{n-1}1^{n-1}\cdot\frac{1}{n}+C_n^{n-2}\cdot1^{n-2}\cdot\frac{1}{n^2}+C_n^{n-3}\cdot1^{n-3}\cdot\frac{1}{n^3}+C_n^{n-4}\cdot1^{n-4}\cdot\frac{1}{n^4}+$$

$$C_n^{n-5}\cdot1^{n-5}\cdot\frac{1}{n^5}+C_n^{n-6}\cdot1^{n-6}\cdot\frac{1}{n^6}+C_n^{n-7}\cdot1^{n-7}\cdot\frac{1}{n^7}+C_n^{n-8}\cdot1^{n-8}C_n^{n-9}\cdot1^{n-9}\cdot\frac{1}{n^9}+\dots$$

$$+C_n^1\cdot1^1\cdot\frac{1}{n^{n-1}}+\frac{1}{n^n}=1+\frac{n!}{(n-1)!}\cdot\frac{1}{n}+\frac{n!}{(n-2)!\cdot2!}\cdot\frac{1}{n^2}+\frac{n!}{(n-3)!\cdot3!}\cdot\frac{1}{n^3}+$$

$$\frac{n!}{(n-4)!\cdot4!}\cdot\frac{1}{n^4}+\frac{n!}{(n-5)!\cdot5!}\cdot\frac{1}{n^5}+\frac{n!}{(n-6)!\cdot6!}\cdot\frac{1}{n^6}+\frac{n!}{(n-7)!\cdot7!}\cdot\frac{1}{n^7}+$$

$$\frac{n!}{(n-8)!\cdot8!}\cdot\frac{1}{n^8}+\frac{n!}{(n-9)!\cdot9!}\cdot\frac{1}{n^9}+\dots+\frac{n!}{(n-1)!}\cdot\frac{1}{n^{n-1}}+\frac{1}{n^n}=1+n\cdot\frac{1}{n}+$$

$$\frac{n(n-1)}{2}\cdot\frac{1}{n^2}+\frac{n(n-1)(n-2)}{6}\cdot\frac{1}{n^3}+\frac{n(n-1)(n-2)(n-3)}{24}\cdot\frac{1}{n^4}+$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)}{120}\cdot\frac{1}{n^5}+\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{720}\cdot\frac{1}{n^6}+$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)}{5040}\cdot\frac{1}{n^7}+$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)(n-7)}{40320}\cdot\frac{1}{n^8}+$$

$$\frac{(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)(n-7)(n-9)}{362880}\cdot\frac{1}{n^9}+\dots+\frac{1}{n^n}$$

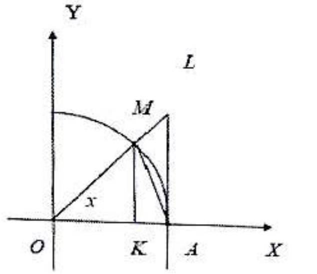
ushbu ifodaning oxirgi tengligidan $n\to\infty$ xar bir hadini limitini hisoblaymiz. $\lim_{n\to\infty}\frac{n(n-1)}{2}\cdot\frac{1}{n^2}=\lim_{n\to\infty}\frac{\left(1-\frac{1}{n}\right)}{2}=\frac{1}{2}$ keyingi hadining limitini hisoblaymiz, u holda $\lim_{n\to\infty}\frac{n(n-1)(n-2)}{6}\cdot\frac{1}{n^3}=\frac{1}{6}$ va xuddi shu kabi keyingi barcha hadlarining limitlari mos ravishda $\frac{1}{24}$, $\frac{1}{120}$, $\frac{1}{720}$, $\frac{1}{5040}$, $\frac{1}{40320}$, $\frac{1}{362880}$ ga teng qiymatlar qabul qiladi. Endi bu qiymatlar yig'indisini hisoblaymiz. U holda

$1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\frac{1}{720}+\frac{1}{5040}+\frac{1}{40320}+\frac{1}{362880}$ ushbu ifodadan yig'indining kasrlar qatnashgan qismini alohida umumiy maxraj tanlab ishlasak quyidagi ifoda hosil bo'ladi.

$$2+\frac{181440+60480+15120+3024+504+72+9+1}{362880}=2+\frac{260650}{362880}\approx$$

$2+0{,}7182815=2{,}7182815\approx e$ ekanligi kelib chiqadi . Ushbu ajoyib limit funksiyalar uchun ham o'rinli bo'ladi.

Birinchi ajoyib limit $\lim_{x\to\infty}\frac{sinx}{x}=1$ ushbu tenglikni isbotlaymiz. Buning uchun koordinatalar sistemasida markazi O nuqtada bo'lgan va radiusi birga teng bo'lgan birlik aylana chizamiz va markaziy burchagi x ga teng bo'lgan yoyni qaraymiz.



Shakldan quyidagilarga ega bo'lamiz. $S_1 = \Delta MOA$, $S_2 = MOA$ sektor, $S_3 = \Delta LOA$ uchburchak va sektorlarning yuzalari bo'lsa, u holda quyidagi tengsizliklar hosil bo'ladi. $S_1 < S_2 < S_3$ endi yuzalarni hisoblaymiz $S_1 = \frac{1}{2} OA \cdot MK = \frac{1}{2} \cdot 1 \cdot sinx = \frac{1}{2} \cdot sinx$, $S_2 = \frac{1}{2} \cdot OA \cdot MA = \frac{1}{2} \cdot 1 \cdot x = \frac{1}{2} x$, $S_3 = \frac{1}{2} \cdot OA \cdot LA = \frac{1}{2} \cdot 1 \cdot tgx = \frac{1}{2} tgx$, $\frac{1}{2} sinx < \frac{1}{2} < \frac{1}{2} tgx$ ekanligi kelib chiqadi.

Tengsizlikni $sinx > 0$ ga bo'lamiz. Bundan esa $1 < \frac{x}{sinx} < \frac{1}{cosx}$ yoki $cosx < \frac{sinx}{x} < 1$. Endi $x < 0$ bo'lsin. $\frac{\sin(-x)}{-x} = \frac{sinx}{x}$, $\cos\cos(-x) = cosx$ ekanligidan $x < 0$ da ham $cosx < \frac{sinx}{x} < 1$ tengsizlik o'rinli bo'ladi va tengsizlikning ikki chetki hadlari 1 ga intilganligi uchun o'rtadagi had ham 1 ga intiladi. Bundan esa $\lim\limits_{x \to \infty} \frac{sinx}{x} = 1$ ekanligi kelib chiqadi. Isbotlangan ikkita ajoyib limitlarga oid misollarni ko'rib chiqamiz.

Misol. $\lim\limits_{n \to \infty} (\frac{2n+1}{2n-1})^{3n-4}$ ketma-ketlik limitini hisoblang.

Yechish: $\lim\limits_{n \to \infty} (\frac{2n+1}{2n-1})^{3n-4} = \lim\limits_{n \to \infty} (\frac{2n-1+2}{2n-1})^{3n-4} = \lim\limits_{n \to \infty} (1 + \frac{2}{2n-1})^{3n-4} =$

$$\lim_{n \to \infty} (1 + \frac{2}{2n-1})^{\frac{2n-1}{2} \cdot \frac{2}{2n-1}(3n-4)} = \lim_{n \to \infty} e^{\frac{2(3n-4)}{2n-1}} = \lim_{n \to \infty} e^{\frac{6-\frac{8}{n}}{2-\frac{1}{n}}} = e^3$$

Misol. $\lim\limits_{n \to \infty} (\frac{n^2-1}{n^2+1})^{3n-n^2}$ ketma-ketlik limitini hisoblang.

Yechish: Bu limitni ham ketma-ketlik uchun ajoyib limit ko'rinishiga keltiramiz, ya'ni

$$\lim_{n \to \infty} (\frac{n^2-1}{n^2+1})^{3n-n^2} = \lim_{n \to \infty} (1 - \frac{2}{n^2+1})^{3n-n^2}$$

keyingi ifodada kavsni ichidagi ifodani yig'indiga keltirish kerak,chunki formula yig'indi bo'lgan holda berilgan.

$$\lim_{n \to \infty} (\frac{n^2-1}{n^2+1})^{3n-n^2} = \lim_{n \to \infty} (1 - \frac{2}{n^2+1})^{3n-n^2} = \lim (1 + (-\frac{2}{n^2+1}))^{3n-n^2} =$$

$$\lim_{n \to \infty} (1 + (-\frac{2}{n^2+1}))^{-\frac{n^2+1}{2}\left(-\frac{2}{n^2+1}\right)3n-n^2} = \lim_{n \to \infty} e^{-\frac{3n-n^2}{n^2+1}} = \lim_{n \to \infty} e^{\frac{1-\frac{3}{n}}{1+\frac{1}{n^2}}} = e .$$

Funksiyaning birinchi ajoyib limitiga oid misollar qaraymiz.

Misol. $\lim\limits_{x \to 0} \frac{x - sinx}{x + sinx}$ funksiya limitini hisoblang.

Yechish: funksiyaning birinchi ajoyib limiti $\lim\limits_{x \to \infty} \frac{sinx}{x} = 1$ ko'rinishda bo'lganligi uchun, yuqoridagi misolni ham shu ko'rinishga keltiramiz. Ya'ni kasrning surat va maxrajini x argumentga bo'lamiz va:

$\lim\limits_{x \to 0} \frac{x - sinx}{x + sinx} = \lim\limits_{x \to 0} \frac{1 - \frac{sinx}{x}}{1 + \frac{sinx}{x}}$ bu tenglikdan esa limit hisoblansa quyidagi ko'rinishga keladi: $\lim\limits_{x \to 0} \frac{x - sinx}{x + sinx} = \lim\limits_{x \to 0} \frac{1 - \frac{sinx}{x}}{1 + \frac{sinx}{x}} = \frac{1-1}{1+1} = \frac{0}{2} = 0$.

Misol. $\lim\limits_{x \to \pi} \frac{\sin 3x}{\sin 2x}$ funksiya limitini hisoblang.

Yechish: Bu funksiyaning limitini topish uchun argumentni nolga intiltirib ajoyib limitga keltirib hisoblaymiz. Buning uchun $x - \pi = t$ belgilash kiritamiz. Bunda $x \to \pi$, $t \to 0$ bo'ladi va argument $x = t + \pi$ endi topilgan ifodalarni yuqoridagi tenglikka olib borib qo'yib quyidagiga ega bo'lamiz.

$\lim\limits_{x\to\pi}\frac{\sin 3x}{\sin 2x} = \lim\limits_{t\to0}\frac{\sin(3\pi+3t)}{\sin(2\pi+2t)}$ bu tenglikda keltirish formulasi yordamida quyidagi tenglikka kelamiz.

$$\lim_{t\to0}\frac{-\sin 3t}{\sin 2t} = -\lim_{t\to0}\frac{\frac{\sin 3t}{3t}\cdot 3t}{\frac{\sin 2t}{2t}\cdot 2t} = -\frac{3}{2}.$$

Misol. $\lim\limits_{x\to0}(\cos 2x)^{1+ctg^2 x}$ funksiya limitini hisoblang.

Yechish: funksiya limitini hisoblashdan oldin trigonometrik funksiyalarning ba'zi xossalaridan foydalanib soddalashtiramiz. U holda ifoda quyidagi ko'rinishga keladi.

Misol. $\lim\limits_{x\to0}(cos2x)^{1+ctg^2x}$ limitni ajoyib limitlar formulasidan foydalanib yeching.

***Yechish*:**

$$\lim_{x\to0}(\cos 2x)^{1+ctg^2 x} = \lim_{x\to0}(1-2\sin^2 x)^{\frac{1}{\sin^2 x}} = \lim_{x\to0}(1+(-2\sin^2 x))^{\frac{1}{-2\sin^2 x}(-2)} = e^{-2}.$$

Misol. $\lim\limits_{x\to\infty}(1+\frac{k}{x})^x$, $k\in R$ limitni hisoblang.

Yechish: ushbu limitni hisoblash uchun ikkinchi ajoyib limit formulasidan foydalanamiz, ya'ni qavs ichidagi ifodaning o'zgaruvchisi bilan ko'rsatkichdagi o'zgaruvchining birhil lekin teskari bo'lishini ta'minlaymiz.

$$\lim_{x\to\infty}(1+\frac{k}{x})^x = \lim_{x\to\infty}(1+\frac{k}{x})^{\frac{x}{k}k} = \lim_{x\to\infty}[(1+\frac{k}{x})^{\frac{x}{k}}]^k = e^k.$$

Misol. $\lim\limits_{x\to0}(1-3x)^{\frac{2}{x}}$ limitni hisoblang.

***Yechish*:** Bunday limitni hisoblash uchun kavs ichidagi ifodani yig'indi ko'rinishiga keltirish va yuqoridagi misol kabi ko'rsatkichini mutanosib qilish kerak. $\lim\limits_{x\to0}(1-3x)^{\frac{2}{x}} = \lim\limits_{x\to0}(1+(-3x))^{\frac{1}{-3x}(-2)} = e^{-2} = \frac{1}{e^2}.$

Misol. $\lim\limits_{x\to\infty}(\frac{5-x}{6-x})^{x+2}$ limitni hisoblang.

***Yechish*:** bu limitni hisoblashda ham ajoyib limitdan foydalanamiz.

$$\lim_{x\to\infty}(\frac{5-x}{6-x})^{x+2} = \lim_{x\to\infty}(\frac{x-5}{x-6})^{x+2} = \lim_{x\to\infty}(\frac{x-6+1}{x-6})^{x+2} = \lim_{x\to\infty}(1+\frac{1}{x-6})^{x+2} =$$

$$\lim_{x\to\infty}(1+\frac{1}{x-6})^{x-6+8} = \lim_{x\to\infty}(1+\frac{1}{x-6})^{x-6}\cdot(1+\frac{1}{x-6})^8 = \lim_{x\to\infty}(1+\frac{1}{x-6})^{x-6}\cdot$$

$$\lim_{x\to\infty}(1+\frac{1}{x-6})^8 = e\cdot1 = e.$$

Mustaqil yechish uchun misollar:

1-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to\infty}(\frac{x^3}{x^2-3}-x)$, b) $\lim\limits_{x\to\infty}\frac{1+x-x^2}{2x^2+3x}$;

2) $\lim\limits_{x\to0}(1+kx)^{\frac{1}{x}}$, $k\in R$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to3}\frac{x^2-9}{x^2-2x-3}$., $\lim\limits_{x\to\pi}\frac{tgx}{\sin 2x}$.

2-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to3}\frac{\sqrt{-3x+11}-\sqrt{x-1}}{x^2-5x+6}$; b) $\lim\limits_{x\to\infty}\frac{2x^3-x+3}{x^3-8x+5}$;

2) $\lim\limits_{x\to0}\sqrt[x]{1+5x}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{\cos 2x}$. $\lim\limits_{x\to0}\frac{x}{\sqrt{1+3x}-1}$.

3-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to-\infty}(\sqrt{x^2+2}-x)$; b) $\lim\limits_{x\to\infty}\frac{x+5x^2-x^3}{2x^3-x^2+7x}$;

2) $\lim\limits_{x\to0}\sqrt[2x]{1+3x}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$., $\lim\limits_{x\to0}\frac{\sqrt[3]{1+mx}-1}{x}$.

4-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to1}\frac{\sqrt{2-x}-1}{\sqrt{5-x}-2}$;b) $\lim\limits_{x\to\infty}\frac{1-3x^2}{x^2+7x-2}$;

2) $\lim\limits_{x\to0}\sqrt[2x]{1+3x}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$., $\lim\limits_{x\to\pi}\frac{\sqrt{1-tgx}-\sqrt{1+tgx}}{\sin2x}$.

5-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to4}\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$, b) $\lim\limits_{x\to\infty}\frac{x^3+x}{x^4-3x^2+1}$;

2) $\lim\limits_{x\to0}(1+kx)^{\frac{1}{x}}$, $k\in R$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to\infty}\frac{5x^3-7x}{1-2x^3}$., $\lim\limits_{x\to\infty}\frac{x^3-1}{x^2+1}$.

6-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to-8}\frac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}$, b) $\lim\limits_{x\to\infty}\frac{x^5-2x}{2x^3+x^2+1}$;

2) $\lim\limits_{x\to0}(\cos2x)^{\frac{1}{\sin^2x}}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}$., $\lim\limits_{x\to-2}\frac{3x+6}{x^3+8}$.

7-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to0}\frac{\sqrt[4]{x+1}-1}{x(1+\sqrt{x})}$, b) $\lim\limits_{x\to\infty}\frac{(x+1)^2(3-4x)^2}{(2x-1)^4}$;

2) $\lim\limits_{x\to\infty}(\frac{4x-2}{4x-3})^{3x-1}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to-1}\frac{x^2-x-2}{x^3+1}$., $\lim\limits_{x\to\pi+0}\frac{\sqrt{1+\cos x}}{\sin x}$.

8-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to-1}\frac{\sqrt[3]{1+2x}+1}{\sqrt{2+x}+x}$, b) $\lim\limits_{x\to\infty}\frac{10+x\sqrt{x}}{x^2}$;

2) $\lim\limits_{x\to\infty}(\frac{x}{2x+1})^x$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to7}\frac{2-\sqrt{x-3}}{x^2-49}$., $\lim\limits_{x\to1}\frac{x^4-2x^3+2x^2-2x+1}{3x^4-5x^3+2x^2-x+1}$.

9-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to1}\frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$, b) $\lim\limits_{x\to\infty}\frac{1+10x}{2x+\sqrt[3]{x^2}}$;

2) $\lim\limits_{x\to\infty}(\frac{x+5}{x-3})^{x+2}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to 5}\frac{\sqrt{6-x}-1}{3-\sqrt{4+x}}$., $\lim\limits_{x\to 1}\frac{3x-2-\sqrt{4x^2-x-2}}{x^2-3x+2}$.

10-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to 0}\frac{\sqrt[3]{8-x}-2}{x}$, b) $\lim\limits_{x\to\infty}\frac{2x^2-3x-4}{\sqrt{x^4+1}}$;

2) $\lim\limits_{x\to\infty}(\frac{3x+2}{3x-1})^{2x+3}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to 0}\frac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{x^2-x}$., $\lim\limits_{x\to\frac{\pi}{2}}\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos x}$.

11-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to 4}\frac{x^2-16}{\sqrt[3]{5-x}-\sqrt[3]{x-3}}$, b) $\lim\limits_{x\to\infty}\frac{10+x\sqrt{2}}{x^2}$;

2) $\lim\limits_{x\to\infty}(\frac{2x+1}{2x-2})^{3x-1}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to\frac{\pi}{2}}\frac{1-\sin x}{(\frac{\pi}{2}-x)}$., $\lim\limits_{x\to 0}\frac{\sqrt{1+\cos x}-\sqrt{2\cos x}}{\sqrt{3+\cos x}-2\sqrt{\cos x}}$.

12-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to -2}(\frac{1}{x+2}-\frac{12}{x^3+8})$, b) $\lim\limits_{x\to\infty}\frac{1+10x}{2x+\sqrt[3]{x^2}}$;

2) $\lim\limits_{x\to\infty}(\frac{x+2}{x+3})^{x+4}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{h\to 0}\frac{\sin(x+h)-\sin(x-h)}{h}$., $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$.

13-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to -7}(\frac{1}{x+7}+\frac{14}{x^2-49})$, b) $\lim\limits_{x\to\infty}\frac{2x^2-3x-4}{\sqrt{x^4+1}}$;

2) $\lim\limits_{x\to\infty}(\frac{-x+6}{-x+4})^{3x-1}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to 0}\frac{\sin 4x}{\sqrt{x+1}-1}$., $\lim\limits_{x\to 0-0}\frac{\sqrt{1-\cos 2x}}{x}$.

14-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to -1}(\frac{1}{x+1}+\frac{2}{x^2-1})$, b) $\lim\limits_{x\to\infty}(\frac{x^3}{x^2+1}-\frac{3x^2}{3x+1})$;

2) $\lim\limits_{x\to\infty}(\frac{4x+10}{4x+5})^{4x-2}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to 0}\frac{2x\cdot\sin x}{\sec x-1}$., $\lim\limits_{x\to 0}\frac{1-\cos 2x+tg^2x}{x\cdot\sin x}$.

15-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to 3}(\frac{1}{x-3}-\frac{27}{x^3-27})$, b) $\lim\limits_{x\to\infty}\frac{2x-3}{2+|x|}$;

2) $\lim\limits_{x\to\infty}(\frac{5x-3}{5x-2})^{-2x-1}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to -2}\frac{\arcsin(x+2)}{x^2+2x}$., $\lim\limits_{x\to 0}\frac{1-\cos 5x}{1-\cos 3x}$.

16-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to\infty}\frac{x+\sqrt{9x^2+1}}{2x+\sqrt{x^2-1}}$, b) $\lim\limits_{x\to\infty}(\sqrt{x^2+2x+2}-\sqrt{x^2-2x-3})$;

2) $\lim\limits_{x\to\infty}(\frac{4x-2}{4x-3})^{3x-1}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to\infty}\frac{5x^2+2^{\frac{1}{x}}}{1-x^2}$., $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin 2x-\cos 2x-1}{\cos x-\sin x}$.

17-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to\infty}\frac{2x-3}{2+|x|}$, b) $\lim\limits_{x\to-\infty}\frac{x+\sqrt{9x^2+1}}{2x+\sqrt{x^2-1}}$;

2) $\lim\limits_{x\to 0}(1-\sin x)^{\frac{1}{\sin x}}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to\frac{1}{5}}\frac{\arcsin(1-2x)}{4x^2-1}$., $\lim\limits_{x\to 0}\frac{1-\cos mx}{x^2}$.

18-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to+\infty}x(\sqrt{x^2+1}-x)$, b) $\lim\limits_{x\to+\infty}(\sqrt{x^2+4}-x)$;

2) $\lim\limits_{x\to 0}(\cos 2x)^{\frac{1}{\sin^2 x}}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to 2}\left[\frac{\sin(x-2)}{x^2-4}+2^{\frac{-1}{(x-2)^2}}\right]$., $\lim\limits_{x\to 0}\frac{tgx-\sin x}{x^3}$.

19-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$, b) $\lim\limits_{x\to\infty}\frac{1+10x}{2x+\sqrt[3]{x^2}}$;

2) $\lim\limits_{x\to 0}(1+tgx)^{ctgx}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{h\to 0}\frac{\cos(x+h)-\cos(x-h)}{h}$., $\lim\limits_{x\to x_0}\frac{tgx-tgx_0}{x-x_0}$.

20-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to-7}(\frac{1}{x+7}+\frac{14}{x^2-49})$, b) $\lim\limits_{x\to\infty}\frac{2x^2-3x-4}{\sqrt{x^4+1}}$;

2) $\lim\limits_{x\to 0}(1+\sin 2x)^{\frac{1}{arctg3x}}$ ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to\infty}\frac{(2x^3+4x+5)(x^2+x+1)}{(x+2)(x^4+2x^3+7x^2+x-1)}$.,

$\lim\limits_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$.

21-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$, b) $\lim\limits_{x\to\infty}\frac{1+10x}{2x+\sqrt[3]{x^2}}$;

2) $\lim\limits_{x\to 0}\frac{1-\cos 2x}{x\cdot\sin x}$. ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to 1}\frac{x^2-x-2}{x^3+1}$., $\lim\limits_{x\to 3}\frac{9-x^2}{\sqrt{3x}-3}$.

22-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$, b) $\lim\limits_{x\to\infty}\frac{1+10x}{2x+\sqrt[3]{x^2}}$;

2) $\lim\limits_{x\to\frac{\pi}{2}}\frac{\cos 5x}{\sin 4x}$. ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to a}\frac{\sqrt{ax}-x}{x-a}$., $\lim\limits_{x\to\infty}\frac{5x^2-3x+2}{2x^2+4x+1}$.

23-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$, b) $\lim\limits_{x\to\infty}\frac{1+10x}{2x+\sqrt[3]{x^2}}$;

2) $\lim\limits_{x\to 0}\dfrac{\sin^2\frac{x}{2}}{x^2}$. ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{\pi-4x}$., $\lim\limits_{x\to\infty}\dfrac{2x^4+3x^2+5x-6}{x^3+3x^2+7x-1}$.

24-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$, b) $\lim\limits_{x\to\infty}\dfrac{1+10x}{2x+\sqrt[3]{x^2}}$;

2) $\lim\limits_{x\to 0}\dfrac{\sin 4x}{x}$. ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2-x}\right)$., $\lim\limits_{x\to 2}\left(\dfrac{1}{x-2}-\dfrac{12}{x^3-8}\right)$.

25-variant

1) Limitlarni hisoblang. a) $\lim\limits_{x\to 1}\dfrac{x^2-x-2}{x^3+1}$., b) $\lim\limits_{x\to 3}\dfrac{9-x^2}{\sqrt{3x}-3}$.;

2) $\lim\limits_{x\to\infty}\left(\dfrac{x^2-2x+1}{x^2-4x+1}\right)^x$. ajoyib limitlarni hisoblang

3) Limitlarni hisoblang $\lim\limits_{x\to-\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-4x}\right)$., $\lim\limits_{x\to 0}\dfrac{\sin 3x}{\sqrt{x+2}-\sqrt{2}}$.

12-mavzu: Hosila tushunchasi. Elementar funksiyalarning hosilalari. Hosilani hisoblashning qoidalari.

Funksiya hosilasi

> ***Ta'rif.*** Agar $\Delta x\to 0$ da $\dfrac{\Delta y}{\Delta x}$ nisbatning limiti
>
> $$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$
>
> mavjud va chekli bo'lsa, bu limit $f(x)$ funksiyaning x_0 nuqtadagi hosilasi deyiladi. $f(x)$ funksiyaning x_0 nuqtadagi hosilasi $f'(x_0)$ yoki $y'_{x=x_0}$ kabi belgilanadi.

Bu ta'rif funksiya hosilasining to'liq ta'rifi bo'lib, nuqtadagi hosila hisoblanadi. Keyin esa to'plamdagi hosila ta'rifi kiritiladi.

Masalan, birinchi elementar funksiyalardan biri bu $y=c$, $c=const$ ya'ni funksiya o'zgarmas miqdor bo'lgan xolda funksiyaning hosilasi nimaga teng?

Ayirmali nisbatni tuzib olamiz $\dfrac{f(x+h)-f(x)}{h}=\dfrac{c-c}{h}=0$ bu ayirmali nisbatimiz h ga bog'liq bo'lmaganligi uchun funksiya o'zgarmas son bo'lganda hosilasi nolga teng bo'lishini ko'rish qiyin emas.

Asosiy elementar funksiyalardan bittasi, bu chiziqli funksiya $f(x)=kx+b$. Shu funksiyaning hosilasini topamiz. Buning uchun ayirmali nisbat tuzamiz

$$\frac{f(x+h)-f(x)}{h}=\frac{k(x+h)+b-kx-b}{h}=\frac{kx+kh+b-kx-b}{h}=\frac{kh}{h}=k.$$

Bundan ko'rinadiki, $f'(x)=k$ tenglik o'rinli.

Keyingi asosiy elementar funksiyalardan biri $f(x)=x^n$ ko'rinishidagi funksiyadir bu funksiyaning hosilasini topamiz.

Avval ayirmali nisbat tuzamiz, $\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^n-x^n}{h}$. Bu ayirmali nisbatning suratidagi darajali qavsni Nyuton binom formulasidan ochib chiqamiz

$$\frac{(x+h)^n-x^n}{h}=\frac{x^n+nx^{n-1}h+\frac{n(n-1)}{2}x^{n-1}h^2+\ldots+h^n+x^n}{h}=$$

$$\frac{nx^{n-1}h+\frac{n(n-1)}{2}x^{n-1}h^2+\ldots+h^n}{h}$$

va bu ifodaning suratidan h ni qavsdan tashqariga chiqarib soddalashtirgach, quyidagiga kelamiz.

$$\frac{nx^{n-1}h+\frac{n(n-1)}{2}x^{n-1}h^2+\ldots+h^n}{h}=\frac{h(nx^{n-1}+\frac{n(n-1)}{2}x^{n-1}h+\ldots+h^{n-1})}{h}=$$

$$nx^{n-1}+\frac{n(n-1)}{2}x^{n-1}h+\ldots+h^{n-1}$$

Yuqoridagi ifodada $h\to0$ ifodaning qiymati quyidagiga teng bo'ladi, bundan ko'rinadiki $f'(x)=nx^{n-1}$ ekanligi

$$nx^{n-1}+\frac{n(n-1)}{2}x^{n-1}h+\ldots+h^{n-1}=nx^{n-1}$$

kelib chiqadi. Shu bilan birga $f(x)=(x+a)^n$ va $f(x)=(ax+b)^n$ ko'rinishdagi funksiya xosilalarini ko'rib chiqamiz va albatta ayirmali nisbat tuzamiz.

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h+a)^n-(x+h)^n}{h}=\frac{((x+a)+h)^n-(x+a)^n}{h}=$$

$$\frac{(x+a)^n+n(x+a)^{n-1}h+\frac{n(n-1)}{2}(x+a)^{n-2}h^2+\ldots+h^n-(x+a)^n}{h}=$$

$$\frac{n(x+a)^{n-1}h+\frac{n(n-1)}{2}(x+a)^{n-2}h^2+\ldots+h^n}{h}=n(x+a)^{n-1}+$$

$$\frac{n(n-1)}{2}(x+a)^{n-2}h+\ldots+h^{n-1}.$$

Bu ifodadan $h\to0$ da quyidagi tenglikka kelamiz $f'(x)=n(x+a)^{n-1}$.

Funksiyaning darajasi natural son emas balki haqiqiy son bo'lgan holda ham yuqoridagi formula o'rinli ekanligini ko'rib chiqamiz. Buning uchun ajoyib limitdan foydalanamiz.

Ya'ni birinchi ajoyib limitdan kelib chiqadigan quyidagi natijadan foydalanamiz. $\lim_{x\to0}\frac{(1+x)^\alpha-1}{x}=\alpha$ formuladan quyidagini keltirib chiqaramiz.

$$\lim_{\Delta x\to0}\frac{(x+\Delta x)^\alpha-x^\alpha}{\Delta x}=\lim_{\Delta x\to0}\frac{x^\alpha(1+\frac{\Delta x}{x})^\alpha-x^\alpha}{\Delta x}=\lim_{\Delta x\to0}\frac{x^\alpha((1+\frac{\Delta x}{x})^\alpha-1)}{\Delta x}=$$

$$x^\alpha\lim_{\Delta x\to0}\frac{((1+\frac{\Delta x}{x})^\alpha-1)}{\frac{\Delta x}{x}\Delta x}=x^\alpha\Delta\alpha\Delta\frac{1}{x}=\alpha\Delta x^{\alpha-1}$$

bundan esa yuqoridagi formula darajali funksiyaning darajasi ixtiyoriy son bo'lgan holda ham o'rinli ekanligi kelib chiqadi.

Ushbu funksiya hosilasini boshqacha usul bilan ham hisoblash mumkin.

$$\lim_{\Delta x\to0}\frac{(x+\Delta x)^\alpha-x^\alpha}{\Delta x}=\lim_{\Delta x\to0}\frac{x^\alpha((1+\frac{\Delta x}{x})^\alpha-1)}{\Delta x}=\lim_{\Delta x\to0}\frac{x^\alpha((1+\frac{\Delta x}{x})^{\frac{x}{\Delta x}\frac{\Delta x}{x}\alpha}-1)}{\Delta x}=$$

$$\lim_{\Delta x\to0}\frac{x^\alpha(e^{\frac{\alpha\Delta x}{x}}-1)}{\Delta x}=\lim_{\Delta x\to0}\frac{x^\alpha(e^{\frac{\alpha\Delta x}{x}}-1)}{\frac{\alpha\Delta x}{x}\frac{x}{\alpha}}=\frac{\alpha x^\alpha}{x}=\alpha x^{\alpha-1}.$$

Ajoyib limitlardan funksiya $y=x^\alpha$ bo'lgan holda, funksiyaning hosilasi $y'=\alpha x^{\alpha-1}$ ga teng bo'ladi.

Endi $f(x)=(ax+b)^n$ ko'rinishdagi funksiya xosilasini ko'rib chikamiz. Buning uchun yana

$$\frac{f(x+h)-f(x)}{h}=\frac{(a(x+h)+b)^n-(ax+b)^n}{h}=\frac{((ax+ah)+b)^n-(ax+b)^n}{h}=$$

$$\frac{((ax+b)+ah)^n-(ax+b)^n}{h}=$$

$$\frac{(ax+b)^n + n(ax+b)^{n-1}ah + \frac{n(n-1)}{2}(ax+b)^{n-2}h^2 + \ldots + h^n - (ax+b)^n}{h} =$$

$$= \frac{n\left(ax+b\right)^{n-1}(ah) + \frac{n(n-1)}{2}(ax+b)^{n-2}(ah)^2 + \ldots + (ah)^n}{h} = n(ax+b)^{n-1}a +$$

$$\frac{n(n-1)}{2}(ax+b)^{n-2}ah + \ldots + (ah)^{n-1}.$$

Bu ifodada $h \to 0$ da $f'(x) = na(ax+b)^{n-1}$ ekanligini ko'ramiz

Funksiya ko'rsatkichli funksiya bo'lgan holda funksiya hosilasini topamiz $y = a^x$.

Bu funksiyaning hosilasini topishda ham hosila ta'rifidan funksiya orttirmasini argument orttirmasiga nisbatini qaraymiz. $\lim\limits_{\Delta x \to 0} \frac{a^{x+\Delta x} - a^x}{\Delta x} =$

$\lim\limits_{\Delta x \to 0} \frac{a^x(a^{\Delta x} - 1)}{\Delta x} = a^x \ln a$ ekanligini topamiz.

Keyingi funksiyalardan biri bu logarifmik funksiya bo'lib, logarifmik funksiyaning hosilasini topish uchun ham ayirmali nisbat qaraymiz va bu nisbatning limitini topish uchun yana ajoyib limitlar va ajoyib limitlardan kelib chiqadigan natijalardan foydalanamiz. $\lim\limits_{\Delta x \to 0} y = \log_a x$ бу funksiyaning ayirmali nisbati quyidagicha bo'ladi.

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\log_a(x+\Delta x) - \log_a x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\log_a(1+\frac{\Delta x}{x})}{\Delta x} =$$

$$\lim_{\Delta x \to 0} \frac{x}{\Delta x} \cdot \frac{1}{x} \cdot \log_a\left(1+\frac{\Delta x}{x}\right) = \lim_{\Delta x \to 0} \frac{1}{x} \cdot \log_a\left(1+\frac{\Delta x}{x}\right)^{\frac{x}{\Delta x}} = \frac{1}{x} \cdot \log_a x = \frac{1}{x \ln a}$$

bulardan logarifmik funksiyaning hosilasi $y' = \frac{1}{x \ln a}$ ekanligi kelib chiqadi.

Endi trigonometrik funksiyalarning hosilasini topish bilan shug'ullanamiz. Masalan $y = \sin x$ bu funksiyaning hosilasini topish uchun yana funksiya orttirmasini tekshiramiz. $\lim\limits_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin x}{\Delta x}$ bundan limitda

kasr suratiga trigonometriyaning yana bir hossasini qo'llab ya'ni ayirmani ko'paytmaga aylantirish formulasidan quyidagiga ega bo'lamiz:

$$\lim_{\Delta x \to 0} \frac{2\sin\frac{\Delta x}{2} \cdot \cos(x+\frac{\Delta x}{2})}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sin\frac{\Delta x}{2} \cdot \cos(x+\frac{\Delta x}{2})}{\frac{\Delta x}{2}} =$$

$$= \lim_{\Delta x \to 0} \frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}} \cdot \cos(x+\frac{\Delta x}{2}) = \cos x.$$

Bundan esa $y' = \cos x$ ekanligi kelib chiqadi. Huddi shu kabi $y = \cos x$ funksiyaning hosilasi $y' = -\sin x$ ekanligini ham huddi shu kabi ko'rsatishimiz mumkin.

1. Trigonometrik funksiyalardan yana biri ya'ni $y = tgx$ ning hosilasini topamiz. Bu funksiyaning hosilasini topishda ikki usuldan foydalanishimiz mumkin. Birinchi usul bu albatta o'zimiz biladigan funksiya hosilasi ta'rifidan foydalanib hisoblash. Ikkinchi usulidan foydalanish uchun hosila olish qoidalarni bilish kerak. Bu usulni o'rgatish asnosida o'quvchilarga hosila olish qoidalari haqida ham tushuncha berishimiz mumkin. Shuning uchun hosila olish qoidalarida to'htalamiz.

a) Ikki funksiyaning algebraik yig'indisining hosilasi, shu funksiyalar hosilalarining algebraik yig'indilariga teng. Ya'ni $y = f(x) \pm g(x)$ ko'rinishdagi funksiya berilgan bo'lsa, u holda funksiya hosilasi quyidagicha bo'ladi. $y' = f'(x) \pm g'(x)$ ga teng bo'ladi. Buni isbotlash uchun ham hosila ta'rifidan foydalanamiz.

$$\lim_{\Delta x \to 0} \frac{\left(f(x+\Delta x) \pm g(x+\Delta x)\right) - (f(x) \pm g(x))}{\Delta x} =$$

$$\lim_{\Delta x \to 0} \frac{(f(x+\Delta x) - f(x)) \pm (g(x+\Delta x) - g(x))}{\Delta x} == \lim_{\Delta x \to 0} \frac{(f(x+\Delta x) - f(x))}{\Delta x} \pm$$

$$\lim_{\Delta x \to 0} \frac{(g(x+\Delta x) - g(x))}{\Delta x} = f'(x) + g'(x).$$

b) Ikki funksiyaning ko'paytmasining hosilasini topamiz. YA'ni funksiya $y = f(x) \cdot g(x)$ ko'rinishda berilgan bo'lsa, bu funksiyaning hosilasini topamiz. $y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$ bu formula ikki funksiya ko'paytmasining hosilasini beradi. Shuni isbotlaymiz. Bu formulani isbotlash uchun ham funksiya orttirmasini argument orttirmasiga nisbatini qaraymiz.

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)g(x+\Delta x)-f(x)g(x)}{\Delta x}=$$

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)g(x+\Delta x)-f(x+\Delta x)g(x)+f(x+\Delta x)g(x)-f(x)g(x)}{\Delta x}=$$

$$\lim_{x\to 0}\frac{f(x+\Delta x)(g(x+\Delta x)-g(x))+g(x)(f(x+\Delta x)-f(x))}{\Delta x}=$$

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)(g(x+\Delta x)-g(x))}{\Delta x}+\lim_{\Delta x\to 0}\frac{g(x)(f(x+\Delta x)-f(x))}{\Delta x}=$$

$$f'(x)\cdot g(x)+f(x)\cdot g'(x)$$

ekanligi kelib chiqadi.

c) Endi ikki funksiya bo'linmasining hosilasini qaraylik, $y'=\left(\frac{f(x)}{g(x)}\right)'=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)}$ bo'linmaning hosilasi ushbu formula bilan topiladi. Bu formulaning isboti ham juda oddiy va maktab o'quvchilariga tushuntirishda o'qituvchilar qiyinchilikka uchramaydi. Bu formulani isbotlashda ham hosila ta'rifidan foydalanamiz. Buning uchun ayirmali nisbatni ko'rib chiqamiz.

d) $$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\frac{f(x+\Delta x)}{g(x+\Delta x)}-\frac{f(x)}{g(x)}}{\Delta x}=$$

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)g(x)-g(x+\Delta x)f(x)}{\Delta x\cdot g(x+\Delta x)\cdot g(x)}=$$

$$\lim_{x\to 0}\frac{f(x+\Delta x)g(x)-f(x)g(x)+f(x)g(x)-g(x+\Delta x)f(x)}{\Delta x\cdot g(x+\Delta x)\cdot g(x)}=$$

$$\lim_{\Delta x\to 0}\frac{g(x)(f(x+\Delta x)-f(x))-f(x)(g(x+\Delta x)-g(x))}{\Delta x\cdot g(x+\Delta x)\cdot g(x)}=$$

$$\lim_{\Delta x\to 0}\frac{1}{g(x+\Delta x)\cdot g(x)}\cdot\frac{g(x)(f(x+\Delta x)-f(x))-f(x)(g(x+\Delta x)-g(x))}{\Delta x}=$$

$$\lim_{\Delta x\to 0}\frac{1}{g(x+\Delta x)\cdot g(x)}\cdot\left(\frac{g(x)(f(x+\Delta x)-f(x))}{\Delta x}-\frac{f(x)(g(x+\Delta x)-g(x))}{\Delta x}\right)=$$

$$\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)}$$

Endi yuqoridagi trigonometrik funksiyani hosilasini hisoblaymiz. Yuqorida ta'kidlab o'tganimizdek $y=tgx$ funksiyaning hosilasini ikki hil usulda topishimiz mumkin.

Funksiya orttirmasini argument orttirmasiga nisbatini qaraylik.

$$\lim_{\Delta x\to 0}\frac{tg(x+\Delta x)-tgx}{\Delta x}=\lim_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x\cdot\cos(x+\Delta x)\cos x}=\lim_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x}\cdot\frac{1}{\cos(x+\Delta x)\cos x}=$$

$$\frac{1}{(\cos x)^2}$$

ekanligi kelib chiqadi.

Endi yuqoridagi formula bo'yicha bo'linmaning hosilasi formulasidan foydalanib,

$$y'=\frac{\sin' x\cdot\cos x-\sin x\cos' x}{\cos^2 x}=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}$$

ekanligini keltirib chiqaramiz. Bundan ko'rinadiki har ikki usulda ham natija bir hil bo'ladi.

Elementar funksiyalarning hosilasi jadvali

1) $y=c,\ c=const\quad y'=0$

2) $f(x)=kx+b\quad y'=k$

3) $f(x)=x^{\alpha}\quad y'=\alpha x^{\alpha-1}$

4) $y'=a^x\quad y'=a^x\ln a$

5) $y=e^x\quad y'=e^x$

6) $y=\log_a x\quad y'=\frac{1}{x\ln a}$

7) $y=\ln x\quad y'=\frac{1}{x}$

8) $y=\sin x \quad y'=\cos x$

9) $y=\cos x \quad y'=-\sin x$

10) $y=tgx \quad y'=\frac{1}{\cos^2 x}$

11) $y=ctgx \quad y'=-\frac{1}{\sin^2 x}$

Misol. $y=\frac{x^4}{4}-5\sqrt{x}+\frac{8}{x^2}-\frac{1}{3\sqrt[3]{x}}$ funksiyaning hosilasini toping.

Yechish: ushbu funksiyaning hosilasini topish uchun oldin funksiyaning har bir yig'indisini argumentni darajasi shaklida yozamiz. $y=\frac{x^4}{4}-5\sqrt{x}+\frac{8}{x^2}-\frac{1}{3\sqrt[3]{x}}=\frac{1}{4}x^4-5x^{\frac{1}{2}}+8x^{-2}-\frac{1}{3}x^{-\frac{1}{3}}$ ko'rinishga keltirilgandan keyin darajaning hosilasi formulasi va bir nechta funksiya yig'indisidan olingan hosila xossasidan foydalanib, ushbu funksiya hosilasini topamiz.

$$y'=(\frac{1}{4}x^4-5x^{\frac{1}{2}}+8x^{-2}-\frac{1}{3}x^{-\frac{1}{3}})'=(\frac{1}{4}x^4)'-(5x^{\frac{1}{2}})'+(8x^{-2})'-(\frac{1}{3}x^{-\frac{1}{3}})'=$$

$$\frac{1}{4}\cdot 4x^3-5\cdot\frac{1}{2}x^{-\frac{1}{2}}-8\cdot 2x^{-3}+\frac{1}{3}\cdot\frac{1}{3}x^{-\frac{4}{3}}=x^3-\frac{5}{2\sqrt{x}}-\frac{16}{x^3}+\frac{1}{9\sqrt[3]{x^4}}.$$

Misol. $y=\sin x\cdot e^x$ funksiya hosilasini hisoblang.

Yechish: ushbu funksiya hosilasini hisoblashda ko'paytmaning hosilasi formulasidan $y'=(\sin x\cdot e^x)'=\sin' x\cdot e^x+\sin x\cdot(e^x)'=e^x(\cos x+\sin x)$ ekan-ligi kelib chiqadi.

Murakkab funksiyaning hosilalari

> **Teorema.** $u=\varphi(x)$ funksiya $x=x_0$ nuqtada hosilaga ega bo'lsa, $y=f(u)$ funksiyasi ($x=x_0$ nuqtaga mos keluvchi) $z=z_0$ nuqtada hossilaga ega bo'lsa, u holda bulardan tuzilgan $y=f(\varphi(x))$ murakkab funksiyaning $x=x_0$ nuqtadagi hosilasi $y'=(f(\varphi(x)))'=f'(\varphi(x))\cdot\varphi'(x)$ ga teng bo'ladi.

Misol. $y=\cos(x^2+5x-7)$ ushbu funksiyaning hosilasini hisoblang.

Yechish: bu misolda ko'rinib turibdiki funksiya murakkab funksiya, ya'ni trigonometrik funksiyaning ichida kvadrat funksiya. Bu funksiyadan hosila olish uchun trigonometrik funksiyaning o'zidan hosila olinadi va ichidan hosila olinadi.

$$y'=(\cos(x^2+5x-7))'=\cos'(x^2+5x-7)\cdot(x^2+5x-7)'=$$
$$=-\sin(x^2+5x-7)\cdot(2x+5)=-(2x+5)\sin(x^2+5x-7)$$

Misol. $y=\sin^2 x$ funksiyaning hosilasini toping.

Yechish: bunday olib qaraganda bu funksiya ham murakkab funksiya sanaladi. Chunki kvadrat funksiyaning ichiga trigonometrik funksiya qo'yib hosil qilingan. Lekin bu misolni ikki hil usulda yechish mumkin. YA'ni birinchisi murakkab funksiyadan olingan hosila bo'yicha va trigonometrik funksiyaning xossalarini qo'llash bo'yicha.

A) $y'=(\sin^2 x)'=2\sin x\cdot(\sin x)'=2\sin x\cos x=\sin 2x$

B) $y=\sin^2 x$ funksiyaning darajasini pasaytiramiz. $y=\sin^2 x=\frac{1-\cos 2x}{2}$ endi bu funksiyadan hosila olamiz. Unitmaslik kerakki bu funksiyaning hosilasi ham murakkab funksiyaning hosilasidan kelib chiqadi. Chunki trigonometrik funksiyaning ichida birinchi darajali chiziqli funksiya turibdi. $y'=(\frac{1-\cos 2x}{2})'=\frac{1}{2}(\sin 2x\cdot 2)\sin 2x$ bu yerda ishlash usuliga bog'liq bo'lmagan holda natija bir hil chiqadi.

Funksiya hosilasi. Hosila olish qoidalari va murakkab funksiyalarning hosilalariga oid misollar yechish.

Misol. $y=x^4+5x^3-7x^2+8x-15$ funksiyaning hosilasini toping.

Yechish: bu funksiyaning hosilasini topish uchun hosila topish qoidalaridan funksiyalarning yig'indisining hosilasi funksiyalar hosilalari yig'indisiga tengdir. Shundan foydalanamiz.

$$y'=(x^4+5x^3-7x^2+8x-15)'=(x^4)'+(5x^3)'-(7x^2)'+(8x)'-15'=$$
$$4x^3+15x^2-14x+8$$

Misol. $y=\cos x\cdot\ln x$ funksiyaning hosilasini toping.

Yechish: Bu funksiya ikki funksiyaning ko'paytmasidan tashkil topgan. Shuning uchun ikki funksiya ko'paytmasi hosilasidan formulasidan foydalanamiz.

$$y'=(\cos x\cdot\ln x)'=\cos' x\cdot\ln x+\cos x\cdot\ln' x=-\sin x\cdot\ln x+\frac{\cos x}{x}=$$

$$\frac{\cos x-x\sin x\cdot\ln x}{x}.$$

Misol. $y=\dfrac{\sin x}{e^x}$ funksiyaning hosilasini toping.

Yechish: Funksiya ikki funksiyaning nisbatidan iborat. Shuning uchun bu funksiyaning hosilasini topishda bo'linmaning hosilasi formulasidan foydalanamiz.

$$y'=(\frac{\sin x}{e^x})'=\frac{\sin' x\cdot e^x-\sin x\cdot(e^x)'}{(e^x)^2}=\frac{(\cos x-\sin x)e^x}{e^{2x}}=\frac{(\cos x-\sin x)}{e^x}.$$

Misol. $y=(3x+2)^4$ funksiyaning hosilasini toping.

Yechish: bu funksiyaning hosilasini topishda ikki hil usuldan foydalanish mumkin.Birinchi usul hosila olishning oddiy qoidalaridan foydalanish maqsadida Nyuton binomidan yoki Paskal uchburchagidan foydalanib kavsni ochib chiqish. Hozir shu usuldan foydalanib funksiya hosilasini hisoblaymiz.

$$y=(3x+2)^4=81x^4+4\cdot2\cdot27x^3+6\cdot9x^2\cdot4+4\cdot3x\cdot8+16=$$

$$=81x^4+216x^3+216x^2+96x+16$$

funksiya -darajali funksiya, ko'phad ko'rinishiga keldi. Endilikda bu funksiyaning hosilasini hisoblaymiz.

$$y'=(81x^4+216x^3+216x^2+96x+16)'=(81x^4)'+(216x^3)'+(216x^2)'+(96x)'+$$
$$(16)'==324x^3+648x^2+432x+96.$$

Yuqoridagi funksiya hosilasini topishning ikkinchi usuli bu murakkab funksiya hosilasidan foydalanish demakdir.

$$y'=((3x+2)^4)'=4(3x+2)^3\cdot(3x+2)'=12(3x+2)^3.$$

Yuqoridagi natijani soddalashtirsak ushbu javob bilan bir hil bo'ladi.

Misol. $y=3^{4x^2-5x+1}$ funksiyani hosilasini toping.

Yechish: ko'rinib turibdiki bu funksiya murakkab funksiyadir,

$$y'=(3^{4x^2-5x+1})'=3^{4x^2-5x+1}(4x^2-5x+1)'\cdot\ln3=(8x-5)3^{4x^2-5x+1}\cdot\ln3.$$

Misol. $y=\ln\dfrac{x^3-2x+1}{2x+4}$ funksiyaning hosilasini toping.

Yechish: $y'=(\ln\dfrac{x^3-2x+1}{2x+4})'=\dfrac{1}{\dfrac{x^3-2x+1}{2x+4}}\cdot(\dfrac{x^3-2x+1}{2x+4})'=\dfrac{2x+4}{x^3-2x+1}\cdot$

$$\frac{(3x^2-2x)(2x+4)-2(x^3-2x+1)}{(2x+4)^2}=\frac{2x+4}{x^3-2x+1}\cdot$$

$$\frac{6x^3+12x^2-4x-8-2x^3+4x-2}{(2x+4)^2}=\frac{4x^3+12x^2-10}{2(x+2)(x^3-2x+1)}=\frac{2x^3+12x^2-10}{(x+2)(x^3-2x+1)}$$

Misol. $y=\cos(2^{x^3})$ funksiyaning hosilasini toping.

Yechish: bu funksiyaning hosilasini olish uchin ham murakkab funksiya hosilasidan foydalaniladi. Lekin bu funksiyada ikkita funksiya emas balki uchta funksiya ichma- ich joylashgan. Bunday holatda ham hosila murakkab funksiyaniki kabi bajariladi. YA'ni $y=f(g(\varphi(x)))$ bo'lgan holatda ham hosila huddi shunday amalga oshiriladi.

$y'=(f(g(\varphi(x))))'=f'(g(\varphi(x)))\cdot g'(\varphi(x))\cdot\varphi'(x)$ formula orqali.

Demak

$$y'=(\cos(2^{x^3}))'=\sin(2^{x^3})\cdot(2^{x^3})'\cdot(x^3)'=2^{x^3}\cdot\ln2\cdot\sin(2^{x^3})\cdot3x^2=$$

$$3\ln2\cdot x^2\cdot2^{x^3}\cdot\sin(2^{x^3}).$$

Mustaqil yechish uchun misollar:

1-variant

1) $y = x^3 + 3x^2 - 4$ funksiya hosilasini ta'rif yordamida hisoblang.

2) Berilgan funksiyalarning hosilalarini toping. $y = \sin 2x$, $y = \frac{1}{2}arctg\frac{x}{2}$

$y = \ln(\sin 5x) - \frac{4x^2}{\pi} + \frac{4}{5}$,

3) $y'(x_0) = ?$ $x_0 = \frac{\pi}{10}$

2-variant

1) $y = 2x^4 - \frac{2}{3}x^3 - 4x + 5$ funksiya hosilasini ta'rif yordamida hisoblang.

2) Berilgan funksiyalarning hosilalarini toping. $y = \arcsin 4x$, $y = arctge^x$

$y = \ln(\sin 5x) - \frac{4x^2}{\pi} + \frac{4}{5}$,

3) $y'(x_0) = ?$ $x_0 = \frac{\pi}{10}$

3-variant

1) $y = 3x^5 - 3\sqrt[3]{x} - \frac{1}{2x^3} + \frac{10}{\sqrt[5]{x^4}}$ funksiya hosilasini ta'rif yordamida hisoblang.

2) Berilgan funksiyalarning hosilalarini toping. $y = \log_3(2x-5)$, $y = \frac{1}{2}\ln\frac{x-1}{x+1}$

$y = \ln\sqrt{(x-4)^3} + (x-4)^3$,

3) $y'(x_0)$, $x_0 = 5$

4-variant

1) $y = \sin x \cdot e^x$ funksiya hosilasini ta'rif yordamida hisoblang.

2) Berilgan funksiyalarning hosilalarini toping. $y = tg(5x+1)$, $y = \ln(x + \sqrt{x^2+1})$

$y = \ln\sqrt{(x-4)^3} + (x-4)^3$,

3) $y'(x_0)$, $x_0 = 5$

5-variant

1) $y = xarctgx$ funksiya hosilasini ta'rif yordamida hisoblang.

2) Berilgan funksiyalarning hosilalarini toping. $y = (3x-8)^7$, $y = \ln(x + \sqrt{x^2+1})$

3) $y = e^{x-1}(4x-5)$, $y'(x_0)$, $x = \ln 2$

6-variant

1) $y = \sqrt[4]{x}\ln x$ funksiya hosilasini ta'rif yordamida hisoblang.

2) Berilgan funksiyalarning hosilalarini toping. $y = \ln\sqrt{x}$, $y = \ln\sin x$

3) $y = e^{x-1}(4x-5)$, $y'(x_0)$, $x = \ln 2$

7-variant

1) $y = x^2 tgx$ funksiya hosilasini ta'rif yordamida hisoblang.

2) Berilgan funksiyalarning hosilalarini toping. $y = \sin^2 x$, $y = \ln\cos x$

3) $y = (x+1)arctge^{-2x}$, $y'(x_0)$, $x = 0$

8-variant

1) $y=\frac{\cos x}{\ln x}$ funksiya hosilasini ta'rif yordamida hisoblang.

2) Berilgan funksiyalarning hosilalarini toping. $y=\cos^3 x$, $y=\sqrt{1-x^2}$

3) $y=(x+1)arctge^{-2x}$, $y'(x_0)$, $x=0$

9-variant

1) $y=\frac{3^x}{2x+1}$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=\arcsin x^3$, $y=\sqrt[5]{4-3x^5}+4^x\frac{2}{\ln 4}$ berilan funksiyalarni $x=1$ nuqtadagi hosilasini toping.

3) $y=\ln\frac{2+tgx}{2-tgx}$, $y'(x_0)$, $x_0=\frac{\pi}{3}$

10-variant

1) $y=\frac{x^2+1}{x^2-1}$ funksiya hosilasini ta'rif yordamida hisoblang.

2) Berilgan funksiyalarning hosilalarini toping. $y=3x^2+5\sqrt[3]{x^5}-\frac{4}{x^3}$, $y=\frac{1}{2}e^x(\sin x+\cos x)$

3) $y=\ln\frac{2+tgx}{2-tgx}$, $y'(x_0)$, $x_0=\frac{\pi}{3}$

11-variant

1) $y=\frac{1+e^x}{1-e^x}$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=\frac{x^2}{x^2+1}$, $y=\ln\ln x$ Berilgan funksiyalarning hosilalarini toping.

3) $y=\arcsin\frac{x-1}{x}$, $y'(x_0)$, $x=5$

12-variant

1) $y=\frac{\ln x}{\sin x}+xctgx$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=\sqrt{1-x^2}$, $y=\sin x\cdot\ln x$. Berilgan funksiyalarning hosilalarini toping.

3) $y=\arcsin\frac{x-1}{x}$, $y'(x_0)$, $x=5$

13-variant

1) $y=\frac{1}{x}$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=x^3\sin x.$, $y=x\cdot\arccos x.$ Berilgan funksiyalarning hosilalarini toping.

3) $y=(4x^2-3x+1)^3$, $y'(x_0)$, $x_0=0$

14-variant

1)funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=\sqrt{1+5\cos x}$, $y=\frac{x^4+1}{x^4-1}$ Berilgan funksiyalarning hosilalarini toping.

3) $y=(4x^2-3x+1)^3$, $y'(x_0)$, $x_0=0$

15-variant

1) $y=\frac{1}{x^2}$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=\frac{1}{2}e^x(\sin x+\cos x)$, $y=x^2 ctgx$ Berilgan funksiyalarning hosilalarini toping.

3) $y=(4x^2-3x+1)^3$, $y'(x_0), x_0=0$

16-variant

1) $y=\frac{1}{\sqrt{x}}$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=\ln\ln x$, $y=3x^3\ln x-x^3$ Berilgan funksiyalarning hosilalarini toping.

3) $y=\sin(7x^2+x)$, $y'(x_0)$, $x_0=0$

17-variant

1) $y=x+\frac{1}{x^2}-\frac{1}{5x^5}$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=arc\sin\sqrt{x}$, $y=e^x arctgx$ Berilgan funksiyalarning hosilalarini toping.

3) $y=\sin(7x^2+x)$, $y'(x_0)$, $x_0=0$

18-variant

1) $y=\sin 2x$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=\arcsin\frac{x}{5}$, $y=\frac{\cos x}{1-\sin x}$. Berilgan funksiyalarning hosilalarini toping.

3) $y=\sin(7x^2+x)$, $y'(x_0)$, $x_0=0$

19-variant

1) $y=2x^3+5x^2$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=\ln 3x$, $y=\frac{xtgx}{1+x^2}$ Berilgan funksiyalarning hosilalarini toping.

3) $y=e^{4x^2-5x}$, $y'(x_0)$, $x_0=0$

20-variant

1) $2y=x^2$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=\frac{\sqrt{x}}{\sqrt{x}+1}$, $y=\frac{\ln x}{\sin x}+x\cdot ctgx$. Berilgan funksiyalarning hosilalarini toping.

3) $y=e^{4x^2-5x}$, $y'(x_0)$, $x_0=0$

21-variant

1) $2y=x^2$ funksiya hosilasini ta'rif yordamida hisoblang.

$y=\frac{\cos x}{x^2}$., $y=x^2\log_3 x$ Berilgan funksiyalarning hosilalarini toping.

2) $y=e^{4x^2-5x}$, $y'(x_0)$, $x_0=0$

22-variant

1) $f(x)=x^3-7x^2+8$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=arctgx-arcctgx$, . $y=\frac{x^3}{3}$ $x=-1$ nuqtada berilgan funksiyalarning hosilalarini toping.

3) $y=\arcsin\frac{1}{x}$, $y'(x_0)$, $x=2$

23-variant

1) $f(x)=x^3-7x^2+8$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=(x^2+2x+2)e^{-x}$ $y=\frac{8}{4+x^2}$ $x=2$. Nuqtada berilgan funksiyalarning hosilalarini toping

3) $y=\arcsin\frac{1}{x}$, $y'(x_0)$, $x=2$

24-variant

1) $2y=8-x^2$ funksiya hosilasini ta'rif yordamida hisoblang.

2) $y=x^2\cdot\sin x+2x\cos x-2\sin x.$, $y=\frac{x^7-5x^4+1}{x^2+1}$ Berilgan funksiyalarning hosilalarini toping.

3) $y=(1+\sqrt[3]{x})^2$, $y=\frac{\cos x}{1+2\sin x}$.

25-variant

1) $2y=8-x^2$ funksiya hosilasini ta'rif yordamida hisoblang.

4) $y=\sqrt[5]{x}arctgx$, $y=\frac{2^{3x}}{3^{2x}}$ Berilgan funksiyalarning hosilalarini toping.

5) $y=(1+\sqrt[3]{x})^2$, $y=\frac{\cos x}{1+2\sin x}$.

13-mavzu: Yuqori tartibli hosila va differensiallar.

Funksiya differensiali.Yuqori tartibli hosilalar. $y=f(x)$ funksiya uchun birinchi tartibli hosilasi y' aniqlangan bo'lsin. Birinchi hosiladan olingan hosila ikkinchi tartibli hosila yoki boshlang'ich funksiyaning *ikkinchi tartibli hosilasi deyiladi* va y'' yoki $f''(x)$ bilan belgilanadi: $y''=(y')'=f''(x)$.

Ikkinchi tartibli hosiladan olingan hosila uchinchi tartibli hosila yoki boshlang'ich funksiyaning *uchinchi tartibli hosilasi deyiladi* va y''' yoki $f'''(x)$ bilan belgilanadi.

Umuman, $f(x)$ funksiyaning n-tartibli hosilasi deb uning $(n-1)$-tartibli hosilasidan (birinchi tartibli) hosilasiga aytiladi va $y^{(n)}$ yoki $f^{(n)}(x)$ bilan belgilanadi:

$$y^{(n)}=(y^{(n-1)})=f^{(n)}(x).$$

Bunda ushbu formulalar o'rinli:

$$(U+V)^{(n)}=U^{(n)}+V^{(n)};$$

$$(CU)^{(n)}=C\cdot U^{(n)};$$

$$(U\cdot V)^{(n)}=U^{(n)}V+nU^{(n-1)}V'+\frac{n(n-1)}{1\cdot 2}U^{(n-2)}V''+...+UV^{(n)}$$ (Leybnits formulasi).

Yuqori tartibli differensiallar ham shunday ta'riflanadi:

n-tartibli differensial deb (n-*1*)-tartibli differensialning birinchi differensialiga aytiladi: $d^n y=d(d^{n-1}y)=\lfloor f^{(n-1)}(x)dx^{n-1}\rfloor dx$ yoki $d^n(y)=f^{(n)}(x)dx^n$.

Misol. $y=\frac{1}{3}\sin^3\sqrt{x}-\frac{2}{5}\sin^5\sqrt{x}+\frac{1}{7}\sin^7\sqrt{x}$ funksiya hosilasini toping.

Yechish: $y'=\frac{1}{3}\cdot 3\sin^2\sqrt{x}\cdot\cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}-\frac{2}{5}\cdot 5\sin^4\sqrt{x}\cdot\cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}+$

$+\frac{1}{7}\cdot 7\sin^6\sqrt{x}\cdot\cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2\sqrt{x}}\sin^2\sqrt{x}\cdot\cos\sqrt{x}(1-2\sin^2\sqrt{x}+\sin^4\sqrt{x})=\frac{1}{2\sqrt{x}}\sin^2\sqrt{x}\cdot\cos^5\sqrt{x}.$

Misol: Quyidagi funksiyaning n-tartibli hosilalari topilsin: $y=\ln x$

$$y'=\frac{1}{x}=x^{-1},\ y''=-1\cdot x^{-2},\ y'''=-1\cdot(-2)\cdot x^{-3},\ y^{(IV)}=1\cdot 2\cdot 3\cdot x^{-4},$$

$$y^{(n)}=1\cdot 2\cdot 3\cdot\ldots\cdot(n-1)\cdot(1)^{n-1}\cdot x^{-n}=(1)^{n-1}\cdot\frac{(n-1)!}{x^{n}}.$$

Misol. Oshkor bo'lmagan $x^3+\ln y-x^2e^y=0$ ko'rinishda berilgan funksiya hosilasini toping.

$$3x^2+\frac{y'}{y}-x^2e^y y'-2xe^y=0 \text{ ya'ni } y'=\frac{(2xe^y-3x^2)y}{1-x^2ye^y}.$$

Mustaqil yechish uchun mashqlar

1-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=\ln\left(2x^3+3x^2\right)$, $y=\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}}$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\frac{3}{4}x\sqrt[3]{x}$, $y=x^{\frac{1}{\ln x}}$.

2-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=\sqrt{4x+\sin 4x}.$, $y=\sqrt[3]{x\sqrt{x\sqrt{x}}}$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\left(x^2+2x+2\right)e^{-x}$, $y=\log_{\cos x}\sin x$.

3-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=\sqrt{\frac{x}{2}+\sin\frac{x}{2}}.$, $y=x^x$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\ln\left(2x^3+3x^2\right)$, $y=\log_a\left(x+\sqrt{x^2+9}\right)$

4-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=\sqrt{1-3x^2}$, $y=\log_x e$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\ln tg\frac{2x+1}{4}$, $y=2\left(tg\sqrt{x}-\sqrt{x}\right)$

5-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=e^{\frac{x}{a}}\cdot\cos\frac{x}{a}.$, $y=xe^x(\sin x-\cos x)+e^x\cos x$.

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a}$, $y=arctg\sqrt{4x^2-1}$

6-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=\cos^3\left(\frac{x}{3}\right).$, $y=\log_2\sin^2 x$.

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=e^{-x}-\sin e^{-x}\cos e^{-x}.$, $y=\frac{\ln x}{x^5}+\frac{1}{5x^5}$

7-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=-ctg^2\frac{x}{2}-2\ln\left(\sin\frac{x}{2}\right).$, $y=\arccos\sqrt{1-2^x}$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\ln\frac{x^5}{x^5+2}$, $y=\ln\frac{\sqrt{x^2+2x}}{x+1}$

8-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=\arccos\frac{9-x^2}{9+x^2}$, $y=\frac{x^2e^{x^2}}{x^2+1}$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\frac{x-e^{2x}}{x+e^{2x}}$, $y=3x\sin^3 x+3\cos x-\cos 3x.$

9-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=1-e^{\sin^2 3x}\cdot\cos^2 3x.$, $y=arctg\frac{2x^4}{1-x^2}$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=x\sqrt[2]{x}$, $y=\frac{1}{2x+1}$

10-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=e^{\sqrt{2x}}\left(\sqrt{2x}-1\right)$, $y=2^{\cos 3x-3\cos x}$.

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=5-3\cos^2 x.$, $y=2^x+2^{-x}$

11-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=\arcsin\sqrt{1-0.2x^2}$, $y=\ln tg\frac{x}{2}+\cos x+\frac{1}{3}\cos^3 x.$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\frac{ax+b}{cx+d}$, $y=e^{kx}$.

12-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=\frac{1}{\sqrt{1-mx^2}}$, $y=\frac{\sin x}{1+\ln\sin x}$.

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\sin^2 x.$, $y=tgx.$

13-variant

1) Berilgan funksiyalarning 2- tartibli hosilalarini toping $y=arctg(x+1)+\frac{x+1}{x^2+2x+2}$ $y=\ln\left(1-\frac{1}{x}\right)+\frac{1}{x}$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\sqrt{1+x^2}.$, $y=x\ln x.$

14-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=\ln\ln x(\ln\ln\ln x-1)$, $y=tg^3 tgx+3tgtgx$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\frac{1+tgx}{1-tgx}$, $y=\ln\sqrt{\frac{1+tgx}{1-tgx}}-x$.

15-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y = e^{-\frac{x}{a}}$, $y = \sqrt{x}$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y = \sin^3 2x$,
$y = \ln\sqrt{\frac{1-sinx}{1+cosx}}$.

16-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y = x^n$, $y = \sin x$.

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y = e^{1+\ln^2 x}$,
$y = \ln(e^x + \sqrt{1+e^{2x}})$.

17-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y = \cos^2 x.$,
$y = \ln tg\frac{x}{2} + \cos x + \frac{1}{3}\cos^3 x.$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y = tg\ln\sqrt{x}$,
$y = \frac{\ln x}{\sqrt{x^2-1}}$.

18-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y = \arcsin\sqrt{1-0.2x^2}$,
$y = arctg\frac{x}{a}$.

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping
$y = sin\sqrt{1+x^2}$, $y = tg^2(x^3+1)$.

19-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y = x\sin x.$,
$y = x(\ln x - 1)$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y = \cos\ln^2 x$,
$y = \sqrt[3]{tg^2 3x}$.

20-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y = \sqrt[4]{x^2+3x} - \sqrt{(6x-1)^2}$, $y = \frac{x}{2}\sqrt{x^2+4}$.

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y = \frac{1+sin3x}{1-sin3x}$,
$y = 5^{-\frac{1}{\sin^2 x}}$.

21-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y = \arcsin\sqrt{1-0.2x^2}$,
$y = arctg\frac{x}{a}$.

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping
$y = sin\sqrt{1+x^2}$, $y = tg^2(x^3+1)$.

22-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y = \frac{2x}{\sqrt{1+x}} - 4\sqrt{1+x}$,, $y = 2\sqrt[3]{(2-x^3)^2}$.

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping
$y = \sin^6 10x + \cos^6 10x$, 24. $y = \sqrt[3]{tg6x+1}$.

23-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y = \sqrt{x+\sqrt{x}}$, $y = \sqrt[4]{\frac{1+x^4}{1-x^4}}$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping
$y = e^{-x^2}\cos^3(2x+3)$, 11. $y = \frac{1+e^{-x}}{1-e^{-x}}$.

24-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=\frac{1}{x+\sqrt{1+x^2}}$, $y=\frac{x-1}{x+1}\sqrt{x^2-6}$

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\sin^2 3x$, $y=\sqrt{1-\ln^3 x}$.

25-variant

1) Berilgan funksiyalarning 2- trtibli hosilalarini toping $y=\sqrt{x+\sqrt{x}}$, $y=\sqrt{\frac{1+x}{1-x}}$.

2) Berilgan funksiyalarning 3 va 4- tartibli hosilalarini toping $y=\frac{1}{3}tg^3x-tgx+x$, $y=tg^2(x^3+1)$.

14-mavzu: Boshlang'ich funksiya va aniqmas integral. Aniqmas integral jadvali. Aniqmas integralning ba'zi bir xossalari.

Agar berilgan $\rho=X$ oraliqning barcha nuqtalarida $f(x)$ funksiya $F(x)$ ning hosilasi, ya'ni $F'(x)=f(x)$ bo'lsa, u holda $F(x)$ funksiya berilgan oraliqda $f(x)$ funksiyaning boshlang'ich funksiyasi yoki aniqmas integrali deyiladi.

Teorema. Agar biror (chekli yoki cheksiz, ochiq yoki yopiq) X oraliqda $F(x)$ funksiya $f(x)$ funksiyaning boshlang'ich funksiyasi bo'lsa, u holda $F(x)+C$ ham (bu yerda C ihtiyoriy o'zgarmas son) boshlang'ich funksiya bo'ladi. Aksincha, X oraliqda $f(x)$ ning har bir boshlang'ich funksiyasini shu ko'rinishda yozish mumkin.

Isbot: $F(x)$ funksiya bilan birgalikda $F(x)+C$ funksiya ham $f(x)$ ning boshlang'ich funksiyasi ekanligini ko'rsatish uchun har ikkala funksiyadan hosila olishning o'zi yetarli va bulardan ko'rinadiki $(F(x)+C)'=F(x)'=f(x)$ bo'ladi bu esa teoremani isbotlaydi.

Ushbu teoremadan berilgan $f(x)$ funksiyaning hamma boshlang'ich funksiyalarini topish uchun faqat bitta boshlang'ich funksiyani topish yetarli ekanligi kelib chiqadi, chunki ular bir biridan o'zgarmas qo'shiluvchigagina farq qiladilar. Bunga ko'ra $F(x)+C$ ifoda, bu yerda C ihtiyoriy o'zgarmas son, $f(x)$ hosilaga ega bo'lgan funksiyaning umumiy ko'rinishi bo'ladi. Bu ifoda $f(x)$ ning aniqmas integrali deyiladi va $\int f(x)dx$ kabi belgilanadi.

Yuqoridagi ta'rif va teoremalarni o'quvchilarga tushuntirgach boshlang'ich funksiya yoki aniqmas integral hisoblashning ba'zi hossa va qoidalar tushuntirilishi shart. Buning sababini har bir xossa va qoidalarni tushuntirib berayotganda yoritib o'tamiz.

1^0 $d\int f(x)dx=f(x)dx$ ya'ni d va $\int$ belgilar birinchisi ikkinchisidan oldin yozilgan bo'lsa, o'zaro qisqaradi. YA'ni integrallashdan olingan hosila shu integral belgisi ostidagi funksiyaning o'ziga teng bo'ladi.

2^0 $F(x)$ funksiya $F'(x)$ ning boshlang'ich funksiyasi bo'lgani uchun $\int F(x)'dx = F(x)+C$ ga ega bo'lamiz, buni $\int dF(x) = F(x)+C$ ko'rinishda ham yozish mumkin. Bundan $F(x)$ oldidagi d va $\int$ belgilar d belgi $\int$ dan keyin kelsa ham qisqaradi, faqat bunda $F(x)$ ga ixtiyoriy o'zgarmas son qo'shish kerak.

Aniqmas integralning ba'zi asosiy xossalari.

I. Agar a o'zgarmas son bo'lsa bu holda $\int af(x)dx = a\int f(x)dx$. Ya'ni o'zgarmas ko'paytuvchini integral belgisi ostidan chiqarish mumkin.

II. $\int (f(x) \pm g(x))dx = \int f(x)dx + \int g(x)dx$ arifmetik yig'indining integrali integrallar arifmetik yig'indisiga teng.

Bu ikkala formula haqida quyidagini aytib o'taylik. Bularga ikkita har biri ihtiyoriy o'zgarmasni o'z ichiga oladigan aniqmas integral kiradi. Bu tipdagi tenglikni o'ng va chap tomoni orasidagi ayirma o'zgarmasga teng degan ma'noda tushuniladi. Bu tengliklarni asl ma'noda tushunish ham mumkin lekin u vaqtda bunda ishtirok etgan integrallardan bittasi ihtiyoriy boshlang'ich funksiya bo'lmay qoladi: tenglikdagi o'zgarmas son boshqa integrallardagi o'zgarmaslarni topilgandan keyin aniqlanadi.

III. Agar $\int f(t)dt = F(t)+C$ bo'lsa, u holda: $\int f(ax+b)dx = \frac{1}{a}F(ax+b)+C'$ bo'ladi.

Ushbu xossalar asosida quyida misollarda taxlil qilamiz.

Misol. $\int (6x^2-3x+5)dx$ aniqmas integralni hisoblang.

Yechish: bu misolni yechish uchun yuqoridagi qoida va xossalardan foydalanamiz:

$$\int (6x^2-3x+5)dx = \int 6x^2dx - \int 3xdx + \int 5dx = 6\int x^2dx - 3\int xdx + 5\int dx =$$

$$2x^3 - \frac{3}{2}x^2 + 5x + C.$$

Ba'zi elementar funksiyalarning boshlang'ich funksiyasining jadvalini tuzamiz.

Masalan:

1) $\int 0dx = C$ бу formulani isbotlash uchun yuqorida ta'kidlaganimizdek funksiya uchun hosila formulasidan foydalanamiz. Ya'ni o'zgarmas sonning hosilasi 0 ga teng bo'ladi.

2) Keyingi elementar funksiyalarimizdan biri bu o'zgarmas sondir, o'zgarmas sonning boshlang'ich funksiyasi $\int kdx = kx + C$ ga teng bo'ladi. Shuni isbotlasak. $(kx+C)' = (kx)' + C' = k$ demak haqiqatdan ham formula o'rinli ekan.

3) Navbatdagi elementar funksiyalardan biri bu darajali funksiya, ya'ni darajasi ixtiyoriy son bo'lgan elementar funksiyadir. Bu funksiyaning boshlang'ich funksiyasi $\int x^{\alpha}dx = \frac{x^{\alpha+1}}{\alpha+1} + C$ ga ya'ni, funksiya darajasiga birni qo'shib shu darajaga bo'linadi. Bu formulaning isboti ham yuqoridagi kabi hosila olish yordamida amalga oshiraladi.

4) Keyingi elementar funksiyalardan yana biri bu darajasi / ga teng bo'lgan funksiyadir ya'ni, $\int \frac{1}{x}dx = \ln x + C$ formula o'rinli. Bu formulaning isboti ham yuqoridagi kabi hosila olish orqali isbotlanadi. Yuqoridagi va ushbu formulalarni maktab o'quvchilariga mustaqil ish sifatida tagdim qilish mumkin. Huddi shu kabi quyidagi

5) $\int \frac{1}{1+x^2}dx = arctgx + C$

6) $\int \frac{1}{\sqrt{1-x^2}}dx = \int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$

7) $\int \sin xdx = -\cos x + C$

8) $\int \cos xdx = \sin x + C$

9) $\int \frac{1}{\cos^2 x}dx = tgx + C$ formulalarni maktab o'quvchilariga funksiya hosilasi jadvalidan foydalanib isbotlashni mustaqil ish sifatida berish mumkin. Endi boshlang'ich funksiya topish yoki integrallash jadvalida yana bitta funksiyaning boshlang'ichini topish formulasi borki buni o'qituvchining o'zi isbotlab ko'rsatishi lozim. U ham bo'lsa ko'rsatkichli funksiyadir.

10) $\int a^x dx = \frac{a^x}{\ln a} + C$ bu formulani isbotlashda $\frac{a^x}{\ln a} + C$ funksiyadan hosila olamiz. $(\frac{a^x}{\ln a} + C)'$ ushbu ifodada $\frac{1}{\ln a}$ o'zgarmas son bo'lganligi uchun, hosilada o'zgarmas sonni kavsdan tashqariga chiqarib hisoblash mumkin degan qoidadan kavsdan tashqariga chiqarib yuboramiz. $(\frac{a^x}{\ln a} + C)' = \frac{1}{\ln a}(a^x + c_1)'$ bu yerda $c_1 = C\ln a$ ga teng. Endi yana hosila olish qoidasidan yig'indining har bir hadidan hosila olamiz va $\frac{1}{\ln a}(a^x + c_1)' = \frac{1}{\ln a}\cdot a^x \cdot \ln a + \frac{1}{\ln a}\cdot 0 = a^x$ bundan esa formulaning o'rinli ekanligi kelib chiqadi.

Ushbu jadval va aniqmas integral topish xossalaridan foydalanib bir nechta misollarni taxlil qilamiz.

Misol. $\int (3x-5)^2 dx$ aniqmas integralni hisoblang.

Yechish: Ushbu aniqmas integralni hisoblash uchun integral ostidagi ifodani kavsni ochamiz va soddalashtirib keyin integrallaymiz. $(3x-5)^2 = 9x^2 - 30x + 25$ ga teng. Endi bu funksiyaning boshlang'ich funksiyasini topamiz:
$\int (3x-5)^2 dx = \int (9x^2 - 30x + 25)dx = \int 9x^2 dx - \int 30x dx + \int 25 dx = \frac{9x^2}{3} - \frac{30x^2}{2} + 25x + C = 3x^3 - 15x^2 + 25x + C.$

Misol. $\int \frac{\sqrt{x^4 + x^{-4} + 2}}{x^3}dx$ aniqmas integralni hisoblang.

Yechish: ushbu aniqmas integralni hisoblash uchun, integral ostidagi ifodani soddalashtiramiz: $\sqrt{x^4 + x^{-4} + 2} = \sqrt{\frac{x^8 + 2x^4 + 1}{x^4}} = \sqrt{\frac{(x^4+1)}{x^4}} = \frac{x^4+1}{x^2}$.

Soddalashtirgach integralga qo'yib aniqmas integralni topamiz:

$$\int \frac{\sqrt{x^4 + x^{-4} + 2}}{x^3}dx = \int \frac{\frac{x^4+1}{x^2}}{x^3}dx = \int \frac{x^4+1}{x^5}dx = \int (\frac{1}{x} + \frac{1}{x^5})dx = \int (\frac{1}{x} + x^{-5})dx =$$

$$\ln x - \frac{1}{4x^4} + C.$$

Misol. $\int (\sin\frac{x}{2} + \cos\frac{x}{2})^2 dx$ aniqmas integralni hisoblang.

Yechish: $\int (\sin\frac{x}{2} + \cos\frac{x}{2})^2 dx = \int (\sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2} + \cos^2\frac{x}{2})dx =$

$$\int (1 + \sin x)dx = x - \cos x + C.$$

Misol. $\int ctg^2 x dx$ aniqmas integralni hisoblang.

Yechish:

$$\int ctg^2 x dx = \int \frac{\cos^2 x}{\sin^2 x}dx = \int \frac{1 - \sin^2 x}{\sin^2 x}dx = \int (\frac{1}{\sin^2 x} - 1)dx = -ctgx - x + C.$$

Misol. $\int \frac{dx}{1 + \cos 2x}$ aniqmas integralni hisoblang.

Yechish:

$$\int \frac{dx}{1+\cos 2x} = \int \frac{dx}{1 + \cos^2 x - \sin^2 x} = \int \frac{dx}{2\cos^2 x} = \frac{1}{2}\int \frac{dx}{\cos^2 x} = \frac{1}{2}tgx + C.$$

Misol. $\int \sin^2\frac{x}{4}\cos^2\frac{x}{4}dx$ aniqmas integralni hisoblang.

Yechish: $\int \sin^2\frac{x}{4}\cos^2\frac{x}{4}dx$ integralni hisoblash uchun trigonometrik funksiyalarning xossalaridan foydalanamiz. YA'ni

$$\sin^2\frac{x}{4}\cos^2\frac{x}{4}=\frac{1}{4}\cdot 4\cdot\sin^2\frac{x}{4}\cos^2\frac{x}{4}=\frac{1}{4}(2\sin\frac{x}{4}\cos\frac{x}{4})^2=\frac{1}{4}\sin^2\frac{x}{2}=\frac{1-\cos x}{8}$$

olingan natijaga ko'ra integralni hisoblaymiz.

$$\int\sin^2\frac{x}{4}\cos^2\frac{x}{4}dx=\int\frac{1-\cos x}{8}dx=\frac{1}{8}\int(1-\cos x)dx=\frac{1}{8}(x-\sin x)+C$$

Aniqmas integral hisoblash usullari. O'zgaruvchi almashtirish usuli. O'zgaruvchilarni almashtirish yo'li bilan integrallash: funksiyalarni integrallashda kuchli usullardan biri bo'lgan o'zgaruvchilarni almashtirish yoki o'rniga qo'yish usulini byon qilamiz. Buning asosida quyidagi sodda izoh yotadi:

Agar $\int g(t)dt=G(t)+C$ ekani ma'lum bo'lsa, u holda: $\int g(\omega(x))\omega'(x)dt=G(\omega(x))+C$.

Bu to'g'ridan- to'g'ri murakkab funksiyani differensiallash usulidan kelib chiqadi: $(G(\omega(x)))'=G'(\omega(x))\cdot\omega'(x)=g(\omega(x))\cdot\omega'(x)$

Buni, $dG(t)=g(t)dt$ munosabat t erkli o'zgaruvchining $\omega(x)$ funksiya bilan almashtirganda ham o'z kuchini saqlaydi deb, yana boshqacha ifodalash mumkin. $\int f(x)dx$ integralni hisoblash talab etilsin, deylik. Ko'p hollarda yangi o'zgaruvchi sifatida x ning funksiyasini tanlash mumkin bo'ladi: $t=\omega(x)$ va bunda integral ostidagi ifoda $f(x)dx=g(\omega(x))\cdot\omega'(x)dx$ shaklda yoziladi, bu yerdagi $g(t)$ funksiyani integrallash $f(x)$ ni integrallashga qaraganda qulayroq bo'ladi. U vaqtda yuqorida aytilgandek, $\int g(t)dt=G(t)+C$ integralni topish yetarli, unda $t=\omega(x)$ almashtirishni bajarib izlangan integral topiladi.

Masalan. $\int\sin^3 x\cos xdx$ integralni hisoblang.

Ushbu misolni ikki hil usulda hisoblash mumkin. Bu ikkala usul ham o'zgaruvchilarni almashtirish usuliga tayanadi.

1) Birinchi usul shundan iboratki integral belgisi ostidagi ifodaning bir qismini differensiallash belgisi ostiga kiritamiz. $\int\sin^3 x\cos xdx=\int\sin^3 xd(\sin x)$ бу ifodadan $t=\sin x$ belgilash kiritamiz. Bundan integral quyidagi ko'rinishga

keladi. $\int\sin^3 xd(\sin x)=\int t^3dt$ bu esa oddiy darajali funksiyaning integrali ekanligi ko'rinib turibdi. Demak $\int t^3dt=\frac{t^4}{4}+C=\frac{\sin^4 x}{4}+C$ ekanligi kelib chiqadi va misol ishlandi.

2) Endi bu misolni ishlashning ikkinchi usuliga to'xtaladigan bo'lsak ikkinchi usulda xuddi shu qilingan ishlar bajariladi faqat differensiallash belgisi ostiga kiritilmaydi ya'ni, $t=\sin x$ belgilash kiritiladi va $dt=d(\sin x)=\cos xdx$ ekanligida bajargan belgilashlarimizni o'rniga qo'yib chiqamiz. $\int\sin^3 x\cos xdx=\int t^3dt=\frac{t^4}{4}+C=\frac{\sin^4 x}{4}+C$ ekanligi kelib chiqadi. Bu ikki usul bir hil ma'noni anglatadi va farq qilmaydiganday tuyuladi lekin aslida ba'zida belgilash kiritganda ikkinchi usuli qo'l keladi. Sababi birinchi belgilash kiritish usulida integral ostidagi qaysi ifodani differensiallash belgisi ostiga kiritish kerakligini ajratib olish qiyin bo'lib qolishi mumkin.

Masalan. $\int\sqrt{1-x^2}dx$ aniqmas integralni hisoblang.

Ushbu integralni hisoblashda x o'zgaruvchini $x=\sin t$ kabi belgilash kiritish qulayroq. Chunki irratsional ifodaning ostidan ma'lum bir ifoda chiqadi. YA'ni $dx=d(\sin t)=\cos tdt$ belgilashlarni o'z o'rniga olib borib qo'yilsa u holda $\int\sqrt{1-x^2}dx=\int\cos t\cdot\cos tdt=\int\cos^2 tdt$ ifodaga kelamiz va bu integralni triganametrik funksiyalar xossalaridan foydalanib ya'ni daraja pasaytirib xisoblaymiz. $\int\cos^2 tdt=\int\frac{1+\cos 2t}{2}dt$ bu ifodadan yuqoridagi xossalardan o'zgarmas sonni integral belgisi tashqarisiga chiqarish mumkinligidan, $\int\frac{1+\cos 2t}{2}dt=\frac{1}{2}\int(1+\cos 2t)dt=\frac{1}{2}(t+\frac{1}{2}\sin 2t+C)$ ekanligini topamiz. Oxirgi ifodada belgilash kiritilganligi uchun o'zgaruvchilarni o'z o'rniga qo'yishdan oldin soddalashtirib olamiz. $x=\sin t$ belgilash kiritilgan edi bundan $t=\arcsin x$ ekanligi kelib chiqadi. $\frac{1}{2}(t+\frac{1}{2}\sin 2t+C)=\frac{1}{2}t+\sin t\cos t+C=\frac{1}{2}\arcsin x+x\sqrt{1-x^2}+C$ demak javob $\int\sqrt{1-x^2}dx=\frac{1}{2}\arcsin x+x\sqrt{1-x^2}+C$ ekanligi kelib chiqdi.

Ushbu usulni yana boshqa misollardagi talqinlarini ko'rib o'rganaylik.

Misol. $\int ctgxdx$ aniqmas integralni hisoblang.

Yechish: ushbu integralni hisoblash uchun quyidagicha belgilash va o'zgartirish kiritamiz:

$$\int ctgxdx=\int\frac{\cos x}{\sin x}dx=\int\frac{1}{\sin x}\cdot\cos xdx=\int\frac{1}{\sin x}d(\sin x)=\int\frac{1}{t}dt=\ln t+C=$$

$$\ln|\sin x|+C.$$

Misol. $\int\frac{\arccos x}{\sqrt{1-x^2}}dx$ aniqmas integralni hisoblang.

Yechish: bunday integrallarni hisoblashda qaysi qismini differensiallash belgisi ostiga kiritish kerakligini bilish lozim.

$$\int\frac{\arccos x}{\sqrt{1-x^2}}dx=\int\arccos x\cdot\frac{1}{\sqrt{1-x^2}}dx=\int\arccos xd(\arccos x)=\int tdt=\frac{t^2}{2}+C=$$

$$\frac{\arccos^2 x}{2}+C$$

Misol. $\int\frac{\sqrt{1+\ln x}}{x}dx$ aniqmas integralni hisoblang.

Yechish:

$$\int\frac{\sqrt{1+\ln x}}{x}dx=\int\sqrt{1+\ln x}\cdot\frac{1}{x}dx=\int\sqrt{1+\ln x}d(1+\ln x)=\int\sqrt{t}dt=\int t^{\frac{1}{2}}dt=\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+C=$$

$$\frac{2}{3}t\sqrt{t}+C=\frac{2}{3}\ln x\sqrt{\ln x}+C.$$

Misol. $\int\frac{dx}{(x+1)\sqrt{x}}$ aniqmas integralni toping.

Yechish:

$$\int\frac{dx}{(x+1)\sqrt{x}}=\int\frac{1}{(x+1)}\cdot\frac{1}{\sqrt{x}}dx=2\int\frac{d(\sqrt{x})}{(x+1)}=2\int\frac{dt}{(t^2+1)}=2arctgt+C=$$

$$2arctg\sqrt{x}+C.$$

Bo'laklab integrallash usuli Bo'laklab integrallash: faraz qilaylik $u=f(x)$ va $v=g(x)$ lar x ning ikkita funksiyasi va ular $u'=f'(x)$ va $v'=g'(x)$ uzluksiz hosilalarga ega bo'lsin. U vaqtda ikki funksiya ko'paytmani hosilasini olish qoidasiga ko'ra $(uv)'=u'v+uv'$ yoki $d(uv)=udv+vdu$ yoki $udv=d(uv)-vdu$ bo'ladi; $d(uv)$ ifoda uchun, shubhasiz, uv boshlang'ich funksiya bo'ladi: shuning uchun $\int udv=uv-\int vdu$ formula o'rinlidir.

Bu formula bo'laklab integrallash qoidasini ifodalaydi. Yuqoridagi keltirib chiqargan formulamiz bo'laklab integrallash formulasini beradi va bu formula ikki funksiya ko'paytmasining hosilasidan kelib chiqqan. Endi bu formulani misollarda tushuntirsak.

Masalan: $\int x\cos xdx$ aniqmas integralni hisoblang.

Bu integralni hisoblashda bo'laklab integrallash formulasidan foydalanamiz. Ya'ni $\cos x$ funksiyani differensiallash belgisi ostiga kiritamiz va integral quyidagi ko'rinishga keladi. $\int x\cos xdx=\int xd(\sin x)$ bu ifodada $u=x$, kabi belgilaymiz va yuqoridagi formuladan foydalanamiz. $\int xd(\sin x)=x\sin x-\int\sin xdx=x\sin x+\cos x+C$ ekanligi kelib chiqadi. Odatda integrallash belgisi ostidagi ifodada qaysi birini u funksiya va qaysi birini v funksiya sifatida qabul qilish mumkin kabi savollar paydo bo'ladi. Yuqoridagi kabi funksiyalarda odatda trigonometrik funksiyalarni differensiallash belgisi ostiga kiritib / funksiya siqatida olish mumkin. O'z- o'zidan ma'lumki darajasi oshib boruvchi funksiyalarni differensiallash belgisi ostiga kiritmagan ma'qul. Yani nomini o'zgartiruvchi yoki umuman o'zgarmaydigan funksiyalarni differensiallash belgisi ostiga kiritib / funksiya sifatida olgan ma'qul.

Misollar yordamida ko'rib chiqamiz.

Misol. $\int\ln(x+8)dx$ aniqmas integralning hisoblang.

Yechish: $\int\ln(x+8)dx=\begin{cases}u=\ln(x+8) & du=\frac{1}{x+8}\\ dv=dx & v=x\end{cases}$ belgilashlar kiritamiz.

Endi bo'laklash formulasidan foydalanadigan bo'lsak, quyidagiga kelamiz.

$\int\ln(x+8)dx=x\ln(x+8)-\int x\cdot d(\ln(x+8))=x\ln(x+8)-\int x\cdot\frac{1}{x+8}dx=x\ln(x+8)-$

$$\int\frac{x+8-8}{x+8}dx = x\ln(x+8)-\int(1-\frac{8}{x+8})dx = x\ln(x+8)+8\ln(x+8)-x+C .$$

Misol. $\int(x^2-3)\cos x dx$ aniqmas integralni hisoblang.

Yechish: ushbu misolni yechish uchun trigonometrik funksiyani differensiallash belgisi ostiga kiritib, keyin ikkita funksiyaga keltiriladi. $\int(x^2-3)\cos x dx = \int(x^2-3)d(\sin x)$ ko'rinishiga keltirib keyin quyidagicha belgilashlar kiritiladi.

$$\int(x^2-3)d(\sin x) = \begin{cases} u = x^2-3 \ du = 2xdx \\ dv = \cos x dx \ v = \sin x \end{cases}$$

$\int(x^2-3)d(\sin x) = \sin x\cdot(x^2-3)-\int 2x\cdot\sin x dx$ integralimiz ko'rinishini o'zgartirdi endi integralning ikkinchi qismidagi integralni hisoblash uchun yana bir marta bo'laklash amalga oshiramiz. Bu bo'laklashda ham yuqoridagi kabi trigonametrik funksiyani differensiallash belgisi ostiga kiritamiz va yuqoridagi kabi belgilashni amalga oshiramiz.

$$\int 2x\cdot\sin x dx = -\int 2xd(\cos x) = -2x\cdot\cos x + 2\int\cos x dx = -2x\cos x + 2\sin x$$

ushbu topilgan oxirgi natija bilan oldingi natijani birlashtirib quyidagini topamiz:

$$\int(x^2-3)\cos x dx = \sin x\cdot(x^2-3)-(-2x\cos x+2\sin x) = (x^2-3)\sin x + 2x\cos x - .$$

$$2\sin x + C .$$

Misol. $\int e^x\sin 2x dx$ aniqmas integralni hisoblang.

Yechish: bu integralni topish uchun qaysi qunksiyani differensial belgisi ostiga kiritishning ahamiyati yo'q.

$\int e^x\sin 2x dx = \int\sin 2x de^x = \begin{cases} u = \sin 2x \ du = 2\cos 2x \\ dv = e^x dx \ v = e^x \end{cases}$ belgilash kiritgach bo'laklab integrallash formulasiga qo'yamiz:

$$\int\sin 2x de^x = \begin{cases} u = \sin 2x \ du = 2\cos 2x \\ dv = e^x dx \ v = e^x \end{cases} = e^x\cdot\sin 2x - 2\int e^x\cos 2x dx$$

Integral shu ko'rinishga kelgach integralning ikkinchi qismi uchun yana bo'laklash bajaramiz. $\int e^x\cos 2x dx = e^x\cos 2x + 2\int e^x\sin 2x dx$ yuqoridagi va keyingi natijalarni birlashtiradigan bo'lsak, quyidagi natijaga kelamiz. $\int\sin 2x de^x = e^x\cdot\sin 2x - 2(e^x\cos 2x + 2\int e^x\sin 2x dx) = e^x\cdot\sin 2x - 2e^x\cos 2x -$

$4\int e^x\sin 2x dx$ bundan ko'rinadiki integralishiz takrorlanuvchi yoki qaytariluvchi integrallardan ekan. Bunday integrallarni hisoblashda quyidagicha amal bajaramiz. $\int\sin 2x de^x = e^x\cdot\sin 2x - 2e^x\cos 2x - 4\int e^x\sin 2x dx$ aniqmas integralni biror noma'lum bilan belgilab tenglama ishlaymiz. $\int e^x\sin 2x dx = y$ va quyidagiga kelamiz.

$$y = e^x\cdot\sin 2x - 2e^x\cos 2x - 4y$$
$$5y = e^x\cdot\sin 2x - 2e^x\cos 2x \quad .$$
$$y = \frac{e^x\cdot\sin 2x - 2e^x\cos 2x}{5}$$

Demak, $\int e^x\sin 2x dx = \frac{e^x\cdot\sin 2x - 2e^x\cos 2x}{5} + C$ bo'ladi.

Misol. $\int\frac{x}{\sin^2 x}dx$ aniqmas integralni hisoblang.

Yechish: ushbu misolni yechishda ham bo'laklab integrallash formulasidan foydalanib ishlaymiz. $\int\frac{x}{\sin^2 x}dx = -\int xd(ctgx)$ integralni kabi ko'rinishga keltirib olamiz va quyidagicha belgilashlarini amalga oshiramiz.

$-\int xd(ctgx) = \begin{cases} u = x, du = dx \\ v = ctgx, dv = -\frac{1}{\sin^2 x} \end{cases}$ belgilashlarni kiritganimizdan so'ng formulaga qo'yamiz $-\int xd(ctgx) = -xctgx + \int ctgx dx = -xctgx + \ln(\sin x) + C$.

Ba'zi trigonometrik funksiyalarni integrallash Ba'zi trigonometrik funksiyalarni integrallashga oid misollarni ko'rib chiqaylik.

Misol. $\int\sin 2x\cos 3x dx$ aniqmas integralni hisoblang.

Yechish: bu integralni topish uchun trigonometriyaning formulalaridan biri bo'lgan ko'paytmani yig'indiga almashtirish formulasidan foydalanamiz. YA'ni $\sin\alpha\cos\beta=\frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$ formuladan foydalanib, integral ostidagi ifodani soddalashtiramiz. $\sin 2x\cos 3x=\frac{1}{2}(\sin 5x-\sin x)$ ekanligi topdik endi

$$\int\sin 2x\cos 3x dx=\int\frac{1}{2}(\sin 5x-\sin x)dx=\frac{1}{2}(-\frac{1}{5}\cos 5x+\cos x)+C$$

ekanligini topamiz va misolning javobi chiqdi.

Misol. $\int\sin^4 x\cos^2 x dx$ aniqmas integralni hisoblang.

Yechish: Ushbu integralni hisoblash uchun trigonametriyaning asosiy ayniyatlaridan biri bo'lgan daraja pasaytirish formulasidan foydalanib olding integral ostini soddalashtiramiz.

$$\sin^4 x\cos^2 x=\frac{(1-\cos 2x)^2}{4}\cdot\frac{1+\cos 2x}{2}=\frac{(1-\cos^2 2x)(1-\cos 2x)}{8}=$$

$$\frac{(1-\cos 4x)(1-\cos 2x)}{16}=\frac{1-\cos 4x-\cos 2x+\cos 4x\cos 2x}{8}=$$

$$\frac{1-\cos 2x-\cos 4x+\frac{1}{2}(\cos 6x+\cos 2x)}{16}=\frac{2-\cos 2x-2\cos 4x+\cos 6x}{32}.$$

Ekanligidan $\int\sin^4 x\cos^2 x dx=\int\frac{2-\cos 2x-2\cos 4x+\cos 6x}{32}dx=$

$$\frac{1}{32}(2x-\frac{1}{2}\sin 2x-\frac{1}{2}\sin 4x+\frac{1}{6}\sin 6x)+C.$$

Misol. $\int\frac{dx}{\cos x}$ aniqmas integralni hisoblang.

Yechish: ushbu misolni yechish uchun integral belgisi ostidagi funksiyaning surat va maxrajini funksiyaga ko'paytiramiz va aniqmas integralni quyidagicha ko'rinishga keladi. $\int\frac{dx}{\cos x}=\int\frac{\cos x dx}{\cos^2 x}$ endi ushbu

aniqmas integralni hisoblash uchun kasr suratidagi ifodani differensial belgisi ostiga kiritamiz va quyidagini hosil qilamiz,

$$\int\frac{\cos x dx}{\cos^2 x}=\int\frac{d(\sin x)}{1-\sin^2 x}=\int\frac{d(\sin x)}{(1-\sin x)(1+\sin x)}=\int\frac{dt}{(1-t)(1+t)}=-\int\frac{dt}{(t-1)(1+t)}=-\frac{1}{2}\int(\frac{1}{t-1}-\frac{1}{t+1})dt=$$

$$=-\frac{1}{2}\ln\frac{t-1}{t+1}+C=-\frac{1}{2}\ln\left|\frac{\sin x-1}{\sin x+1}\right|+C$$

bundan ko'rinadiki ushbu integralning javobi $\int\frac{dx}{\cos x}=-\frac{1}{2}\ln\left|\frac{\sin x-1}{\sin x+1}\right|+C$ га тенг bo'ladi.

Mustaqil yechish uchun misollar:

1-variant

Quyidagi aniqmas integrallarni hisoblang.

a) $\int(2x+1)^2 dx$, b) $\int\frac{\cos 5x}{\sqrt{\sin^3 5x}}dx$, c) $\int\ln(x+8)dx$,

d) $\int\sin 2x\cos 3x dx$, e) $\int\frac{dx}{x^4\sqrt{1+x^2}}$.

2-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int ctg^2 x dx$, b) $\int\frac{dx}{\arcsin^4 x\sqrt{1-x^2}}$, c) $\int\sqrt{25-x^2}dx$,

d) $\int tg^4 x dx$, e) $\int\sqrt{x^2-4}dx$.

3-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int\frac{x^2-1}{x}dx$, b) $\int\frac{\sqrt{ctg 4x}}{\sin^2 4x}dx$, c) $\int(x^2-x+1)\ln x dx$,

d) $\int\sin^2 x\cos x dx$, e) $\int\frac{dx}{\sqrt{x}\left(\sqrt[4]{x}+1\right)^{10}}$.

4-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int \frac{dx}{1+\cos 2x}$, b) $\int \frac{\sqrt{arctgx}}{1+x^2}dx$, c) $\int \sqrt{x^2+7}dx$,

d) $\int \sin^4 2xdx$, e) $\int \sqrt[3]{x}\sqrt[4]{2+\sqrt[3]{x^2}}dx$.

5-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int (x+\frac{2}{\sqrt[3]{x}})dx$, b) $\int \frac{dx}{(1+x^2)\sqrt{arctgx}}$, c) $\int (3x-2)\ln^2 xdx$,

d) $\int \sin 5x \sin 2xdx$, e) $\int \frac{dx}{(x+2)\sqrt{x^2+2x}}$.

6-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int \frac{dx}{1-\cos 2x}$, b) $\int e^{\sin x}\cos xdx$, c) $\int \ln^2 xdx$,

d) $\int \frac{\sin^4 x}{\cos^2 x}dx$, e) $\int \frac{dx}{x\sqrt{2x^2-2x-1}}$.

7-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int \frac{2x-3\sqrt{x}+3\sqrt[4]{x^3}}{x}dx$, b) $\int e^{5-2x^2}xdx$, c) $\int (x+8)\sin 3xdx$,

d) $\int \cos 3x cos 4xdx$, e) $\int \frac{3x+2}{\sqrt{x^2+x+2}}dx$.

8-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int \sin\frac{x}{2}\cos\frac{x}{2}dx$, b) $\int 4^{\cos 5x}\sin 5xdx$, c) $\int \sqrt[3]{x}\ln^2 xdx$,

d) $\int \cos 3x cos 4xdx$, e) $\int \sqrt{\frac{1-x}{1+x}}\frac{dx}{x}$.

9-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int \frac{\sqrt{x^4+x^{-4}+2}}{x^3}dx$, b) $\int \frac{2x-4}{x^2+16}dx$, c) $\int (x^2-3)\cos xdx$,

d) $\int \cos 3x cos 4xdx$, e) $\int \frac{\sqrt[6]{x}}{1+\sqrt[3]{x}}$.

10-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int \sin^2\frac{x}{4}\cos^2\frac{x}{4}dx$, b) $\int \frac{2^{\sqrt{x}}}{\sqrt{x}}dx$, c) $\int \frac{x}{\cos^2 x}dx$,

d) $\int \frac{\cos^3 x}{\sin^2 x}dx$, e) $\int \frac{dx}{\sqrt{1-2x}-\sqrt[4]{1-2x}}$.

11-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int \sin^2\frac{x}{2}dx$, b) $\int \frac{dx}{4+9x^2}$, c) $\int x\cos(x-4)dx$,

d) $\int \cos x\cos 3x\cos 5xdx$, e) . $\int \cos lnxdx$.

12-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int e^{x}(3+\frac{e^{-x}}{\cos^{2}x})dx$, b) $\int\sqrt[3]{3+5\cos x}\sin x dx$, c) $\int e^{2x}\cos x dx$,

d) $\int\frac{dx}{\sin x}$, e) . $\int e^{-\frac{x}{2}}\cdot x^{2}dx.$

13-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int\cos^{2}\frac{x}{2}dx$, b) $\int\arcsin x dx$, c) $\int\sin x\cos^{3}x dx$,

d) $\int\frac{dx}{\sin x}$, e) . $\int\frac{\cos 2x}{\cos^{2}x\,\sin^{2}x}dx$

14-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int 4^{x}(3+\frac{4^{-x}}{\sqrt[3]{x^{2}}})dx$, b) $\int\frac{x^{2}dx}{(8x^{3}+125)^{\frac{3}{4}}}$, c) $\int\sin x\cos^{3}x dx$,

d) $\int(5x-1)e^{2x}dx$, e) .. $\int\ln(x^{2}+1)\,dx.$

15-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int(\sin\frac{x}{2}-\cos\frac{x}{2})dx$, b) $\int\frac{xdx}{9+x^{4}}$, c) $\int tgx dx$,

d) $\int arctg2x dx$, e) ... $\int\frac{x\cos x}{\sin^{3}x}dx.$

16-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int(2^{x+1}-5^{x-1})dx$, b) $\int\frac{x^{5}}{\sqrt{x^{12}+3}}dx$, c) $\int x\,arcctgx dx$,

d) $\int\sin^{2}x\cos^{2}x dx$, e) .. $\int\frac{\cos 2x}{\cos^{2}x\cdot\sin^{2x}}dx$

17-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int(\sin\frac{x}{2}-\cos\frac{x}{2})(\sin\frac{x}{2}+\cos\frac{x}{2})dx$, b) $\int ctgx dx$, c) $\int\sin^{2}x dx$,

d) $\int\sin^{2}x\cos^{2}x dx$, e) .. $\int\frac{e^{\sqrt{2x-1}}}{\sqrt{2x-1}}dx$

18-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int(2^{x}+3^{x})^{2}dx$, b) $\int\frac{dx}{x\ln x}$, c) $\int\cos^{2}x dx$,

d) $\int\sin^{2}x\cos^{2}x dx$, e) .. $\int\left(x^{2}+2x+\frac{1}{x}\right)dx$

19-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int\frac{xdx}{x^{2}+1}$, b) $\int\frac{dx}{x\ln x}$, c) $\int(\sqrt{x}+\sqrt[3]{x})dx$,

d) $\int x^{3}(1-2x^{4})^{3}dx$, e) .. $\int\frac{dx}{(x+1)\sqrt{1-x-x^{2}}}$.

20-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int \frac{\sqrt{1+\ln x}}{x} dx$, b) $\int ctg^2 x\, dx$., c) $\int \frac{sin^2 x}{cos^4 x} dx$,

d) $\int x^3(1-2x^4)^3 dx$, e). $\int \frac{dx}{(x+1)\sqrt{x^2+2x+10}}$.

21-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int \frac{\arccos x}{\sqrt{1-x^2}} dx$, b) $\int \frac{x-1}{\sqrt[3]{x^2}} dx$., c) $\int \frac{dx}{x \ln x}$,

d). $\int \frac{dx}{(x+1)\sqrt[3]{\ln(x+1)}}$ e). $\int \frac{dx}{(x+1)\sqrt{x^2+2x+10}}$.

22-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int \cos\frac{1}{x}\frac{dx}{x^2}$, b) $\int \cos^3 x dx$.., c) $\int \frac{dx}{x \ln x}$,

d) $\int \frac{(\sqrt{x}-1)^3}{x} dx$. e) $\int \frac{dx}{\sin^2 x \cos^2 x}$.

23-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int x\sqrt{x} dx$, b) $\int \cos^3 x dx$.., c). $\int \frac{cosx\, dx}{\sqrt{1+2sin^2 x}}$

b) d) $\int \frac{dx}{\sqrt[5]{x}}$.. e) $\int \frac{(1+x^2)}{x(1+x^2)} dx$.

24-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int (\sin\frac{x}{2} - \cos\frac{x}{2})(\sin\frac{x}{2} + \cos\frac{x}{2}) dx$, b) $\int ctgx dx$, c) $\int \sin^2 x dx$,

d) $\int \sin^2 x \cos^2 x dx$, e).. $\int \frac{e^{\sqrt{2x-1}}}{\sqrt{2x-1}} dx$

25-variant

Quyidagi aniqmas integrallarni hisoblang

a) $\int \frac{2x - 3\sqrt{x} + 3\sqrt[4]{x^3}}{x} dx$, b) $\int e^{5-2x^2} x dx$, c) $\int (x+8)\sin 3x dx$,

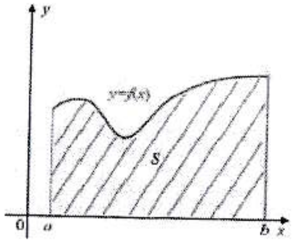
d) $\int \cos 3x cos 4x dx$, e) $\int \frac{3x+2}{\sqrt{x^2+x+2}} dx$.

15-mavzu: Aniq integral tushunchasi. Aniq integralning asosiy xossalari.

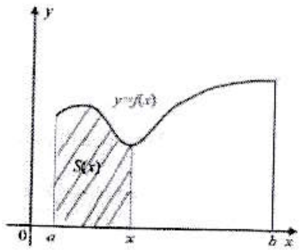
Aniq integrallar. Nyuton- Leybnits formulasi 2-rasmda tasvirlangan shakl egri chiziqli trapetsiya deyiladi. Bu shakl yuqoridan $y=f(x)$ funksiyaning grafigi bilan, quyidan funksiyaning grafigi bilan, quyidan $[a,b]$ kesma bilan, yon tomonlardan esa $x=a$, $x=b$ to‘g‘ri chiziqlaming kesmalari bilan chegaralangan. $[a,b]$ kesma egri chiziqli trepetsiyaning asosi deyiladi.

Egri chiziqli trapetsiyaning yuzini qaysi formulaga ko‘ra hisoblaymiz, degan savol tug‘iladi.

Bu yuzni S deb belgilaylik. S yuzni $y=f(x)$ funksiyaning boshlang‘ich funksiyasi yordamida hisoblash mumkin ekan. Shunga oid mulohazalami keltiramiz.



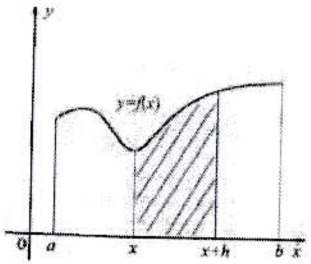
2-rasm.



3-rasm.

$[a,x]$ asosli egri chiziqli trapetsiyaning yuzini asosli egri chiziqli trapetsiyaning yuzini $S(x)$ deb belgilaymiz (3-rasm), bunda x shu $[a,b]$ kesmadagi istalgan nuqta: $x=a$ bo‘lganda $[a,x]$ kesma nuqtaga aylanadi, shuning uchun $S(a)=0$, $x=b$ da $S(b)=S$.

$S(x)$ ni $f(x)$ funksiyaning boshlang‘ich funksiyasi bo‘lishini, ya’ni $S'(x)=f(x)$ ekanini ko‘rsatamiz.



$S(x+h)-S(x)$ ayirmani ko‘raylik, bunda $h>0$ ($h<0$) hol ham xuddi shunday ko‘riladi. Bu ayirma asosi $[x,x+h]$ bo‘lgan egri chiziqli trapetsiyaning yuziga teng (4-rasm). Agar h son kichik bo‘lsa, u holda bu yuz taqriban $f(x)\cdot h$ ga teng, ya’ni $S(x+h)-S(x)\approx f(x)\cdot h$. Demak, $\frac{S(x+h)-S(x)}{h}\approx f(x)$. Bu taqribiy tenglikning chap qismi $h\to 0$ da $S'(x)=f(x)$ tenglik hosil bo‘ladi. Demak, $S(x)$ yuz $f(x)$ funksiya uchun boshlang‘ich funksiyasi ekan.

Boshlang‘ich funksiya $S(x)$ dan ixtiyoriy boshqa boshlang‘ich $F(x)$ funksiya o‘zgarmas songa farq qiladi, ya’ni

$$F(x)=S(x)+C.$$

Bu tenglikdan $x=a$ da $F(a)=S(a)+C$ va $S(a)=0$ bo‘lgani uchun $C=F(a)$. U holda (1) tenglikni quyidagicha yozish mumkin: $S(x)=F(x)-F(a)$. Bundan $x=b$ da $S(b)=F(b)-F(a)$ ekanini topamiz.

Demak, egri chiziqli trapetsiyaning yuzini (2-rasm) quyidagi formula yordam ida hisoblash mumkin:

$$S(b)=F(b)-F(a).$$

Bunda $F(x)$ - berilgan $f(x)$ funksiyaning istalgan boshlang‘ich funksiyasi. Shunday qilib, egri chiziqli trapetsiyaning yuzini hisoblash $f(x)$ funksiyaning $F(x)$ boshlang‘ich funksiyasini topishga, ya’ni $f(x)$ funksiyani integrallashga keltiriladi.

$F(b)-F(a)$ ayirma $f(x)$ funksiyaning $[a,b]$ kesmadagi aniq integrali deyiladi va bunday belgilanadi: $\int_a^b f(x)dx$.

$$\int_a^b f(x)dx=F(b)-F(a)$$

formula Nyuton-Leybnis formulasi deb ataladi.

Misol. $\int_0^2(6x^2-5)dx$ aniq integralni hisobolang.

Yechish: bu aniq integralni hisoblash uchun oldin aniqmas integral hisoblanadi.

$$\int_0^2 (6x^2-5)dx = \frac{6}{3}x^3 - 5x\Big|_0^2 = 2\cdot 8 - 5\cdot 2 = 6$$

Misol. $\int_0^1 \frac{x^2}{x^2+1}dx$ aniq integralni hisoblang.

Yechish: $\int_0^1 \frac{x^2}{x^2+1}dx = \int_0^1 (1-\frac{1}{x^2+1})dx = x - arctgx\Big|_0^1 = 1-\frac{\pi}{4}$.

Misol. $\int_{-\frac{\pi}{4}}^0 tg^2 x dx$ aniq integralni hisoblang.

Yechish:

$$\int_{-\frac{\pi}{4}}^0 tg^2 x dx = \int_{-\frac{\pi}{4}}^0 \frac{\sin^2 x}{\cos^2 x}dx = \int_{-\frac{\pi}{4}}^0 \frac{1-\cos^2 x}{\cos^2 x}dx = \int_{-\frac{\pi}{4}}^0 (\frac{1}{\cos^2 x}-1)dx = tgx - x\Big|_{-\frac{\pi}{4}}^0 =$$

$$0-(-1+\frac{\pi}{4}) = 1-\frac{\pi}{4}.$$

Misol. $\int_0^{\pi/2} \frac{dx}{1+\cos x}$ aniq integralni hisoblang.

Yechish: Ushbu misolni yechish uchun triganometriyaning asosiy ayniyatlari ya'ni yarim burchak formulalaridan foydalanamiz, ya'ni integral ostidagi ifodaning maxrajini soddalashtiramiz. $1+\cos x = 2\cos^2\frac{x}{2}$ ekanligidan quyidagiga kelamiz.

$$\int_0^{\pi/2} \frac{dx}{1+\cos x} = \int_0^{\pi/2} \frac{dx}{2\cos^2\frac{x}{2}} = tg\frac{x}{2}\Big|_0^{\pi/2} = 1-0 = 1.$$

Aniq integrallarda o'zgaruvchi almashtirish va bo'laklash usullari

Misol. $\int_0^1 xe^{x^2}dx$ aniq integralni hisoblang.

Yechish: ushbu aniq integralni hisoblash uchun o'zgaruvchi almashtirish metodidan foydalanamiz.

$$\int_0^1 xe^{x^2}dx = \frac{1}{2}\int_0^1 e^{x^2}d(x^2) = \frac{1}{2}\int_0^1 e^t dt = \frac{1}{2}e^t\Big|_0^1 = \frac{1}{2}e - \frac{1}{2}$$

Misol. $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}dx$ aniq integralni hisoblang.

Yechish: bu integralni hisoblash uchun ham o'zgaruvchi almashtirish metodidan foydalanamiz.

$$\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}dx = \int_1^2 e^{\frac{1}{x}}\cdot\frac{1}{x^2}dx = \int_1^2 e^{\frac{1}{x}}d(\frac{1}{x}) = \int_1^2 e^t dt = e^t\Big|_1^2 = e^{\frac{1}{x}}\Big|_1^2 = e^{0,5} - e.$$

Misol. $\int_1^e x^2 \ln x dx$ aniqmas integralni hisoblang.

Yechish: bu misolni hisoblash uchun bo'laklab integrallashdan foydalanamiz.

$$\int_1^e x^2 \ln x dx = \frac{1}{3}\int_1^e \ln x d(x^3) = \frac{1}{3}x^3\ln x - \frac{1}{3}\int_1^e x^3\cdot\frac{1}{x}dx = \frac{1}{3}x^3\ln x - \frac{1}{3}\int_1^e x^2 dx =$$

$$(\frac{1}{3}x^3\ln x - \frac{1}{9}x^3)\Big|_1^e == \frac{1}{3}e^3 - \frac{1}{9}e^3 + \frac{1}{9} = \frac{2}{9}e^3 + \frac{1}{9}.$$

Misol. $\int_0^1 xe^{-x}dx$ aniq integralni hisoblang.

Yechish: integralni bo'laklab integrallash usuli bilan hisoblaymiz.

$$\int_0^1 xe^{-x}dx = -\int_0^1 xde^{-x} = -xe^{-x} + \int_0^1 e^{-x}dx = -xe^{-x} - e^{-x}\Big|_0^1 = 1-\frac{1}{e}.$$

Misol. $\int_{-3/2}^2 \frac{(x-1)^2}{x^2+3x+4}dx$ aniq integralni hisoblang.

Yechish: $\int_{-3/2}^2 \frac{(x-1)^2}{x^2+3x+4}dx$ bu integralni hisoblash uchun integral belgisi ostidagi ifodaning surat qismini kavsni ochib chiqamiz.

$$\int_{-3/2}^{2}\frac{(x+1)^2}{x^2+3x+4}dx=\int_{-3/2}^{2}\frac{x^2+2x+1}{x^2+3x+4}dx=\int_{-3/2}^{2}\frac{x^2+3x+4-x-3}{x^2+3x+4}dx=$$

$$\int_{-3/2}^{2}1-\frac{x+3}{x^2+3x+4}dx=x-\frac{1}{2}\int_{-3/2}^{2}\frac{3}{x^2+3x+4}+\frac{1}{2}\int_{-3/2}^{2}\frac{d(x^2+3x+4)}{x^2+3x+4}dx=$$

$$x-\frac{1}{2}\int_{-3/2}^{2}\frac{3dx}{(x+\frac{3}{2})^2+\frac{7}{4}}-\frac{1}{2}\int_{-3/2}^{2}\frac{d(x^2+3x+4)}{x^2+3x+4}=$$

$$x-\frac{3}{\sqrt{7}}arcctg\frac{2}{\sqrt{7}}(x+\frac{3}{2})-\ln(x^2+3x+4)\Big|_{-3/2}^{2}=$$

$$2-\frac{3}{\sqrt{7}}arcctg2\sqrt{7}-\ln 14+\frac{3}{2}+\ln\frac{7}{4}.$$

Ekanligini topamiz va oxirgi tenglikning natijasini soddalashtiramiz.

$$2-\frac{3}{\sqrt{7}}arcctg2\sqrt{7}-\ln 14+\frac{3}{2}+\ln\frac{7}{4}=3,5-\frac{3}{\sqrt{7}}arcctg2\sqrt{7}-\ln 14:\frac{7}{4}=$$

$$3,5-\frac{3}{\sqrt{7}}arcctg2\sqrt{7}-\ln 14\cdot\frac{4}{7}=3,5-\frac{3}{\sqrt{7}}arcctg2\sqrt{7}-\ln 8.$$

bundan esa $\int_{-3/2}^{2}\frac{(x-1)^2}{x^2+3x+4}dx=3,5-\frac{3}{\sqrt{7}}arcctg2\sqrt{7}-\ln 8$.

Misol. $\int_{1}^{e}\frac{dx}{x\sqrt{1+\ln x}}$ aniq integralni hisoblang.

Yechish: $\int_{1}^{e}\frac{dx}{x\sqrt{1+\ln x}}$ bu aniq integralni hisoblash uchun o'zgaruvchi kiritish usulidan foydalanamiz. Ya'ni integral ostidagi $\frac{1}{x}$ ifodani differensial belgisi ostiga kiritamiz.

$$\int_{1}^{e}\frac{dx}{x\sqrt{1+\ln x}}=\int_{1}^{e}\frac{1}{x}\cdot\frac{1}{\sqrt{1+\ln x}}dx=\int_{1}^{e}\frac{1}{\sqrt{1+\ln x}}d(1+\ln x)=$$

$$\int_{1}^{e}\frac{dt}{\sqrt{t}}=\int_{1}^{e}t^{-1/2}dt=2t^{1/2}\Big|_{1}^{e}=2(1+\ln x)^{1/2}\Big|_{1}^{e}=2\sqrt{2}-2.$$

Misol. $\int_{1}^{4}\frac{dx}{x+x^2}$ aniq integralni hisoblang.

Yechish: ushbu aniq integralni hisoblash uchun integral ostidagi ifodani ikkita kasr ayirmasi shaklida yozamiz ya'ni,

$$\int_{1}^{4}\frac{dx}{x+x^2}=\int_{1}^{4}\frac{dx}{x(1+x)}=\int_{1}^{4}(\frac{1}{x}-\frac{1}{x+1})dx=(\ln x-\ln(x+1))\Big|_{1}^{4}=\ln\frac{x}{x+1}\Big|_{1}^{4}=\ln\frac{4}{5}-\ln\frac{1}{2}=$$

$$\ln\frac{4}{5}:\frac{1}{2}=\ln\frac{8}{5}.$$

Misol. $\int_{1}^{2}\sqrt{x}\ln x dx$ aniq integralni hisoblang.

Yechish: Ushbu integralni hisoblash uchun aniq integrallarda bo'laklash usulidan foydalanamiz.

$$\int_{1}^{2}\sqrt{x}\ln x dx=\frac{2}{3}\int_{1}^{2}\ln x dx^{3/2}=\frac{2}{3}x^{3/2}\ln x-\frac{2}{3}\int_{1}^{2}x^{3/2}\cdot\frac{1}{x}dx=\frac{2}{3}x^{3/2}\ln x-\frac{2}{3}\int_{1}^{2}x^{1/2}dx=$$

$$\frac{2}{3}x^{3/2}\ln x-\frac{4}{9}x^{3/2}\Big|_{1}^{2}=\frac{4\sqrt{2}}{3}\ln 2-\frac{8\sqrt{2}}{9}+\frac{4}{9}.$$

Mustaqil yechish uchun misollar.

1-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_{0}^{2}(6x^2-5)dx$, b) $\int_{0}^{1}\frac{arctgx}{1+x^2}dx$, c) $\int_{-2}^{-1}(6x^2+2x-10)dx$,

2) $\int_1^b (b-4x)dx \ge 6-5b$ tengsizlik bajariladigan $b>1$ sonlarni toping.

2-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_1^2 (5x^2-\frac{3}{x^2})dx$, b) $\int_1^e x^2 \ln x dx$, c) $\int_0^2 (2x^2-5x+3)dx$.

2) $\int_1^b (b-4x)dx \ge 6-5b$ tengsizlik bajariladigan $b>1$ sonlarni toping.

3-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_0^1 \frac{dx}{x^2-2x+2}$, b) $\int_0^2 \frac{3x^5}{\sqrt{x^6+1}}dx$, c) $\int_{-2}^0 x(x+3)(2x-3)dx$

2) $\int_1^b (b-4x)dx \ge 6-5b$ tengsizlik bajariladigan $b>1$ sonlarni toping.

4-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_0^\pi \sin 3x \cos 2x dx$, b) $\int_0^1 xe^{-x}dx$, c) $\int_1^3 \frac{3x-1}{\sqrt[3]{x}}dx$

2) $\int_1^b (b-4x)dx \ge 6-5b$ tengsizlik bajariladigan $b>1$ sonlarni toping.

5-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_{-1/2}^{1/2} \frac{dx}{\sqrt{1-x^2}}$, b) $\int_1^2 \frac{e^x}{e^x-1}dx$, c) $\int_0^9 (x-3\sqrt{x})dx$

2) b ning qanday qiymatlarida $\int_{b/2}^b \frac{1+2x}{4}dx = \frac{5}{2}$ tenglik bajariladi

6-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_{-\pi/4}^0 tg^2 x dx$, b) $\int_{\pi/6}^{\pi/2} \frac{\cos x}{\sin^3 x}dx$, c) $\int_{-\pi}^{\pi} \sin^2 x dx$

2) b ning qanday qiymatlarida $\int_{b/2}^b \frac{1+2x}{4}dx = \frac{5}{2}$ tenglik bajariladi

7-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_0^{\pi/3} \cos^2 \frac{x}{2}dx$, b) $\int_0^{\ln 5} \frac{e^x\sqrt{e^x-1}}{e^x+3}dx$, c) $\int_{-1}^0 (x-1)(x^2-5)dx$

2) b ning qanday qiymatlarida $\int_{b/2}^b \frac{1+2x}{4}dx = \frac{5}{2}$ tenglik bajariladi

8-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_0^2 (6x^2-5)dx$, b) $\int_{-1}^1 \frac{dx}{e^x+e^{-x}}$, c) $\int_1^9 \left(2x-\frac{3}{\sqrt{x}}\right)dx$

2) b ning qanday qiymatlarida $\int_{b/2}^b \frac{1+2x}{4}dx = \frac{5}{2}$ tenglik bajariladi

9-variant

1) berilgan aniq integrallarni hisoblang

a) $\int_0^1 \frac{x^2}{1+x^2}dx$, b) $\int_0^{\sqrt{\pi}/2} \frac{xdx}{\cos^2 x^2}$, c) $\int_1^4 \frac{x+1}{\sqrt{x}}dx$

2) a ning qanday qiymatlarida $\int_{a/2}^{a} \frac{1-2x}{3}dx = -\frac{4}{3}$ tenglik o'rinli bo'ladi.

10-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_0^1 xe^{x^2}dx$, b) $\int_0^2 e^{3x}dx$, c) $\int_0^{\frac{\pi}{2}} \text{sinxcosx}\, dx$

2) a ning qanday qiymatlarida $\int_{a/2}^{a} \frac{1-2x}{3}dx = -\frac{4}{3}$ tenglik o'rinli bo'ladi.

11-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_1^e \frac{\sin(\ln x)}{x}dx$, b) $\int_{-1}^{1} (x^2+1)dx$, c) $\int_0^4 \frac{5}{\sqrt{2x+1}}dx$

2) a ning qanday qiymatlarida $\int_{a/2}^{a} \frac{1-2x}{3}dx = -\frac{4}{3}$ tenglik o'rinli bo'ladi.

12-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_{-1/2}^{1/2} \frac{dx}{(1-x^2)\sqrt{1-x^2}}$, b) $\int_1^3 \left(x+\frac{1}{x}\right)^3 dx$, c) $\int_0^{\log_3 2} 3^{0,5x}dx$

2) a ning qanday qiymatlarida $\int_{a/2}^{a} \frac{1-2x}{3}dx = -\frac{4}{3}$ tenglik o'rinli bo'ladi.

13-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_0^{\pi/2} \frac{\cos ydy}{4+\sqrt{\sin y}}$, b) $\int_4^9 \frac{1}{\sqrt{x}}dx$, c) $\int_0^7 \frac{4}{\sqrt{x+2}}dx$

2) a ning qanday qiymatlarida $\int_{a/2}^{a} \frac{1-2x}{3}dx = -\frac{4}{3}$ tenglik o'rinli bo'ladi.

14-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_0^{\pi/2} \sin^6 xdx$, b) $\int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)dx$, c) $\int_0^{\log_2 3} 2^{3x}dx$

2) a ning qanday qiymatlarida $\int_{a/2}^{a} \frac{1-2x}{3}dx = -\frac{4}{3}$ tenglik o'rinli bo'ladi.

15-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_1^2 \frac{e^{1/x}}{x^2}dx$, b) $\int_0^{\pi} (\sin^4 x + \cos^4 x)dx$, c) $\int_{0,5b}^{b} \frac{1}{2\ln x}dx$

2) a ning qanday qiymatlarida $\int_{a/2}^{a} \frac{1-2x}{3}dx = -\frac{4}{3}$ tenglik o'rinli bo'ladi.

16-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_0^{\pi} x\sin xdx$, b) $\int_1^2 4e^{4x}dx$, c) $\int_{-3}^{0} (x-2)e^{-x/3}dx$

2) a ning qanday qiymatlarida $\int_{a/2}^{a} \frac{1-2x}{3}dx = -\frac{4}{3}$ tenglik o'rinli bo'ladi.

17-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_0^{\pi} \sin x \cos^2 x dx$, b) $\int_1^3 \frac{1}{x^3} dx$, c) $\int_{-\pi/2}^{\pi} |-2\cos x| dx$

2) a ning qanday qiymatlarida $\int_{0,5a}^{a} e^{2x} dx = 1$ tenglik o'rinli bo'ladi.

18-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_1^7 \sqrt{x} dx$, b) $\int_{-3\pi}^{0} \cos 3x dx$, c) $\int_{-1}^{0} (|x| + 2) dx$

2) a ning qanday qiymatlarida $\int_{0,5a}^{a} e^{2x} dx = 1$ tenglik o'rinli bo'ladi.

19-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_2^3 \frac{1}{x^2} dx$, b) $\int_0^1 (e^x - 2) dx$, c) $\int_1^4 |x - 3| dx$

2) a ning qanday qiymatlarida $\int_{0,5a}^{a} e^{2x} dx = 1$ tenglik o'rinli bo'ladi.

20-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_{-\pi}^{2\pi} \cos x \, dx$, b) $\int_{-1}^{2} (1 - 3x^2) dx$, c) $\int_1^4 |x^2 - 1| dx$

2) a ning qanday qiymatlarida $\int_{0,5a}^{a} e^{2x} dx = 1$ tenglik o'rinli bo'ladi.

21-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_0^1 x dx$, b) $\int_0^{\ln 2} e^x dx$, c) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx$

2) $\int_1^a 2x dx > 3$ tengsizlikni qanoatlantiruvchi barcha a larni toping.

22-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_0^3 x^2 dx$, b) $\int_{-2\pi}^{\pi} \sin 2x dx$, c) $\int_{-1}^{0} |3^x - 3^{-x}| dx$

2) $\int_1^a 2x dx > 3$ tengsizlikni qanoatlantiruvchi barcha a larni toping.

23-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_{-1}^{2} 3x^2 dx$, b) $\int_1^e \frac{1}{x} dx$, c) $\int_{-1}^{1} x^2 |x| dx$

2) $\int_1^a 2x dx > 3$ tengsizlikni qanoatlantiruvchi barcha a larni toping.

24-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_{-2}^{3} 2x dx$, b) $\int_{-2\pi}^{\pi} \sin x dx$, c) $\int_{-2}^{1} |x^2 + 2x + 5| dx$

2) $\int_2^a 2x dx < 5$ tengsizlikni qanoatlantiruvchi barcha a larni toping.

25-variant

1) Berilgan aniq integrallarni hisoblang

a) $\int_{-3}^{2}(2x-3)dx$, b) $\int_{-2}^{-1}(5-4x)dx$, c) $\int_{-2}^{1}\left|x^2+2x+5\right|dx$

2) $\int_{2}^{a}2xdx<5$ tengsizlikni qanoatlantiruvchi barcha a larni toping.

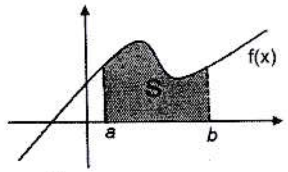
16-mavzu: Aniq integralning tadbiqlari. Tekis figuralarning yuzalarini va hajmlarini hisoblash.

Endi esa biz aniq integrallarning tadbiqlarini ko'rib o'tamiz.

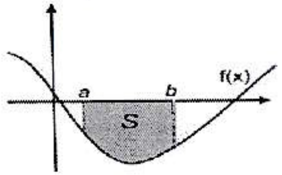
Birinchi tadbiqlaridan biri bu yoy uzunligini topish, ikkinchi tadbiqlaridan biri bu egri chiziqli trapetsiya yuzini topish yoki bir nechta funksiyalar bilan chegaralangan shaklning yuzini topish, uchinchi tadbiqlaridan biri esa aylanma jismlarning sirti yuzasini topish, to'rtinchi tadbiqlaridan biri o'zgaruvchi kuchning bajargan ishi va yana bir tadbiqlaridan inersiya momenti bo'lib biz aniq integrallarning yuza topishga doir misollarini ko'rib chiqamiz.

Egri chiziqli trapetsiyaning yuzi

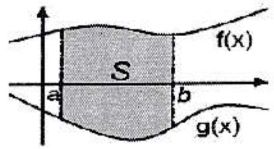
$x\in[a;b]$ да $y=f(x)$ funksiyaning grafigi va Ox o'qi bilan chegaralangan yuza - $S=\int_{a}^{b}f(x)dx$.



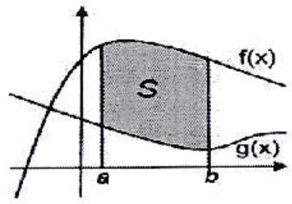
Agar $y=f(x)$ funksiyaning grafigi $x\in[a;b]$ da Ox o'qidan pastda joylashgan bo'lsa, u holda yuza - $S=-\int_{a}^{b}f(x)dx$



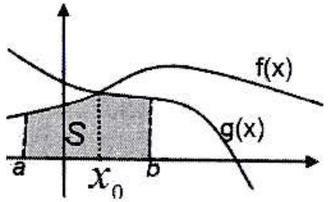
$x\in[a;b]$ da $y=f(x)$ va $y=g(x)$ funksiyalar grafiglari bilan chegaralangan yuza - $S=\int_{a}^{b}(f(x)+g(x))dx$.



1. $x\in[a;b]$ da $y=f(x)$ va $y=g(x)$ funksiyalar grafiklari bilan chegaralangan

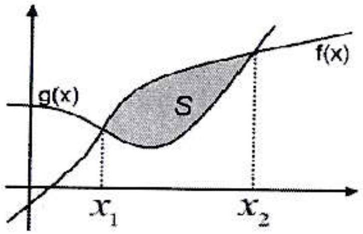


yuza - $S=\int_a^b\big(f(x)-g(x)\big)dx$.



2. $x\in[a;b]$ da $y=f(x)$ va $y=g(x)$ funksiyalar grafiklari va Ox o'qi bilan chegaralangan yuza - $S=\int_a^{x_0}\big(f(x)-g(x)\big)dx$

Bu yerda x_0 - $f(x)=g(x)$ tenglamaning $x\in[a;b]$ dagi ildizi.

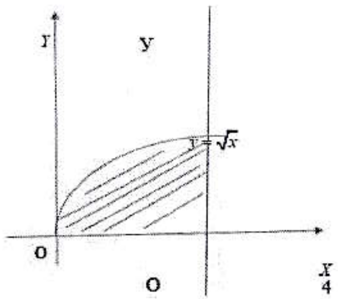


$y=f(x)$ va $y=g(x)$ funksiyalar grafiklari bilan chegaralangan yuza - $S=\int_{x_1}^{x_2}\big(f(x)-g(x)\big)dx$

Bu yerda x_1 va x_2 lar $f(x)=g(x)$ tenglamaning $x\in[a;b]$ dagi ildizlari.

Misol. $y=\sqrt{x}$, $y=0$, $x=4$ chiziqlar bilan chegaralangan figuraning yuzini toping.

Yechish: ushbu misolni yechish uchun aniq integralning chegaralarini aniqlashtirib olishimiz kerak buning uchun $y=\sqrt{x}$, $y=0$ funksiyalarni tenglashtirib o'zgaruvchining qiymatlarini topamiz va yechim aniq integralning quyi chegarasi bo'ladi. $\int_0^4\sqrt{x}dx=\frac{3}{2}x\sqrt{x}\Big|_0^4=\frac{3}{2}\cdot4\cdot2-0=12$



Misol. $y=\sin x$, $y=0$, $x=\pi$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

Yechish: $\sin x=0$ tenglamani yechamiz va quyi chegarasi tenglamaning yechimi bo'ladigan aniq integral tuziladi. $\int_0^{\pi}\sin xdx=-\cos x\Big|_0^{\pi}=-(-1)+0=1$.

Mustaqil yechish uchun misollar:

1-variant

1) $y=\sqrt{x^3}$ funksiya grafigining $0\le x\le4$ oraliq bilan chegaralangan qismining yuzini toping.

2) Ushbu $y=4x^2$, $y=\frac{3}{x}$, va $x=e$ chiziqlar bilan chegaralangan figuraning yuzini toping.

2-variant

1) $y=\frac{x^2}{4}$, funksiya grafigining $0\le x\le2$ oraliq bilan chegaralangan qismining yuzini toping.

2) $y=5x^2$, $y=2x+1$ chiziqlar bilan chegaralangan sohaning yuzini toping.

3-variant

1) $y=\frac{4}{5}\sqrt[5]{x^4}$ funksiya grafigining $0\le x\le9$ oraliq bilan chegaralangan qismining yuzini toping.

2) $y=3x^3$ ва $y=4\sqrt{x}$ chiziqlar bilan chegaralangan shaklning yuzini toping.

4-variant

1) $y=\ln x$ funksiya grafigining $2\sqrt{2}\le x\le 2\sqrt{6}$ oraliq bilan chegaralangan qismining yuzini toping.

2) $4x-6y+10=0$, $y=0$, $x=4$ va $x=5$ chiziqlar bilan chegaralangan figuraning yuzini toping

5-variant

1) $y=\ln\cos x$ funksiya grafigining $0\le x\le\frac{\pi}{4}$ oraliq bilan chegaralangan qismining yuzini toping.

2) $y=x$, $y=\frac{4}{x}$, $y=0$ va $x=e$ chiziqlar bilan chegaralangan figuraning yuzini toping.

6-variant

1) $y=e^x$ funksiya grafigining $0\le x\le\ln 7$ oraliq bilan chegaralangan qismining yuzini toping.

2) $x=0$, $y=16-x^2$ va $y=x^2+3$ chiziqlar bilan chegaralangan sohaning yuzini toping.

7-variant

1) $y=\frac{x^2}{4}-\frac{1}{2}\ln x$ funksiya grafigining $1\le x\le e$ oraliq bilan chegaralangan qismining yuzini toping.

2) $y=2-|x-3|$ va $y=|x-3|$ chiziqlar bilan chegaralangan sohaning yuzini toping.

8-variant

1) $y=2\ln\frac{4}{4-x^2}$ funksiya grafigining $-1\le x\le 1$ oraliq bilan chegaralangan qismining yuzini toping.

2) $x\in[0;\pi]$ da $y=\cos x$ funksiyaning grafigi va x o'qi bilan chegaralangan yuzani toping.

9-variant

1) $y=\sqrt{2x-x^2}-1$ funksiya grafigining $\frac{1}{4}\le x\le 1$ oraliq bilan chegaralangan qismining yuzini toping.

2) $y=16-x^2$ va $y=2x^2-8$ chiziqlar bilan chegaralangan sohaning yuzini toping.

10-variant

1) $y=\frac{x}{4}\sqrt{2-x^2}$ funksiya grafigining $0\le x\le 1$ oraliq bilan chegaralangan qismining yuzini toping.

2) Ushbu $y=-3x^2$, $y=0$, $x=1$ ва $x=2$ chiziqlar bilan chegaralangan figuraning yuzini toping.

11-variant

1) $y=2\sqrt{x}$ funksiya grafigining $1\le x\le 2$ oraliq bilan chegaralangan qismining yuzini toping.

2) $y=\sqrt{x+2}$, $y=x-4$ va $y=0$ chiziqlar bilan chegaralangan shaklning yuzini toping.

12-variant

1) $y=\frac{1}{x}$, $x=1$, $x=4$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

2) $y=x^2+1$ parabola va *y=0, x=-1, x=4* to'g'ri chiziqlar bilan chegaralangan figuraning yuzini toping

13-variant

1) $y=\sqrt{x}$, $x=0$, $y=1$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

2) $y=(x-1)^2$ va $x^2-\frac{y^2}{2}=1$ chiziqlar bilan chegaralangan figuraning yuzini toping.

14-variant

1) $y^2=6x$, $y=0$, $x=3$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

2) $y^4=4x^3$ egri chiziqning $O(0,0)$ dan $V(\sqrt{3};\ 2\sqrt{3})$ gacha bo'lgan yoyining uzunligini toping.

15-variant

1) $y=x^2$, $y=\sqrt{x}$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

2) $y=\ln(\sin x)$ egri chiziqning $x=\frac{\pi}{3}$ dan $x=\frac{\pi}{2}$ gacha bo'lgan yoyining uzunligini toping.

16-variant

1) $y=3\sqrt{1-x^2}$, $y=1-x^2$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

2) . $xy=4,\ x=1,\ x=4,\ y=1,$ chiziqlar bilan chegaralangan figurani *Ox* o'qi atrofida aylantirishdan hosil bo'lgan jismning hajmini hisoblang.

17-variant

1) $y=\frac{e^x+e^{-x}}{2}$, $y=0$, $x=-1$, $x=1$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

2) $y=\sqrt{x+1}$, $x+y=1$, $y=0$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

18-variant

1) $y=\cos x$, $y=1-x$, $y=0$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

2) $y=4-x^2$, $x=0$, $y=0$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

19-variant

1) $y=\sqrt{x}$, $y=0$ va $x=3$ chiziqlar bilan chegaralangan figuraning yuzini toping.

2) $y=-2x+5$, $y=0$ va $x=2$ chiziqlar bilan chegaralangan yuzani hisoblang.

20-variant

1) Ushbu chiziqlar bilan chegaralangan figuraning yuzini hisoblang. $y=\sin 2x$, $y=0$, $x=0$ ва $x=\frac{\pi}{4}$

2) Quyidagi chiziqlar bilan chegaralangan yuzani hisoblang. $y=-\frac{x}{2}$, $y=0$ ва $x=5$

21-variant

1) $y=\ln\cos x$ funksiya grafigining $0\le x\le\frac{\pi}{4}$ oraliq bilan chegaralangan qismining yuzini toping.

2) $y=x$, $y=\frac{4}{x}$, $y=0$ va $x=e$ chiziqlar bilan chegaralangan figuraning yuzini toping.

22-variant

1) $y=2\ln\frac{4}{4-x^2}$ funksiya grafigining $-1\le x\le 1$ oraliq bilan chegaralangan qismining yuzini toping.

2) $x\in[0;\pi]$ da $y=\cos x$ funksiyaning grafigi va x o'qi bilan chegaralangan yuzani toping.

23-variant

1) $y=3\sqrt{1-x^2}$, $y=1-x^2$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

2) . $xy=4,\ x=1,\ x=4,\ y=1,$ chiziqlar bilan chegaralangan figurani Ox o'qi atrofida aylantirishdan hosil bo'lgan jismning hajmini hisoblang.

24-variant

1) $y=\ln\cos x$ funksiya grafigining $0\le x\le\frac{\pi}{4}$ oraliq bilan chegaralangan qismining yuzini toping.

2) $y=x$, $y=\frac{4}{x}$, $y=0$ va $x=e$ chiziqlar bilan chegaralangan figuraning yuzini toping.

25-variant

1) $y=\frac{1}{x}$, $x=1$, $x=4$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

2) $y=x^2+1$ parabola va $y=0$, $x=-1$, $x=4$ to'g'ri chiziqlar bilan chegaralangan figuraning yuzini toping

IV BOB ODDIY DIFFERENSIAL TENGLAMALAR BO'LIMI MAVZULARINI O'QITISHDA KASBGA YO'NALTIRISH ISHLARINI TASHKIL QILISH

17-mavzu: Masalaning qo'yilishi. Birinchi tartibli differensial tenglamalar. O'zgaruvchilari ajralgan va unga keltiriladigan differensial tenglamalar.

Differensial tenglama deb, erkli o'zgaruvchi *x*, noma'lum funksiya *y(x)* va uning turli tartibli hosilalari yoki differensiallarini bog'lovchi tenglamaga aytiladi va $F(x,y,y',y'',...,y^{(n)})=0$ ko'rinishda yoziladi.

Agar noma'lum funksiya birgina erkli o'zgaruvchiga bog'liq bo'lsa, bunday differensial tenglama oddiy differensial tenglama deyiladi. Birinchi tartibli differensial tenglama $F(x,y,y')=0$ yoki hosilaga nisbatan yechilgan bo'lsa $y'=f(x,y)$ ko'rinishda yoziladi.

Birinchi tartibli differensial tenglamalarning umumiy yechimi deb, bitta ixtiyoriy o'zgarmas *C* miqdorga bog'liq bo'lgan hamda quyidagi shartlarni qanoatlantiruvchi $y=\varphi(x,C)$ funksiyaga aytiladi.

a) bu funksiya differensial tenglamani *C* o'zgarmas miqdorning har qanday aniq qiymatida ham qanoatlantiradi.

b) $x=x_0$ bo'lganda $y=y_0$ boshlang'ich shart har qanday bo'lganda ham *C* miqdorning shunday $C=C_0$ qiymatini topish mumkinki, $y=\varphi(x,C_0)$ funksiya berilgan boshlang'ich shartni qanoatlantiradi.

$y=\varphi(x,C_0)$ berilgan tenglamaning xususiy yechimi bo'ladi.

Biz o'zgaruvchilari ajratilgan va o'zgaruvchilari ajraladigan hamda bir jinsli va chiziqli differensial tenglamalarni qaraymiz.

$M(x)dx+N(y)dy=0$ ko'rinishdagi tenglama o'zgaruvchilari ajralgan tenglama deyiladi. Bu tenglama yechimini topish uchun har bir o'zgaruvchi bo'yicha integrallanadi.

I. $M(x)N(y)dx+P(x)Q(y)dy=0$ ko'rinishdagi tenglama o'zgaruvchilari ajraladigan tenglama deyiladi. Bu tenglamada oldin o'zgaruvchilar ajratiladi, so'ngra integrallanadi.

Misollar keltiramiz:

Misol. $y'=y\cdot ctgx, (0<x<\pi, -\infty<y<+\infty)$ differensial tenglamaning $x_0=\frac{\pi}{6}, y_0=2$ shartni qanoatlantiruvchi yechimini toping.

Yechish: $\frac{dy}{dx}=y\cdot ctg\,x,\quad \frac{dy}{y}=ctg\,x\cdot dx,\quad \ln y=\ln\sin x+\ln C,\quad y=C\cdot\sin x$ umumiy yechim.

$x_0=\frac{\pi}{6}, y_0=2$ shartlarni qo'yamiz, $2=C\cdot\sin\frac{\pi}{6},\quad C=4,\quad y=4\cdot\sin x$ xususiy yechim.

Misol. $xdx+ydy=0$ o'zgaruvchilari ajralgan differensial tenglamani yeching.

Yechish: O'zgaruvchilari ajralgan differensial tenglamani har bir o'zgaruvchi bo'yicha integrallaymiz: $\int xdx+\int ydy=C,\quad \frac{x^2}{2}+\frac{y^2}{2}=C,$ $x^2+y^2=C_1^2$ tenglamaning umumiy yechimi bo'lib, markazi koordinata boshida yotgan, radiusi C_1 ga teng bo'lgan aylanalar oilasining tenglamasidir.

Misol. $(1+x)ydx+(1-y)xdy=0$ differensial tenglama yeching.

Yechish: Bu o'zgaruvchilari ajraladigan differensial tenglamadir. O'zgaruvchilarni ajratish uchun tenglamaning har bir hadini $xy\neq 0$ ga bo'lamiz.

$\frac{1+x}{x}dx+\frac{1-y}{y}dy=0,$ integrallaymiz: $\ln|x|+x+\ln|y|-y=\ln C,$

$\ln\left|\frac{xy}{C}\right|=y-x,\quad \frac{xy}{C}=e^{y-x},\quad xy=Ce^{y-x}$ bu tenglamaning umumiy integrali.

II $\frac{dy}{dx}=\varphi(\frac{y}{x})$ ko'rinishdagi tenglamaga bir jinsli differensial tenglama deyiladi.

$\frac{y}{x}=u$ almashtirish bilan integrallanadi. $\frac{y}{x}$ - nol o'lchovli bir jinsli funksiya.

Misol. $\frac{dy}{dx}=\frac{2x^2y}{x^3-y^3}$ differensial tenglamani yeching.

Yechish: Tenglama bir jinsli differensial tenglama, bu tenglamani $y=ux$ almashtirish bajarib, integrallaymiz, $y=ux$ dan $\frac{dy}{dx}=u+x\frac{du}{dx}$ ni topamiz va tenglamaga qo'yamiz:

$$u+x\frac{du}{dx}=\frac{x^2\cdot ux}{x^3-u^3x^3};\ x\frac{du}{dx}=\frac{u}{1-u^3}-u;\ x\frac{du}{dx}=\frac{u^4}{1-u^3}.$$

Tenglamani yechish uchun o'zgaruvchilarni ajratamiz: $\frac{dx}{x}=\frac{1-u^3}{u^4}du$ bu tenglamani integrallaymiz $\ln x+\ln C=-\frac{1}{3u^3}-\ln u$, $\ln(C\cdot x\cdot u)=-\frac{1}{3u^3}$, $\ln(C\cdot x\cdot\frac{y}{x})=-\frac{x^3}{3y^3}$, $\ln Cy=-\frac{x^3}{3y^3}$ tenglamaning umumiy integrali.

III. Chiziqli differensial tenglamalar. Birinchi tartibli chiziqli differensial tenglama deb

$$y'+P(x)y=Q(x) \quad (1)$$

ko'rinishdagi tenglamaga aytiladi. Birinchi tartibli chiziqli tenglama ikki usulda integrallanadi.

1-usul. (1) tenglama yechimi ikki funksiya ko'paytmasi shaklida qidiriladi:

$$y=u(x)\,v(x) \quad (2)$$

u, v lardan birini topish ixtiyoriy.

2-usul. O'zgarmasni variatsiyalash usuli (1) tenglamaning umumiy yechimini topishda $Q(x)=0$ deb olib, uning umumiy yechimidagi o'zgarmas sonni, *x* ning funksiyasi deb qaraladi va $y'+P(x)y=0$ bir jinsli chiziqli tenglama yechiladi.

Mislollar keltiramiz:

Misol. $\frac{dy}{dx}-2xy=x-x^3$ chiziqli tenglamaning umumiy yechimini toping.

Yechish: $P(x)=-2x,\ Q(x)=x-x^3$.

Tenglamaning umumiy yechimini

$$y=uv \quad (2)$$

ko'rinishda qidiramiz va bu tenglikni differensiallaymiz:

$$\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx} \quad (3)$$

(2), (3) ni berilgan tenglamaga qo'yamiz:

$$u\frac{dv}{dx}+v\frac{du}{dx}-2xuv=x-x^3,\ u(\frac{dv}{dx}-2xv)+v\frac{du}{dx}=x-x^3 \quad (4)$$

u, v lardan birini topish ixtiyoriy bo`lgani uchun *v* ni $\frac{dv}{dx}-2xv=0$ tenglikdan topamiz.

$\frac{dv}{v}=2xdx$, $\ln|v|=x^2+\ln C$, $v=Ce^{x^2}$ xususiy holda C=1 deb olsak, $v=e^{x^2}$ ga teng bo'ladi. Bularni (4) ga qo'yib, o'zgaruvchilarni ajratib, integrallaymiz:

$$du=(x-x^3)e^{x^2}dx,\ u=\int(x-x^3)e^{x^2}dx=\int xe^{x^2}dx-\int x^3e^{x^2}dx.$$

Bu integrallarni bo'laklab, integrallaymiz:

$$u=e^{x^2}(1-\frac{1}{2}x^2)+C,\ y=uv=e^{x^2}\cdot(e^{x^2}(1-\frac{1}{2}x^2)+C).$$

Misol. $y'+2xy=x\,e^{-x^2}$ tenglamaning umumiy yechimini toping.

Yechish: $Q(x)=x\,e^{-x^2}=0$ deb olib,

$y'+2xy=0$ tenglamani hosil qilamiz:

$$\frac{dy}{dx}=-2xy,\ \frac{dy}{y}=-2xdx,\ \ln|y|=-x^2+\ln C,\ y=Ce^{-x^2}.$$

Tenglamaning umumiy yechimidagi C ni x ning funksiyasi deb qarab, y ni x bo'yicha integrallaymiz:

$\frac{dy}{dx}=\frac{dC}{dx}e^{-x^2}-2Ce^{-x^2}$ ni berilgan tenglamaga qo'yamiz.

$$\frac{dC}{dx}e^{-x^2}-2Ce^{-x^2}+2Ce^{-x^2}=xe^{-x^2},\ \frac{dC}{dx}e^{-x^2}=xe^{-x^2},\ dC=xdx,\ C=\frac{x^2}{2}+C_1,$$

$y=Ce^{-x^2}=(\frac{x^2}{2}+C_1)e^{-x^2}$ tenglamaning umumiy yechimi hosil bo'ladi.

Mustaqil yechish uchun mashqlar

1-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $xy\,dx+(x+1)dy=0$.

2) Quyidagi bir jinsli tenglamalarni yeching. $x\frac{dy}{dx}+y=0$.

3) Tenglamalarni integrallang. $z'=10^{x+z}$.

2-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $\sqrt{y^2+1}\ dx=xy\,dy$

2) Quyidagi bir jinsli tenglamalarni yeching. $xdy=(x+y)dx$.

3) Tenglamalarni integrallang $e^{-s}(1+\frac{ds}{dt})=1$.

3-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $(x^2-1)y'+2xy^2=0$ $y(0)=1$

2) Quyidagi bir jinsli tenglamalarni yeching $(x+2y)dx-xdy=0$.

3) Tenglamalarni integrallang $x\frac{dx}{dt}+t=1$.

4-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping.. $y'\,ctgx+y=2$ $y(0)=-1$.

2) Quyidagi bir jinsli tenglamalarni yeching. $(x-y)dx+(x+y)dy=0$.

3) Tenglamalarni integrallang. $(y^2-2xy)dx+x^2dy=0$.

5-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping $y'=3\sqrt[3]{y^2}$, $x=2$ ga $y=0$.

2) Quyidagi bir jinsli tenglamalarni yeching. $(x-y)dx+(x+y)dy=0$.

3) Tenglamalarni integrallang $2x^3y'=y(2x^2-y^2)$.

6-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $2x^2y\,y'+y^2=2$.

2) Tenglamalarni integrallang $y'-y=2x-3$.

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $xy'-2y=2x^4$.

7-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping.. $y'\,ctgx+y=2$ $y(0)=-1$.

2) Quyidagi bir jinsli tenglamalarni yeching. $(x-y)dx+(x+y)dy=0$.

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $(x^2+y^2)y'=2xy$.

8-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping $y'=3\sqrt[3]{y^2}$, $x=2$ ga $y=0$.

2) Quyidagi bir jinsli tenglamalarni yeching . $(x-y)dx+(x+y)dy=0$.

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $xy'-y=x\,tg\,\frac{y}{x}$.

9-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $2x^2yy'+y^2=2$.

2) Tenglamalarni integrallang $y'-y=2x-3$.

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $x^2y'+xy+1=0$.

10-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $xy\,dx+(x+1)dy=0$.

2) Quyidagi bir jinsli tenglamalarni yeching. $x\frac{dy}{dx}+y=0$.

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $xy'+(x+1)y=3x^2e^{-x}$.

11-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $\sqrt{y^2+1}\;dx=xy\,dy$

2) Quyidagi bir jinsli tenglamalarni yeching. $xdy=(x+y)dx$.

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $(2x+1)\,y'=4x+2y$.

12-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $(x^2-1)\,y'+2xy^2=0\; y(0)=1$

2) Quyidagi bir jinsli tenglamalarni yeching $(x+2y)dx-xdy=0$.

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $x(y'-y)=e^x$.

13-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $xy\,dx+(x+1)dy=0$.

2) Quyidagi bir jinsli tenglamalarni yeching. $x\frac{dy}{dx}+y=0$.

3) Tenglamalarni integrallang . $z'=10^{x+z}$.

14-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $\sqrt{y^2+1}\;dx=xy\,dy$

2) Quyidagi bir jinsli tenglamalarni yeching. $xdy=(x+y)dx$.

3) Tenglamalarni integrallang $e^{-s}(1+\frac{ds}{dt})=1$.

15-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $(x^2-1)\,y'+2xy^2=0\; y(0)=1$

2) Quyidagi bir jinsli tenglamalarni yeching $(x+2y)dx-xdy=0$.

3) Tenglamalarni integrallang $x\frac{dx}{dt}+t=1$.

16-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping.. $y'\,ctgx+y=2\quad y(0)=-1$.

2) Quyidagi bir jinsli tenglamalarni yeching . $(x-y)dx+(x+y)dy=0$.

3) Tenglamalarni integrallang . $(y^2-2xy)dx+x^2dy=0$.

17-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich

shartlarni qanoatlantiruvchi yechimni toping $y'=3\sqrt[3]{y^2}$, *x=2 ga y= 0.*

2) Quyidagi bir jinsli tenglamalarni yeching. *(x-y)dx+(x+y)dy =0.*

3) Tenglamalarni integrallang $2x^3 y' = y(2x^2-y^2)$.

18-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $2x^2 y y' + y^2 = 2$.

2) Tenglamalarni integrallang $y' - y = 2x - 3$.

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $xy' - 2y = 2x^4$.

19-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping.. $y' ctgx + y = 2$ $y(0) = -1$.

2) Quyidagi bir jinsli tenglamalarni yeching. *(x-y)dx+(x+y)dy =0.*

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $(x^2 + y^2) y' = 2xy$.

20-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping $y'=3\sqrt[3]{y^2}$, *x=2 ga y= 0.*

2) Quyidagi bir jinsli tenglamalarni yeching. *(x-y)dx+(x+y)dy =0.*

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $xy' - y = x\, tg\, \frac{y}{x}$.

21-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $2x^2 y y' + y^2 = 2$.

2) Tenglamalarni integrallang $y' - y = 2x - 3$.

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $x^2 y' + xy + 1 = 0$.

22-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $xy\,dx + (x+1)dy = 0$.

2) Quyidagi bir jinsli tenglamalarni yeching. $x\frac{dy}{dx} + y = 0$.

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $(xy' - 1)\ln x = 2y$

23-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $\sqrt{y^2+1}\ dx = xy\,dy$

2) Quyidagi bir jinsli tenglamalarni yeching. *xdy=(x+y)dx.*

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $(2x+1)\, y' = 4x + 2y$.

24-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping. $(x^2-1)\, y' + 2xy^2 = 0$ $y(0) = 1$

2) Quyidagi bir jinsli tenglamalarni yeching *(x+2y)dx-xdy=0.*

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $x(y' - y) = e^x$.

25-variant

1) Quyidagi differensial tenglamalarni integrallang va boshlang'ich shartlarni qanoatlantiruvchi yechimni toping $y'=3\sqrt[3]{y^2}$, *x=2 ga y= 0.*

2) Quyidagi bir jinsli tenglamalarni yeching. *(x-y)dx+(x+y)dy =0.*

3) Quyidagi chiziqli differensial tenglamalarni yuqoridagi har ikki usulda yeching. $y + x(y' - x\cos x)$.

18-mavzu: Birinchi tartibli bir jinsli va chiziqli differensial tenglamalar. Bernulli tenglamasi.

Chiziqli differensial tenglamalar. Bernulli tenglamasi

$$y' + P(x)y = Q(x)y^n$$

ko'rinishdagi tenglama Bernulli tenglamasi deyiladi, $P(x)$, $Q(x)$ - x ning uzluksiz funksiyalari. Bu tenglamaning $n\neq 0$, $n\neq 1$ bo'lganda tenglamaning har ikki tomonini y^n ga bo'lib osongina birinchi tartibli chiziqli tenglama ko'rinishiga keltiriladi.

Misol. $\frac{dy}{dx}+\frac{y}{x}=-xy^2$ Bernulli tenglamasini integrallang.

Yechish: Tenglamaning har ikki tomonini y^2 ga bo'lamiz. $\frac{1}{y^2}\frac{dy}{dx}+\frac{1}{xy}=-x$, $\frac{1}{y}=z$ belgilash kiritamiz: $\frac{1}{y}=z$ ni x bo'yicha differensiallaymiz $-\frac{1}{y^2}\frac{dy}{dx}=\frac{dz}{dx}$, $\frac{dz}{dx}-\frac{z}{x}=x$ chiziqli tenglama hosil bo'ladi,

$z=uv$, $\frac{dz}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$ $u\frac{dv}{dx}+v\frac{du}{dx}-\frac{uv}{x}=x$, $u(\frac{dv}{dx}-\frac{v}{x})+v\frac{du}{dx}=x$ $\frac{dv}{dx}-\frac{v}{x}=0$,

$\frac{dv}{dx}=\frac{v}{x}$, $v=x$ $x\frac{du}{dx}=x$, $u=x+C$, $z=x(x+C)$, $\frac{1}{y}=x(x+C)$, $y=\frac{1}{x(x+C)}$

umumiy yechim.

II. ***To'la differensialli tenglama.*** Agar

$$M(x,y)dx+N(x,y)dy=0$$

tenglamaning chap qismi biror $F(x,y)$ funksiyaning to'la differensiali bo'lsa berilgan tenglama to'la differensialli tenglama deyiladi, bunda

$$dF=M(x,y)dx+N(x,y)dy$$

(2) tenglama to'la differensialli bo'lishi uchun $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ shart bajarilishi kerak.

Misol. $(2x+3x^2y)dx+(x^3-3y^2)dy=0$ differensial tenglama to'la differensialli tenglama ekanini aniqlang va tenglamani yeching.

Yechish: $M=2x+3x^2y,\ N=x^3-3y^2$ $\frac{\partial M}{\partial y}=3x^2,\ \frac{\partial N}{\partial x}=3x^2$ $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

tenglik o'rinli, demak berilgan tenglama to'la differensialli tenglama. To'la differensialli

$dF=F_x'dx+F_y'dy$ bo'ladigan $F(x,y)$ funksiyani topamiz. $F_x'=2x+3x^2y$, $F_y'=x^3-3y^2$ tenglamaning birinchisini o'zgaruvchi y ni o'zgarmas deb olib, x bo'yicha integrallaymiz:

$$F=\int(2x+3x^2y)dx=x^2+x^3y+\varphi(y).$$

Bu ifodani ikkinchi tenglamaga qo'yib $[x^2+x^3y+\varphi(y)]_y'=x^3-3y^2$, bu tenglikdan $x^3+\varphi'(y)=x^3-3y^2$, $\varphi'(y)=-3y^2$ hosil bo'ladi. Integrallasak $\varphi(y)=-\frac{3y^3}{3}+C=-y^3+C$. Demak, $F(x,y)=x^2-x^3y-y^3+C$ tenglamaning umumiy yechimi hosil bo'ladi.

Mustaqil yechish uchun mashqlar

1-variant

1) Berilgan Bernulli tenglamalarini integrallang. $\frac{dy}{dx}-\frac{4y}{x}=x\sqrt{y}$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $2xydx+(x^2-y^2)dy=0$.

3) Berilgan differensial tenglamalarni integrallang. $y'=\frac{y}{3x-y^2}$.

2-variant

1) Berilgan Bernulli tenglamalarini integrallang. $\frac{1}{y}\frac{dy}{dx}-\frac{1}{x}=2y^3x$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $(2-9xy^2)xdx+(4y^2-6x^3)ydy=0$.

3) Berilgan differensial tenglamalarni integrallang. $y'=y^2e^{x^2}-2xy$.

3-variant

1) Berilgan Bernulli tenglamalarini integrallang. . $2x^2yy'+3x^2y^2+7=0$. .

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $e^{-y}dx-(2y+xe^{-y})dy=0$

3) Berilgan differensial tenglamalarni integrallang. $2x(1+\sqrt{x^2-y})dx-\sqrt{x^2-y}\,dy=0$.

4-variant

1) Berilgan Bernulli tenglamalarini integrallang. . $2x^2yy'+3x^2y^2+7=0$. .

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $\frac{y}{x}dx+(y^3+\ln x)dy=0$.

3) Berilgan differensial tenglamalarni integrallang. $y'=\frac{y}{3x-y^2}$.

5-variant

1) Berilgan Bernulli tenglamalarini integrallang. $3xy^2\,y'-2y^3=x^2$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching . $\frac{3x^2+y^2}{y^2}dx-\frac{2x^3+5y}{y^3}dy=0$.

3) Berilgan differensial tenglamalarni integrallang. $(1+y^2\sin 2x)dx-2y\cos^2x\,dy=0$.

6-variant

1) Berilgan Bernulli tenglamalarini integrallang. $\frac{dy}{dx}-\frac{4y}{x}=x\sqrt{y}$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $2xydx+(x^2-y^2)dy=0$.

3) Berilgan differensial tenglamalarni integrallang. $y'=\frac{y}{3x-y^2}$.

7-variant

1) Berilgan Bernulli tenglamalarini integrallang. $\frac{1}{y}\frac{dy}{dx}-\frac{1}{x}=2y^3x$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $(2-9xy^2)xdx+(4y^2-6x^3)ydy=0$.

3) Berilgan differensial tenglamalarni integrallang. $3x^2(1+\ln y)dx=(2y-\frac{x^3}{y})dy$

8-variant

1) Berilgan Bernulli tenglamalarini integrallang. . $2x^2yy'+3x^2y^2+7=0$. .

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $e^{-y}dx-(2y+xe^{-y})dy=0$

3) Berilgan differensial tenglamalarni integrallang. $2\,y'+y\,tgx=y^2\sin x$.

9-variant

1) Berilgan Bernulli tenglamalarini integrallang. . $2x^2yy'+3x^2y^2+7=0$. .

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $\frac{y}{x}dx+(y^3+\ln x)dy=0$.

3) Berilgan differensial tenglamalarni integrallang. $(\frac{x}{\sin y}+2)dx+\frac{|x^2+1|\cos y}{\cos 2y-1}dy=0$.

10-variant

1) Berilgan Bernulli tenglamalarini integrallang. $3xy^2\, y' - 2y^3 = x^2$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $\frac{3x^2+y^2}{y^2}dx - \frac{2x^3+5y}{y^3}dy = 0$.

3) Berilgan differensial tenglamalarni integrallang. $xy\,dy = (y^2+x^2)dx$.

11-variant

1) Berilgan Bernulli tenglamalarini integrallang. $\frac{dy}{dx} - \frac{4y}{x} = x\sqrt{y}$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $2xydx + (x^2-y^2)dy = 0$.

3) Berilgan differensial tenglamalarni integrallang. $y' = \frac{y}{3x - y^2}$.

12-variant

1) Berilgan Bernulli tenglamalarini integrallang. $\frac{1}{y}\frac{dy}{dx} - \frac{1}{x} = 2y^3x$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $(2-9xy^2)xdx + (4y^2-6x^3)ydy = 0$.

3) Berilgan differensial tenglamalarni integrallang. $y' = y^2e^{x^2} - 2xy$.

13-variant

1) Berilgan Bernulli tenglamalarini integrallang. . $2x^2y\,y' + 3x^2y^2 + 7 = 0$. .

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $e^{-y}dx - (2y + xe^{-y})dy = 0$

3) Berilgan differensial tenglamalarni integrallang. $2x(1+\sqrt{x^2-y})dx - \sqrt{x^2-y}\,dy = 0$.

14-variant

1) Berilgan Bernulli tenglamalarini integrallang. . $2x^2y\,y' + 3x^2y^2 + 7 = 0$. .

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $\frac{y}{x}dx + (y^3 + \ln x)dy = 0$.

3) Berilgan differensial tenglamalarni integrallang. $y' = \frac{y}{3x - y^2}$.

15-variant

1) Berilgan Bernulli tenglamalarini integrallang. $3xy^2\, y' - 2y^3 = x^2$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $\frac{3x^2+y^2}{y^2}dx - \frac{2x^3+5y}{y^3}dy = 0$.

3) Berilgan differensial tenglamalarni integrallang. $(1+y^2\sin 2x)dx - 2y\cos^2 x\, dy = 0$.

16-variant

1) Berilgan Bernulli tenglamalarini integrallang. $\frac{dy}{dx} - \frac{4y}{x} = x\sqrt{y}$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $2xydx + (x^2-y^2)dy = 0$.

3) Berilgan differensial tenglamalarni integrallang. $y' = \frac{y}{3x - y^2}$.

17-variant

1) Berilgan Bernulli tenglamalarini integrallang. $\frac{1}{y}\frac{dy}{dx} - \frac{1}{x} = 2y^3x$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $(2-9xy^2)xdx + (4y^2-6x^3)ydy = 0$.

3) Berilgan differensial tenglamalarni integrallang. $3x^2(1+\ln y)dx = (2y - \frac{x^3}{y})dy$

18-variant

1) Berilgan Bernulli tenglamalarini integrallang.. $2x^2y\,y' + 3x^2y^2+7=0$. .

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $e^{-y}dx-(2y+xe^{-y})dy=0$

3) Berilgan differensial tenglamalarni integrallang. $2\,y' + y\,tgx = y^2 sinx$.

19-variant

1) Berilgan Bernulli tenglamalarini integrallang.. $2x^2y\,y' + 3x^2y^2+7=0$. .

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $\frac{y}{x}dx + (y^3 + \ln x)dy = 0$.

3) Berilgan differensial tenglamalarni integrallang. $\left(\frac{x}{\sin y}+2\right)dx + \frac{|x^2+1|\cos y}{\cos 2y - 1}dy = 0$

20-variant

1) Berilgan Bernulli tenglamalarini integrallang. $3xy^2\,y' - 2y^3 = x^2$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $.\frac{3x^2+y^2}{y^2}dx - \frac{2x^3+5y}{y^3}dy = 0$.

3) Berilgan differensial tenglamalarni integrallang. $xy\,dy = (y^2+x^2)dx$.

21-variant

1) Berilgan Bernulli tenglamalarini integrallang. $3xy^2\,y' - 2y^3 = x^2$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $.\frac{3x^2+y^2}{y^2}dx - \frac{2x^3+5y}{y^3}dy = 0$.

3) Berilgan differensial tenglamalarni integrallang. $(1+y^2\sin 2x)dx - 2y\cos^2 x\, dy = 0$.

22-variant

1) Berilgan Bernulli tenglamalarini integrallang. $\frac{dy}{dx} - \frac{4y}{x} = x\sqrt{y}$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $2xydx + (x^2-y^2)dy=0$.

3) Berilgan differensial tenglamalarni integrallang. $y' = \frac{y}{3x - y^2}$.

23-variant

1) Berilgan Bernulli tenglamalarini integrallang. $\frac{1}{y}\frac{dy}{dx} - \frac{1}{x} = 2y^3x$.

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $(2-9xy^2)xdx + (4y^2-6x^3)ydy=0$.

3) Berilgan differensial tenglamalarni integrallang. $3x^2(1+\ln y)dx = (2y - \frac{x^3}{y})dy$

24-variant

1) Berilgan Bernulli tenglamalarini integrallang.. $2x^2y\,y' + 3x^2y^2+7=0$. .

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $e^{-y}dx-(2y+xe^{-y})dy=0$

3) Berilgan differensial tenglamalarni integrallang. $2\,y' + y\,tgx = y^2 sinx$.

25-variant

1) Berilgan Bernulli tenglamalarini integrallang.. $2x^2y\,y' + 3x^2y^2+7=0$. .

2) Quyidagi differensial tenglamalarni to'la differensialli tenglama ekanini aniqlang va tenglamalarni yeching $\frac{y}{x}dx + (y^3 + \ln x)dy = 0$.

3) Berilgan differensial tenglamalarni integrallang. $\left(\frac{x}{\sin y}+2\right)dx + \frac{|x^2+1|\cos y}{\cos 2y - 1}dy = 0$.

19-mavzu: Yuqori tartibli differensial tenglamalar. N-tartibli o'zgarmas koeffitsientli bir jinsli chiziqli differensial tenglamalar.

Yuqori tartibli differensial tenglama tushunchasi. n-chi tartibli differensial tenglamani $F(x,y,y',y'',...,y^{(n)})=0$ ko'rinishda yoki agar uni n-chi tartibli hosilaga nisbatan yechish mumkin bo'lsa,

$$y^{(n)}=f(x,y,y',y'',...,y^{(n-1)}) \qquad (1)$$

deb yozish mumkin.

Tartibi n bo'lgan tenglama uchun boshlang'ich shartlar $x=x_0$ da n ta ya'ni

$$\left.\begin{array}{l} y(x_0)=y_0 \\ y'(x_0)=y_0' \\ y''(x_0)=y_0'' \\ \ldots\ldots \\ y^{(n-1)}=y_0^{(n-1)} \end{array}\right\} \text{ yoki } \left.\begin{array}{l} y\big|_{x=x_0}=y_0 \\ y'\big|_{x=x_0}=y_0' \\ \ldots\ldots\ldots\ldots \\ y^{(n-1)}\big|_{x=x_0}=y_0^{(n-1)} \end{array}\right\} \qquad (2)$$

кo'rinishida beriladi.

(1) differensial tenglamaning (2) boshlang'ich shartlarni qanoatlantiruvchi xususiy yechimini topish Koshi masalasi deyiladi.

Ikkinchi tartibli $y''=f(x,y,y')$ tenglama uchun boshlang'ich shart quyidagicha bo'ladi.

$$y\big|_{x=x_0}=y_0, \qquad y'\big|_{x=x_0}=y_0' \qquad (2^1)$$

Bu yerda x_0, y_0, y_0' ma'lum sonlardir. $P_0(x_0,y_0)$ nuqtadan o'tuvchi birgina egri chiziqqa o'tkazilgan urinmaning OX o'qi bilan tashkil etgan burchagining tangensi: $y_0'=tg\,\alpha=k$. Bundan ko'rinadiki y_0' turli qiymatlar qabul qilganda $P_0(x_0,y_0)$ nuqtadan o'tuvchi og'maning burchaklari turlicha bo'lgan cheksiz ko'p integral egri chiziqlar to'plamini hosil qilamiz.

Tartibini pasaytirish mumkin bo'lgan differensial tenglamalar. Eng sodda n-tartibli tenglamani qaraymiz:

$$y^{(n)}=f(x) \qquad (3)$$

Bu tenglamaning umumiy integralini topamiz. Ikkala qismini x bo'yicha integrallab va $y^{(n)}=\left(y^{(n-1)}\right)'$ ekanligini hisobga olib,

$$y^{(n-1)}(x)=\int_{x_0}^{x} f(x)dx+C_1$$ ifodani hosil qilamiz.

Yana bir marta integrallasak:

$$y^{(n-2)}=\int_{x_0}^{x}\left(\int_{x_0}^{x} f(x)dx+C_1\right)dx+C_2$$

Integrallashni shu tartibda davom ettirsak, n marta integrallashdan so'ng

$$y(x)=\int_{x_0}^{x}\ldots\int_{x_0}^{x} f(x)dx\ldots dx+\frac{C_1(x-x_0)^{n-1}}{(n-1)!}+\frac{C_2(x-x_0)^{n-2}}{(n-2)!}+\ldots+C_n$$

ifodani hosil qilamiz.

Misol. $y'''=\sin x-\cos x$ tenglamani yeching.

Yechish: Tenglamaning ikkala qismini x bo'yicha ketma-ket uch marta integrallaymiz:

$$y''=\int(\sin x-\cos x)dx+C_1=-\cos x-\sin x+C_1$$

$$y'=\int(-\cos x-\sin x+C_1)dx=-\sin x+\cos x+C_1x+C_2$$

$$y=\int(-\sin x+\cos x+C_1x+C_2)dx=\cos x-\sin x+C_1\frac{x^2}{2}+C_2x+C_3$$

Misol. Ushbu $y''=\sin(kx)$ tenglamaning $y\big|_{x=0}=0,\ y'\big|_{x=0}=1$ boshlang'ich shartlarni qanoatlantiruvchi xususiy yechimini toping.

Yechish: Ketma-ket ikki marta integrllaymiz.

$$y'=\int_{0}^{x}\sin kx dx+C_1=-\frac{1}{k}\cos kx+C_1$$

$$y(x) = -\frac{1}{k}\int_0^x \cos kx dx + C_1\int_0^x dx + C_2 = -\frac{\sin kx}{k^2} + C_1 x + C_2$$

bu tenglamaning umumiy yechimidir.

Endi xususiy yechimini topamiz. Boshlang'ich shartdan foydalansak:

$$\begin{cases} 1 = -\frac{\cos 0 - 1}{k} + C_1 \\ 0 = -\frac{\sin 0}{k^2} + \frac{0}{k} + C_1 \cdot 0 + C_2 \end{cases}$$

bundan $C_1 = 1$, $C_2 = 0$ ekanini topamiz.

U holda tenglamaning xususiy yechimi: $y = -\frac{\sin kx}{k^2} + x$

Ikkinchi tartibli differensial tenglamalarning ba'zi turlari. Tartibini pasaytirish mumkin bo'lgan ikkita eng sodda ikkinchi tartibli differensial tenglamani qaraymiz.

1. Izlanayotgan y funksiyani oshkor holda o'z ichiga olmagan tenglamani qaraymiz:

$$F(x, y', y'') \text{ yoki } y'' = f(x, y) \qquad (4)$$

Shu tenglamani integrallash bilan shug'ullanamiz. $y' = p$ deb belgilasak $y'' = p'$ bo'ladi, bu yerda

$$y' = \frac{dy}{dx} \qquad y'' = p' = \frac{dp}{dx} \qquad (5)$$

U holda (4) ning o'rnida noma'lum p funksiyaga nisbatan birinchi tartibli tenglama hosil bo'ladi:

$$p' = f(x, p) \text{ yoki } \frac{dp}{dx} = f(x, p(x))$$

Oxirgi ifodani integrallab uning umumiy yechimini topamiz:

$$p = p(x, C_1)$$

Endi $\frac{dy}{dx} = p$ munosabatdan

$$\frac{dy}{dx} = p(x, c_1)$$

hosil bo'ladi. Bu o'zgaruvchilari ajraladigan tenglamadir:

$$dy = p(x, C_1)dx$$

Ikkala qismini integrallab, tenglamaning umumiy yechimini topamiz:

$$y = \int p(x, C_1)dx + C_2 = f_1(x, C_1, C_2) \qquad (6)$$

(4) tenglamani integrallash usuli quyidagi xususiy hollar uchun ham o'rinlidir:

$$F(y', y'') = 0, \; F(x, y'') = 0, \; F(y'') = 0 \qquad (7)$$

Ikkinchi tartibli differensial tenglamaning tartibini pasaytirish usuli bilan yechishni ikkita birinchi tartibli tenglamalar sistemasiga keltirib yechish usuli bilan almashtirish ham mumkin ya'ni,

$$\begin{cases} y' = p \\ F(x, p, p') = 0 \end{cases}$$

Misol. $y'' = y' + x$ tenglama $x = 0$ bo'lganda, $y = 3$, $y' = 0$ shartda integrallang.

Yechish: $y' = p$, $y'' = p'$ deb belgilash kiritib, $p' = p + x$ tenglamani hosil qilamiz. Bu chiziqli tenglamadir. 17§ (2) va (3) munosabatlarga asosan yechamiz:

$$p = uv \;, \; p' = u'v + uv'$$

U holda

$$u'v + uv' + uv = x$$

tenglamani hosil qilamiz.

Bundan

$$\begin{cases} u' - u = 0 \\ uv' = x \end{cases}.$$

Birinchi tenglama o'zgaruvchilari ajraladigan tenglamadir.

$$u' = u, \frac{du}{dx} = u, \qquad \frac{du}{u} = dx.$$

Oxirgi ifoda o'zgaruvchilari ajralgan tenglamadir. Ikkala qismini integrallasak xususiy yechim quyidagicha bo'ladi:

$$u = e^x.$$

Buni ikkinchi tenglamaga qo'yamiz:

$$e^x v' = x, \ \frac{dv}{dx} = xe^{-x}, \ dv = xe^{-x}dx.$$

Oxirgi ifodani bo'laklab integrallaymiz, ya'ni $\int udv = u \cdot v - \int v \cdot du$ formulaga asoslanib, va

$$dv = e^{-x}dx, \ u = x$$

$$v = -e^{-x}, \ du = dx$$

ifodalarni hisobga olib:

$$v = -xe^{-x} + \int e^{-x}dx + C_1 = -xe^{-x} - e^{-x} + C_1$$

ni hosil qilamiz. U holda,

$$p = uv = e^x\left[-e^{-x}(1+x) + C_1\right] = -x - 1 + C_1 e^x$$

$y' = p$ belgilashdan

$$y' = C_1 e^x - x - 1 \ \text{ yoki } \ dy = \left(C_1 e^x - x - 1\right)dx.$$

Bu ifodani integrallasak umumiy yechimini hosil qilamiz:

$$y = C_1 e^x - \frac{x^2}{2} - x + C_2.$$

Endi xususiy yechimni topamiz. Buning uchun boshlang'ich shartdan foydalanib, umumiy yechimdagi C_1 va C_2 ni topish uchun ushbu sistemani hosil qilamiz:

$$\begin{cases} C_1 + C_2 = 3 \\ C_1 - 1 = 0 \end{cases} \Rightarrow \begin{cases} C_1 = 1 \\ C_2 = 2 \end{cases}$$

Shunday qilib tenglamning xususiy yechimini topamiz:

$$y = e^x - \frac{x^2}{2} - x + 2.$$

Misol. $xy'' \cdot \ln x = y$ tenglamaning umumiy yechimini toping.

Yechish: $y' = p, y'' = p'$ deb belgilash bilan quyidagi tenglamani hosil qilamiz:

$$xp' \ln x = p$$

Bu o'zgaruvchilari ajraladigan tenglamalardir:

$$xdp\ln x = pdx$$

yoki

$$\frac{dp}{p} = \frac{dx}{x\ln x}.$$

Bu munosabatni integrallash uchun $\ln x = t, \ \frac{dx}{x} = dt$ deb olamiz. Natijada

$$\ln|P| = \int\frac{dt}{t} \Rightarrow \ln|p| = \ln|t| + \ln C_1 \Rightarrow p = C_1 t \Rightarrow p = C_1 \ln x$$

Shunday qilib $y' = p$ dan y funksiyani topish uchun $y' = C_1 ln x$ tenglamani hosil qilamiz. Bundan

$$y = C_1\int \ln x dx + C_2$$

hosil bo'lgan integralni bo'laklab integrallaymiz. U holda ushbu o'rniga qo'yishdan

$$u = \ln x, \ du = \frac{dx}{x}, \ dv = dx, \ v = x.$$

$y = C_1(x\ln x - \int x \cdot \frac{dx}{x}) + C_2 = C_1 x(\ln x - 1) + C_2$ - umumiy yechimni topamiz.

2. x argumentni o'z ichiga olmagan ikkinchi tartibli differensial tenglamani qaraymiz:

$$F(y,y',y'')=0 \text{ yoki } y''=f(y,y') \qquad (8)$$

Bu ko'rinishdagi tenglamalarni integrallash uchun belgilash kiritamiz:

$$y'=p(y), \quad y''=p\,p' \qquad (9)$$

U holda (8) tenglama

$$pp'=f(y,p) \qquad (10)$$

ko'rinishga keladi.

Bu yerda

$$p'=\frac{dp}{dy}, \qquad y'=\frac{dy}{dx}$$

(10) ni integrallab, uning yechimini $p=\varphi(y,C_1)$ ko'rinishda hosil qilamiz.

Endi berilgan (8) tenglamaning yechimini quyidagi o'zgaruvchilari ajraladigan tenglamadan topamiz:

$$\frac{dy}{dx}=p=\varphi(y,C_1), \quad \frac{dy}{\varphi(y,C_1)}=dx.$$

Natijada

$$\int\frac{dy}{\varphi(y,C_1)}=x+C_2.$$

Misol. $2yy''+y'^2=0$ tenglamani yeching.

Yechish: $y'=p(y)$, $y''=p\,p'$ bo'lgani uchun

$$2ypp'=-p^2 \text{ yoki } 2yp'=-p$$

Bu o'zgaruvchilari ajraladigan tenglamadir $\frac{dp}{p}=-\frac{dy}{2\cdot y}$.

$$\ln|p|=-\frac{1}{2}\ln|y|+\ln C_1 \qquad p=\frac{C_1}{\sqrt{y}}.$$

Endi $y'=\frac{C_1}{\sqrt{y}}$ tenglamadan $y=(C_1x+C_2)^{\frac{2}{3}}$ umumiy yechimini topamiz.

O'zgarmas koeffitsentli ikkinchi tartibli bir jinsli tenglama. Bir jinsli tenglamani qaraymiz:

$$L(y)=0 \qquad (11)$$

L- operator uchun quyidagi tenglik o'rinli:

$$L(e^{kx})=e^{kx}p(k) \qquad (12)$$

Bu yerda $k=const$, $p(k)$-ikkinchi tartibli algebraik ko'p had bo'lib, $p(k)=k^2+a_1k+a_2$ xarakteristik funksiya deb yuritiladi.

Endi (12) munosabatning to'g'riligini isbotlaymiz:

$$L(e^{kx})=(e^{kx})''+a_1(e^{kx})'+a_2e^{kx}=k^2e^{kx}+a_1ke^{kx}+a_2e^{kx}=e^{kx}(k^2+a_1k+a_2)=e^{kx}p(k).$$

$y=e^{kx}$ funksiyani (11) tenglamaga qo'ysak: $L(e^{kx})=e^{kx}p(k)=0$ bo'ladi.

Bu yerda $e^{kx}\neq 0$ bo'lgani sababli

$$p(k)=k^2+a_1k+a_2=0 \qquad (13)$$

hosil bo'ladi va bu xarakteristik tenglama deb ataladi.

Umumiy yechim quyidagi ko'rinishda bo'ladi.

$$y=C_1e^{k_1x}+C_2e^{k_2x} \qquad (14)$$

Misol. $y''-7y'+12y=0$ tenglamaning umumiy yechimini toping.

Yechish: Dastavval xarakteristik tenglama tuzamiz:

$$k^2-7k+12=0,$$

$$k_{1,2}=\frac{7\pm\sqrt{7^2-4\cdot1\cdot12}}{2\cdot1}=\frac{7\pm\sqrt{49-48}}{2}=\frac{7\pm1}{2},$$

$$k_1=\frac{7-1}{2}=\frac{6}{2}=3, \qquad k_2=\frac{7+1}{2}=\frac{8}{2}=8.$$

Xususiy yechimlar:

$$y_1 = e^{3x} \text{ va } y_2 = e^{4x}$$

ko'rinishda bo'lib fundamental sistema hosil qiladi. Haqiqatdan, xususiy yechimlar va ularning birinchi tartibli hosilalaridan tuzilgan Vronskiy determinanti noldan farqli, ya'ni

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{3x} & e^{4x} \\ 3e^{3x} & 4e^{4x} \end{vmatrix} = e^{3x} - e^{4x} \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = e^{(4+3)x}(4-3) = e^{7x} \cdot 1 = e^{7x} \neq 0.$$

Demak, (14) ga asosan qaralayotgan tenglamaning umumiy yechimi quydagicha bo'ladi:

$$y = C_1 e^{3x} + C_2 e^{4x}.$$

Misol. $y'' - y = 0$ tenglamani yeching.

Yechish: $k^2 - 1 = 0,\ (k+1)(k-1) = 0,\ k+1 = 0,\ k_1 = -1,\ k-1 = 0,\ k_2 = 1.$

U holda (14) dan foydalanib berilgan tenglamaning umumiy yechimini yozamiz:

$$y = C_1 e^{-x} + C_2 e^{x}.$$

Xarakteristik tenglamaning ildizlari haqiqiy va bir xil (karrali) bo'lgan hol.

Faraz qilaylik $k_1 = k_2 = k$ bo'lsin. Bu yerda k (14) xarakteristik tenglamaning ustma-ust tushgan karrali ildizidir.

1-usul. y_2 yechim uchun quyidagi formula o'rinli:

$$y_2 = y_1 \int \frac{dx}{y_1 e^{2\int p(x)dx}}$$

Viet teoremasini xarakteristik tenglamaga qo'llab $k_1 + k_2 = -a_1,\ 2k = -a_1,\ a_1 = -2k$ hosil qilamiz. Demak

$$y_2 = e^{kx} \int \frac{dx}{e^{2kx} e^{-\int 2k dx}} = e^{kx} \int \frac{dx}{e^{2kx} e^{-2kx}} = e^{kx} \int \frac{dx}{e^0} = e^{kx} \int dx = xe^{kx}$$ ni hosil qilamiz.

Shunday qilib (11) tenglama yechimlarining fundamental sistemasi:

$$y_1 = e^{kx},\ y_2 = xe^{kx}.$$

U holda (11) ning umumiy yechimi (14)ga asosan:

$$y = e^{kx}(C_1 + C_2 x) \qquad (15.)$$

ko'rinishda bo'ladi.

2-usul. Ikkinchi xususiy yechimni: $y_2 = U(x) e^{k_1 x}$ ko'rinishida izlaymiz.

Ketma-ket ikki marta differensiallab quyidagilarni topamiz:

$$y_2' = U'(x) e^{k_1 x} + k_1 U'(x) e^{k_1 x} = e^{k_1 x}(U' + k_1 U)$$

$$y_2'' = k_1 e^{k_1 x}(U' + k_1 U) + (U'' + k_1 U') e^{k_1 x} = e^{k_1 x}(U'' + 2k_1 U' + k_1^2 U)$$

Hosilalarning bu ifodalarini (11) tenglamaga qo'yib:

$$e^{k_1 x}(U'' + 2k_1 U' + k_1^2 U) + a_1 e^{k_1 x}(U' + kU) + a_2 e^{k_1 x} U =$$
$$e^{k_1 x}\left[U'' + (2k_1 + a_1)U' + (k_1^2 + a_1 k + a_2)U\right] = 0$$

tenglamani hosil qilamiz. k_1 xarakteristik tenglamaning karrali ildizi bo'lgani uchun:

$$k_1^2 + a_1 k_1 + a_2 = 0$$

Bundan tashqari, Viet teoremasiga ko'ra

$$k_1 = k_2 = -\frac{a}{2} \text{ yoki } 2k_1 = -a_1,\ 2k_1 + a_1 = 0.$$

Demak, *U(x)* ni topish uchun $e^{k_1 x} \cdot U'' = 0$ ni hosil qilamiz. $e^{k_1 x} \neq 0$ bo'lgani uchun $U''=0$ bo'ladi. $U''=0$ ni ketma-ket ikki marta integrallab:

$$U' = A,\quad U = A\int dx + B = Ax + B$$ ni hosil qilamiz.

Xususiy holda A=1, B=0 deb olish mumkin. Bundan

$$y_2(x) = U(x) e^{k_1 x} = xe^{k_1 x}$$

bo'ladi. Bu funksiya ikkinchi xususiy yechimdir. Ko'rinib turibdiki

$$\frac{y_2}{y_1} = \frac{xe^{k_1 x}}{e^{k_1 x}} = x \neq const$$

bo'lganligidan bu yechimlar birgalikda chiziqli erklidir. Shuning uchun umumiy yechim $y=C_1e^{k_1x}+C_2e^{k_1x}=e^{k_1x}(C_1+xC_2)$ ko'rinishda bo'ladi.

Misol. $y''-2y'+y=0$ tenglamani yeching.

Yechish: Xarakteristik tenglama formulasi: $k^2-2k+1=0$ bundan $(k-1)^2=0$.

Umumiy yechimi: $y=e^x\left(C_1+C_2x\right)$.

Xarakteristik tenglamaning ildizlari qo'shma kompleks sonlardan iborat bo'lgan hol. $k_1=\alpha+i\beta$, $k_2=\alpha-i\beta$ qo'shma kompleks sonlar xarakteristik tenglamaning ildizlari bo'lsin.

Tenglamaning ikkita xususiy yechimiga egamiz, ya'ni

$$y_1=e^{(\alpha+i\beta)x},\qquad y_2=e^{(\alpha-i\beta)x}$$

Bu yechimlar fundamental sistema tashkil etadi, chunki

$$W(y_1,y_2)=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}=\begin{vmatrix} e^{(\alpha+i\beta)x} & e^{(\alpha-i\beta)x} \\ (\alpha+i\beta)e^{(\alpha+i\beta)x} & (\alpha-i\beta)e^{(\alpha-i\beta)x} \end{vmatrix}=$$

$$=e^{(\alpha+i\beta)x}e^{(\alpha-i\beta)x}\begin{vmatrix} 1 & 1 \\ \alpha+i\beta & \alpha-i\beta \end{vmatrix}=e^{(2\alpha+i\beta-i\beta)x}(\alpha-i\beta-\alpha-i\beta)=$$

$$=e^{2\alpha x}(-2\beta i)\neq 0.$$

bu yerda $\beta\neq 0$, $e^{2\alpha x}\neq 0$.

Endi boshqa fundamental y_1, y_2 sistemaga o'tamiz:

$$y_{1,2}=\frac{y_1\pm y_2}{2}=\frac{e^{(\alpha+i\beta)x}\pm e^{(\alpha-i\beta)x}}{2}=\frac{e^{2x}e^{i\beta x}\pm e^{\alpha x}e^{-i\beta x}}{2}=\frac{e^{\alpha x}\left(e^{i\beta x}\pm e^{-i\beta x}\right)}{2}.$$

Bu yerdan $e^{i\varphi}=\cos\varphi+i\sin\varphi$ bo'ladi.

Eylerning formulasiga asosan:

$$y_1=\frac{e^{\alpha x}}{2}\left(\cos\beta x+i\sin\beta x+\cos\beta x-i\sin\beta x\right)=\frac{e^{\alpha x}}{2}\cdot 2\cos\beta x=e^{\alpha x}\cos\beta x.$$

Xuddi shuningdek, $y_2=\frac{y_1-y_2}{2i}=e^{\alpha x}\sin\beta x$ hosil qilinadi.

Demak, umumiy yechim:

$$y=e^{\alpha x}\left(C_1\cos\beta x+C_2\sin\beta x\right)$$

Mustaqil yechish uchun mashqlar

1-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y''=x+1$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y''-10y'+25y=0$.

2-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y''-x=x^2$

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y''+24y'+144y=0$.

3-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. . $y''-2x=0$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y''-8y'+7y=0$

4-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. . $y''-sinx=0$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y''-7y'+10y=0$.

5-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. . $y''=cos4x$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' + 2y' - 3y = 0$.

6-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. . $y'' = e^{-2x}$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' - y' - 12y = 0$.

7-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. . $y'' = \frac{1}{sin^2x}$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. . $y'' - 4y' - 21y = 0$.

8-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. . $y'' = 4\cos 8x$

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' + 14y' + 49y = 0$.

9-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' = 3x^3 - x^2$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' - 4y' + 10y = 0$.

10-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' - x^2 = 1$. .

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' - 22y' + 121y = 0$.

11-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' = 3x^2 - x^4$. .

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' + 4y' + 4y = 0$.

12-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' = \cos 6x$. .

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' + 2y' = 0$.

13-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' = 3x^2 + 1$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' + 5y = 0$.

14-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' - 7 = x$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' + 4y' = 0$..

15-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' - 2x^4 = 0$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' + 9y' = 0$..

16-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' - 7\sin 2x = 0$. .

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' - 13y = 0$...

17-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' = 7x^2 - 5x$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. . $y'' + 49y = 0$.

18-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. . $y'' = e^{-5x}$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. . $y'' - y' + 2y = 0$.

19-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. . $y'' - 3\sqrt[3]{x} = 0$. .

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. . $y'' - 2y' + 3y = 0$.

20-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' - 2x + \frac{3}{x^2} = 0$. .

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. . $y'' - 4y' + 13y = 0$

21-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' - 2x^4 = 0$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' - 4y' + 20y = 0$

22-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' - 7\sin 2x = 0$. .

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. $y'' - 4y' + 20y = 0$...

23-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. $y'' = 7x^2 - 5x$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. . $y'' - 4y' + 8y = 0$

24-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. . $y'' = e^{-5x}$.

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. . $y'' - 4y' + 8y = 0$.

25-variant

1) Ikkinch tartibli differensial tenglamalarni yeching. . $y'' - 3\sqrt[3]{x} = 0$. .

2) O'zgarmas koeffitsentli differensial tenglamalarni yeching. . $y'' + 6y' + 25y = 0$.

V BOB EXTIMOLLAR NAZARIYASI IQTISODIY VA TEXNIKA MASALALARIGA TADBIQ QILINGAN HOLDA O'QITISHNI TAKOMILLASHTIRISH.

20-mavzu: Kombinatorika elementlari. Ehtimollikning klassik va geometrik ta'rifi.

1.Kombinatorika masalasi. Elementlarning turli kombinatsiyalari va ularning sonini topish bilan bog'liq masalalar *kombinatorika masalalari* deyiladi. Bunday masalalar matematika fanining tarmog'i — kombinatorikada o'rganiladi. Kombinatorika asosan, XVII—XIX asrlarda mustaqil fan sifatida yuzaga kelgan bo'lib, uning rivojiga B.Paskal, P.Ferma, G.Leybnis, Y.Bernulli, L.Eyler kabi olimlar katta hissa qo'shganlar.

Kombinatorikada, asosan, chekli to'plamlar, ularning qism to'plamlari, chekli to'plam elementlaridan tuzilgan kortejlar va ularning sonini topish masalalari o'rganilgani uchun uni to'plamlar nazariyasining bir qismi sifatida qarash mumkin.

2.Yig'indi qoidasi. Kombinatorikada to'plamlar birlashmasi elementlari sonini hisoblash masalasi *yig'indi qoidasi* deb ataladi.

1) Agar $A \cap B = \emptyset$ bo'lsa,

$n(A \cup B) = n(A) + n(B)$ *(1)* bo'ladi.

Ya'ni kesishmaydigan A va B to'plamlar birlashmasi elementlari soni shu to'plamlar elementlari sonlarining yig'indisiga teng.

2) Agar $A \cap B \neq \emptyset$ bo'lsa,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) \quad (2)$$

bo'ladi. Ya'ni umumiy elementga ega ikki to'plam birlashmasi elementlari soni to'plamlarning har biri elementlari sonlari yig'indisidan ularning umumiy elementlari sonining ayrilganiga teng. (2) formula (1)

formulaning umumiy holi bo'lib, (1) formulada $n(A \cap B) = \emptyset$, ya'ni to'plamlarning umumiy elementi yo'q.

3) Yigindi qoidasi umumiy elementga ega bo'lgan uchta A, B, C to'plam uchun quyidagicha yoziladi: agar $A \cap B \cap C = \emptyset$ bo'lsa,

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$ *(3)* bo'ladi.

(1) *formula bilan yechiladigan kombinatorika masalasi umumiy holda quyidagicha ifodalanadi:* agar x elementni k usul, y elementni m usul bilan tanlash mumkin bo'lsa, «x yoki y» elementni k + m usul bilan tanlash mumkin.

Masalan, savatda 8 ta olma va 10 ta nok bor bo'lsa, 1 ta mevani 8 + 10 = 18 usul bilan tanlash mumkin.

(2) formula bilan yechiladigan masala: 40 talabadan 35 tasi matematika imtihonini, 37 tasi rus tili imtihonini topshira oldi. 2-talaba ikkala fandan «2» oldi. Nechta qarzdor talaba bor?

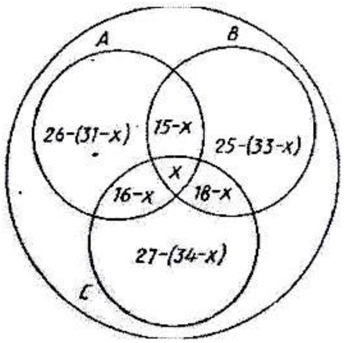
Yechish. A — matematika fanidan «2» olgan, B - rus tili fanidan «2» olgan talabalar to'plami bo'lsin.

$$n(A) = 40 - 35 = 5 \quad n(A \cap B) = 2.$$

$$n(B) = 40 - 37 = 3 \quad n(A \cup B) = 5 + 3 - 2 = 6.$$

Javob: 6 ta qarzdor talaba bor.

(3) formula - yig'indi qoidasi bilan yechiladigan masalani ko`raylik.



Masala. Sinfda 40 o`quvchi bor. Uning 26 tasi basketbol, 25 tasi — suzish, 27 tasi — gimnastika bilan shug`ullanadi, bir vaqtda suzish va gimnastika bilan — 15 ta, basketbol va gimnastika bilan — 16 ta, suzish va gimnastika bilan shug`ullanuvchilar — 18 ta. 1 o`quvchi darsdan ozod. Hamma sport turi bilan nechta o`quvchi shug`ullanadi? Nechta o`quvchi faqat 1 ta sport turi bilan shug`ullanadi?

Yechish. Maslada 3 ta to`plam qaralyapti: A — basketbol bilan shug`ullanuvchilar, B — suzish bilan shug`ullanuvchilar, C — gimnastika bilan shug`ullanuvchilar. Bu uch to`plam kesishadi.

Bu 3 to`plam kesishmasidagi elementlar sonini $x$ bilan belgilasak, quyidagi tenglamaga ega bo`lamiz:

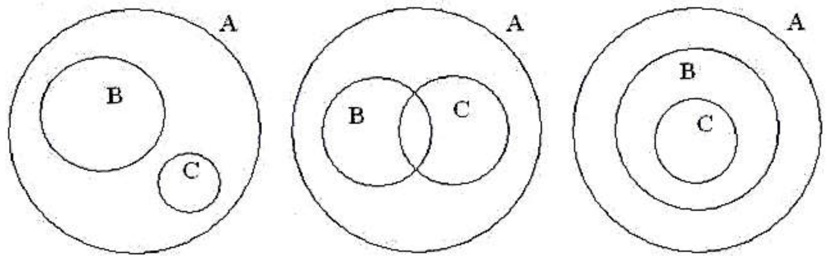
$$26 + 25 — (33 — x) + (18 — x) + 27 — (34 - x) + 1 = 40.$$

Bu yerda $x = 10$. Demak, hamma sport turi bilan 10 ta o`quvchi, faqat 1 ta sport turi bilan 10 ta: basketbol bilan — 5 ta, suzish bilan — 2 ta, gimnastika bilan — 3 ta o`quvchi shug`ullanadi.

Masala. 50 talabadan 20 tasi nemis tilini, 15 tasi inghliz tilini o`rganadi. Ikkala tilni biluvchi va faqat 1 ta tilni biluvchi talabalar soni nechta bo`lishi mumkin?

Yechish. Maslada 2 ta to`plam qaralyapti: $A$ —barcha talabalar to`plami, B — nemis tilini o`rganadigan, $C$ — inghliz tilini o`rganadigan talabalar to`plami. Masala sharti bo`yicha $n(A) = 50$, $n(B) = 20$, $n(C) = 15$.

A, B va To`plamlar orasidagi munosabatlarni Eyler-Venn diagrammalarida quyidagicha tasvirlash mumkin. Ikki tilni biluvchi talabalar soni B va C to`plamlar kesishmasi elementlari sonini topish bilan bog`liq. Faqat 1 ta tilni biluvchi talabalar soni ikki to`plam birlashmasi elementlari sonini topish bilan bog`liq.



$n(B \cap C) = 0$ $n(B \cap C) = 15$

$n(B \cup C) = 35$ $n(B \cup C) = 20$

x—Ikki tilni biluvchi talabalar soni bo`lsa, $0 \le x \le 15$ $(x \in N_0)$. $y$ — 1 ta tilni biluvchi talabalar soni bo`lsa, $20 \le y \le 35$ $(y \in N_0)$.

3.Ko'paytma qoidasi. Chekli to'plamlarning dekart ko'paytmasi elementlari sonini topishga imkon beradigan qoida *ko'paytma qoidasi* deyiladi.

$A = \{a_1, a_2, ..., a_n\}$ va $B = \{b_1, b_2, ..., b_m\}$ to'plamlar elementlaridan nechta tartiblangan (a_i, b_j) juftlik tuzish mumkinligini ko'raylik. Barcha juftliklarni tartib bilan quyidagicha joylashtiramiz:

$$(a_1; b_1), (a_1; b_2), ..., (a_1; b_m),$$
$$(a_2; b_1), (a_2; b_2), ..., (a_2; b_m),$$
$$(a_n; b_1), (a_n; b_2), ..., (a_n; b_m).$$

Bu jadvalda n ta qator va m ta ustun bo'lib, undagi barcha juftliklar soni n·mga teng. Bu yerda $n = n(A)$ va $m = n(B)$.

Ko'paytma qoidasi $n(A \times B) = n(A) \cdot n(B)$ ko'rinishda yoziladi.

Ko'paytma qoidasiga oid kombinatorika masalasining umumiy ko'rinishi: «Agar x elementni m usul, y elementni n usul bilan tanlash mumkin bo'lsa, $(x;y)$ tartiblangan juftlikni mn usul bilan tanlash mumkin».

Ikkitadan ortiq to'plamlar uchun bu formula quyidagicha yoziladi:

$$n(A_1 \times A_2 \times ... \times A_n) = n(A_1) \cdot n(A_2) \cdot ... \cdot n(A_n), (n > 2).$$

Masalan, A shahardan B shaharga 3 yo'l bilan, B shahardan C shaharga ikki yo'l bilan borish mumkin bo'lsa, A shahardan C shaharga necha xil usul bilan borish mumkin?

Yo'lning 1-qismini 3 xil, 2-qismini 2 xil yo'l bilan o'tish mumkin bo'lsa, umumiy yo'lni $3 \cdot 2 = 6$ usul bilan o'tish mumkin.

Umumlashgan ko'paytma qoidasi: «Agar x elementni m usul bilan, y elementni, x ni tanlab bo'lgandan so 'ng, n usul bilan tanlash mumkin bo'lsa, (x;y) juftlikni mn usul bilan tanlash mumkin».

Masala. Nechta turli raqamlar bilan yozilgan ikki xonali sonlar bor?

Yechish. 1-raqamni 9 usul bilan (1, 2, ..., 9), 2-raqamni ham 9 usul bilan (noldan boshlab o'nliklar raqamidan boshqa raqamlar) tanlash mumkin. Hammasi bo'lib 9·9 = 81 ta shunday son bor ekan.

Takrorlanadigan va takrorlanmaydigan o`rinlashtirishlar va o`rin almashtirishlar.

1.Takrorlanadigan o'rinlashtirishlar.

Masala. *m* elementli *X* to'plam elementlaridan tuzilgan *k* uzunlikdagi kortejlar sonini toping.

Yechish. *k* o'rinli kortej $\underbrace{X \times X \times \ldots \times X}_{\text{k marta}}$ dekart ko'paytmaning elementi bo'lib, tartiblangan *k*-likni (ka-lik deb o'qiladi) bildiradi. Masalani yechish uchun $X \times X \times \ldots \times X$ dekart ko'paytma elementlari sonini topish kerak. Bu son $n(X) = m$ bo'lgani uchun

$$n(X \times X \times \ldots \times X) = n(X) \cdot n(X) \cdot \ldots \cdot n(X) = m \cdot m \cdot \ldots \cdot m = m^k \text{ ga teng.}$$

Demak, *m* elementli *X* to'plam elementlaridan tuzilgan *k* o'rinli kortejlar soni m^k ga teng ekan. Kombinatorikada bunday kortejlarni *m elementdan k tadan takrorlanadigan o'rinlashtirishlar* deyiladi. Ularning soni $\overline{A_m^k}$ bilan belgilanadi. (*A* — fransuzcha arrangement so'zining bosh harfidan olingan bo'lib, «o'rnashtirish, joylashtirish ma'nosini bildiradi.) $\overline{A_m^k} = m^k$.

Masala. 6 raqamli barcha telefon nomerlari sonini toping.

Yechish. Telefon nomerlari 0 dan 9 gacha bo'lgan 10 ta raqamdan tuzilgani uchun 10 elementdan tuzilgan barcha tartiblangan 6 o'rinli kortejlar sonini topamiz:

Javob: $\overline{A_{10}^6} = 10^6 = 1000000$. 6 raqamli telefon nomerlari soni 10^6 ga teng.

2.Takrorlanmaydigan o'rinlashtirishlar. Umumiyroq masalani ko'rib chiqaylik: *m* elementli *X* to'plamdan nechta tartiblangan *k* elementli to'plamlar tuzish mumkin?

Bu masalaning oldingi masaladan farqi shundaki, tanlash *k* - elementda tugatiladi. Ularning umumiy soni

$$m(m-1)(m-2) \cdot \ldots \cdot (m-k+1)$$

ko'paytmaga teng. U A_m^k bilan belgilanadi va *m* elementdan *k* tadan *takrorlanmaydigan o'rinlashtirishlar soni* deb ataladi:

$$A_m^k = m(m-1) \cdot \ldots \cdot (m-k+1) = \frac{m!}{(m-k)!}.$$

Bu yerda $m! = m \times (m-1) \times \ldots \times 2 \times 1$.

Masalan, sinfdagi 20 o'quvchidan tozalik va davomat uchun javob beruvchi 2 o'quvchini necha xil usul bilan tanlash mumkin?

$$A_{20}^2 = \frac{20!}{18!} = 20 \cdot 19 = 380 \text{ (usul bilan).}$$

1-ta'rif.-elementli to'plamni turli tartiblashtirishlar takrorsiz o`rin almashtirishlar deyiladi, ularning soni deb belgilanadi va ga teng. Ta'rif bo`yicha deb olinadi

3.Takrorlanmaydigan o'rin almashtirishlar.

- fransuzcha "ermutation" - so`zidan olingan bo`lib, "o`rinalmashtirish" degan ma'noni bildiradi.

Masalan, uchta harfdan 3! = 6 ta o`rinalmashtirish qilish mumkin.

1. Agar chekli *X* to'plam elementlari biror usul bilan nomerlab chiqilgan bo'lsa, *X* to'plam *tartiblangan* deyiladi.

Masalan, $X = \{x_1, x_2, \ldots, x_m\}$. Bitta to'plamni turli usullar bilan tartiblash mumkin.

Masalan, sinf o'quvchilarini yoshiga, bo'yiga, ogirligiga qarab yoki o'quvchilar familiyalari bosh harflarini alifbo bo'yicha tartiblash mumkin.

m elementli *X* to'plamni necha xil usul bilan tartiblash mumkin degan savolga javob beraylik.

Tartiblash — bu elementlarni nomerlash demakdir. 1-nomerni *m* ta elementning istalgan biriga berish mumkin. Shuning uchun

1-elementni m usul bilan, 2-elementni 1-element tanlanib bo'lgandan so'ng m -1 usul bilan tanlash mumkin va hokazo, oxirgi elementni tanlash uchun faqat bitta usul qoladi, xolos. Tartiblashlarning umumiy soni

$m(m-1)(m-2)\cdot...\cdot 2\cdot 1 = m!$ ga teng.

$m!$ — dastlabki m ta natural son ko'paytmasi *(m faktorial* deb o'qiladi). Masalan, $5! = 1\cdot 2\cdot 3\cdot 4\cdot 5 = 120$, $m! = P_m$ bilan belgilanadi va *takrorlanmaydigan o'rin almashtirishlar soni* deb ataladi.

O`rin almashtirishlarni o`rinlashtirishlarning xususiy xoli deb qarash mumkin $m = n$ bo`lgan holi.

P belgisi fransuz tilidagi "permutation", ya'ni "o`rin almashtirish" so`zining 1- harfidan olingan.

Takrorlanadigan o`rinlashtirishlar va o`rin almashtirishlar.

Takrorlanadigan o'rinlashtirishlar *(A—* fransuzcha arrangement — «o'rnashtirish, joylashtirish ma'nosini bildiradi.) $A=m^k$. formula bilan ifodalanadi.

Masala. 6 raqamli barcha telefon nomerlari sonini toping.
Yechish.$A_{10}{}^{b}=10^6=1000000$.

Ehtimollikning klassik va statistik ta'riflari. Tajriba natijasida hodisalarning to'la guruhini tashkil etuvchi va teng imkoniyatli n ta elementar hodisalar ro'y berishi mumkin bo'lsin. Biror A hodisaning ro'y berishi uchun elementar hodisalardan m tasi qulaylik tug'dirsin. U holda, *klassik ta'rif* bo'yicha A hodisaning ehtimolligi

$$P(A)=\frac{m}{n}$$

tenglik bilan aniqlanadi.

Hodisa ehtimolligini ***statistik ehtimolligi*** deb, tajribalar ketma ketligida hodisa ro'y bergan sinovlar sonining o'tkazilgan barcha sinovlar soniga nisbatiga aytiladi:

$$W(A)=\frac{m}{n},$$

bu yerda m – A hodisaning tajribalar ketma ketligida ro'y berishlari soni, n – sinovlarning umumiy soni.

Sinovlar soni yetarlicha katta bo'lganda hodisaning statistik ehtimoli sifatida nisbiy chastota yoki unga yaqinroq son tanlanadi.

Klassik ta'rifdan foydalanib, masalalar yechishda kombinatorika formulalari keng qo'llaniladi. Shuni e'tiborga olib, ba'zi kombinatorika formulalarini keltiramiz.

O'rin almashtirishlar deb n ta turli elementlarning o'rin almashtirishlari soni $P_n = n!(n! = 1\cdot 2\cdot 3\cdots n)$ ga aytiladi.

O'rinlashtirishlar n ta turli elementdan m tadan tuzilgan kombinatsiyalar bo'lib, ular bir-biridan elementlarning tarkibi yoki ularning tartibi bilan farq qiladi.

Ularning soni

$A_n^m = \frac{n!}{(n-m)!}$ yoki $A_n^m = n(n-1)(n-2)\cdots(n-m+1)$ formulalari bilan topiladi.

Guruhlashlar – bir-biridan hech bo'lmaganda bitta elementi bilan farq qiluvchi n ta elementdan m tadan tuzilgan kombinatsiyalardir. Ularning soni $C_n^m = \frac{n!}{m!(n-m)!}$ ga teng.

Misol. Qutida 7 ta oq, 3 ta qora shar bor. Undan tavakkaliga olingan sharning oq bo'lishi ehtimolini toping.

Yechish: A – olingan shar oq ekanligi hodisasi bo'lsin. Bu sinov 10 ta teng imkoniyatli elementar hodisalardan iborat bo'lib, ularning 7 tasi A hodisaga qulaylik tug'diruvchidir. Demak, $P(A)=\frac{7}{10}=0,7$.

Misol. Telefonda raqam terayotgan abonent oxirgi ikki raqamni esdan chiqarib qo'yadi va faqat bu raqamlar har xil ekanligini eslab qolgan holda ularni tavakkaliga terdi. Kerakli raqamlar terilganligi ehtimolini toping.

Yechish: B – ikkita kerakli raqam terilganlik hodisasi bo'lsin, hammasi bo'lib, o'nta raqamdan ikkitadan nechta o'rinlashtirishlar tuzish mumkin bo'lsa, shuncha, ya'ni $A_{10}^2 = 10 \cdot 9 = 90$ ta turli raqamlarni terish mumkin. Demak,

$$P(B) = \frac{1}{A_{10}^2} = \frac{1}{90}.$$

Misol. Qurilma 5 ta elementdan iborat bo'lib, ularning 2 tasi eskirgan. Qurilma ishga tushirilganda tasodifiy ravishda 2 ta element ulanadi. Ishga tushirishda eskirmagan elementlar ulangan bo'lish ehtimolini toping.

Yechish: Sinovning barcha mumkin bo'lgan elementar hodisalari soni C_5^2 ga teng. Bularning ichidan C_3^2 tasi eskirmagan elementlar ulangan bo'lishi hodisasi (A) uchun qulaylik tug'diradi. Shuning uchun $P(A) = \frac{C_3^2}{C_5^2} = \frac{3}{10} = 0,3.$

Misol. Texnik nazorat bo'limi tasodifan ajratib olingan 100 ta kitobdan iborat partiyada 5 ta yaroqsiz kitob topdi. Yaroqsiz kitoblar chiqishining nisbiy chastotasini toping.

Yechish: $W(A) = \frac{5}{100} = 0,05.$

Misol. Nishonga 20 ta o'q uzilgan. Shundan 18 ta o'q nishonga tekkani qayd qilingan. Nishonga tegishlar nisbiy chastotasini toping.

Yechish: $W(A) = \frac{18}{20} = 0,9.$

Mustaqil yechish uchun misollar.

1-variant

1) a) Korxonada 10 erkak va 8 ayol xodim ishlaydi. Shu korxonadan bitta xodimni necha xil usulda tanlab olish mumkin?

b) Quyidagi ifodalarning qiymati topilsin: 1) $\frac{14!}{12!}$; 2) $\frac{16!}{18!}$.

2) A={1,2,3,4,5,6,7,8,9,0} to'plam elementlaridan foydalanib 7 raqamli

barcha passport nomerlari sonini toping.

3) Qutida 5 ta bir xil buyum bo'lib, ularning 3 tasi bo'yalgan. Tavakkaliga 2 ta buyum olinganda ular orasida:

a) bitta bo'yalgan bo'lishi;

b) ikkita bo'yalgan bo'lishi;

c) hech bo'lmaganda bitta bo'yalgan bo'lishi ehtimolini toping.

2-variant

1) a) 10 ta talabadan iborat guruhga ikkita yo'llanma ajratildi. Bu yo'llanmalarni necha xil usul bilan tarqatish mumkin?

b) Quyidagi ifodalarning qiymati topilsin: 1) $8! + \frac{9!}{5! \cdot 4!}$; 2) 9!.

2) 1,2,3,4,5 raqamlardan foydalanib nechta besh xonali son yozish mumkin?

3) Tavakkaliga 20 dan katta bo'lmagan natural son tanlanganda, uning 5 ga karrali bo'lish ehtimolini toping.

3-variant

1) a) Qurilishda 10 ta suvoqchi va 8 ta bo'yoqchi ishlaydi. Ulardan bir suvoqchi va bir bo'yoqchidan iborat juftlikni necha usulda tanlash mumkin?

b) Quyidagilarni isbotlang: $\frac{(m+3)!}{m!} = (m+1)(m+2)(m+3)$;

2) B={1,2,3,4,5,6,7,8,9,0} to'plam elementlaridan foydalanib 4 raqamli barcha transport chiptalarining nomerlari sonini toping.

3) Kartochkalarga 1,2,3,4,5,6,7,8,9 raqamlari yozilgan. Tavakkaliga 4 ta kartochka olinib, ular qator qilib terilganda juft son bo'lishi ehtimolini toping.

4-variant

1) a) Nazoratchi korxonada ishlab chiqarilgan 5 ta maxsulot sifatini ketma-ket tekshirishi kerak. Nazoratchi buni nechta usulda amalga oshirishi mumkin?

b) ushbu ayniyatni isbotlang.

$\frac{n!}{(n-m)!}=n(n-1)\dots(n-m+2)(n-m+1),$ bunda $n>m$.

2) 4 bemor shifokor oldiga necha usul bilan kirish mumkin?

3) Ikkita o'yin soqqasi baravar tashlanganda quyidagi hodisalarning ro'y berish ehtimolini toping:

a) Tushgan ochkolar yig'indisi 8 ga teng.

b) Tushgan ochkolar ko'paytmasi 8 ga teng.

c) Tushgan ochkolar yig'indisi ularning ko'paytmasidan katta

5-variant

1) a) Ishlab chiqarish korxonasini tekshirish uchun besh kishidan iborat guruh ajratildi. Shu besh kishidan tarkibida uch kishi bo'lgan guruhni necha xil usulda tuzish mumkin.

b) Amallarni bajaring: $\frac{1}{n!}-\frac{1}{(n+1)!}$;

2) S={1,2,3,4,5,6,7,8,9,0} to'plam elementlaridan foydalanib 5 raqamli barcha talabalar talabalik guvoxnomasi nomerlari sonini toping.

3) Qutichada 6 ta bir xil (raqamlangan) kubik bor. Tavakkaliga bitta-bittadan barcha kubiklar olinganda kubiklarning raqamlari o'sib borish tartibida chiqishi ehtimolini toping.

6-variant

1) a) Tikuvchilik fabrikasida ishlayotgan xodimga haftaning ixtiyoriy ikki kunini dam olish uchun tanlash imkoni berildi. Xodim dam olish

kunlarini necha usulda tanlashi mumkin?

b) Amallarni bajaring: $\frac{1}{(k-1)!}-\frac{1}{k!}$.

2) 6 o`rinli aylana shaklidagi stolga necha usul bilan odamlarni joylashtirish mumkin?

3) Qutida 12 ta oq va 8 ta qizil shar bor. Tavakkaliga:

a) 2 ta shar olinganda ularning turli rangda bo'lishi ehtimolini toping;

b) 8 ta shar olinganda ularning 3 tasi qizil rangli bo'lishi ehtimolini toping.

7-variant

1) a) Talaba 4 ta fan bo'yicha qo'shimcha tayyorlanish uchun ularning har biriga haftaning bir kunini ajratmoqchi bo'ldi. Talaba hafta kunlarini fanlarga necha usulda taqsimlashi mumkin?

b) To'qqizta har xil qiymatli raqam bilan nechta to'qqiz xonali son yozish mumkin?

2) E={1,2,3,4,5,6,7,8,9,0} to'plam elementlaridan foydalanib 4 raqamli barcha talabalik oylik chiptasining nomerlari sonini toping.

3) Qutida 12 ta oq va 8 ta qizil shar bor. Tavakkaliga:

a) bitta shar olinganda uning oq bo'lishi ehtimolini toping;

b) bitta shar olinganda uning qizil bo'lishi ehtimolini toping;

8-variant

1) a) Xorijiy tillar fakulteti ingliz tili yo'nalishining birinchi kursida 10 ta fan o'qitiladi va har kuni 4 xil dars o'tiladi. Kunlik dars necha usul bilan taqsimlab qo'yilishi mumkin?

b) 12 kishilik ovqat hozirlangan stolga 12 kishini necha turli o'tqazish mumkin?

2) 1,2,3,4,5,6 raqamlardan foydalanib 5 bilan tugaydigan nechta olti xonali son yozish mumkin?

3) Ikkita o'yin soqqasi baravar tashlanganda quyidagi hodisalarning ro'y berish ehtimolini toping:

a) Tushgan ochkolar ko'paytmasi 8 ga teng.

b) Tushgan ochkolar yig'indisi ularning ko'paytmasidan katta.

9-variant

1) a) Butun sonlarning har biri uchta har xil qiymatli raqamlar bilan ifoda qilinadigan bo'lsa, qancha butun son tuzish mumkin?

b) Musobaqada 6 ta talaba qatnashmoqda. O'rinlarni ular o'rtasida necha xil usul bilan taqsimlash mumkin?

2) F={1,2,3,4,5,6,7,8,9,0} to'plam elementlaridan foydalanib 7 raqamli telefon nomerlari soni toping.

3) Ikkita o'yin soqqasi baravar tashlanganda quyidagi hodisalarning ro'y berish ehtimolini toping:

a) Tushgan ochkolar yig'indisi 8 ga teng.

b) Tushgan ochkolar ko'paytmasi 8 ga teng.

10-variant

1) a) Talaba 6 ta kitobdan 4 tasini necha usul bilan ajratishi mumkin?

b) Tenglik to'g'riligini isbotlang: $C_7^4+C_7^3=C_8^4$;

2) Firmada 20 ta qo'y va 24 ta echki bor. Necha xil usul bilan bitta qo'y va bitta echki tanlash mumkin?

3) Qutida 100 ta lampochka bo'lib, ularning 10 tasi yaroqsiz. Tavakkaliga 4 ta lampochka olinadi. Olingan lampochkalar ichida:

a) yaroqsizlar yo'q bo'lishi;

b) yaroqlilari yo'q bo'lishi ehtimolini toping.

11-variant

1) a) Talaba 6 ta kitobdan 4 tasini necha usul bilan ajratishi mumkin?

b) Tenglik to'g'riligini isbotlang: $C_7^4+C_7^3=C_8^4$;

2) N={1,2,3,4,5,6,7,8,9,0} to'plam elementlaridan foydalanib podezddagi eshiklarning 5 raqamli nomerlari soni toping.

3) Yashikda 31 ta birinchi nav va 6 ta ikkinchi nav detal bor. Tavakkaliga 3 ta detal olinadi:

a) Olingan uchala detal birinchi nav bo'lishi ehtimolini toping.

b) Olingan detallarning hech bo'lmaganda bittasi birinchi nav bo'lishi ehtimolini toping.

12-variant

1) a) Ma'lum bo'limda ishlash uchun 20 nafar ishchidan 6 nafar ishchini ajratish kerak. Buni necha usul bilan amalga oshirish mumkin?

b) Tenglik to'g'riligini isbotlang: $C_{10}^5+C_{10}^6=C_{11}^6$.

2) Uch nusxada algebra bo'yicha o'quv kitoblar, yetti nusxada geometriya bo'yicha o'quv kitoblar va yetti nusxada trigonometriya bo'yicha o'quv kitoblari bor. Har bir o'quv kitoblaridan bittadan nusxa olishni necha xil usul bilan tanlash mumkin?

3) N ta buyumdan iborat partiyada M ta standart buyum bor. Partiyadan tavakkaliga n ta buyum olinadi. Bu n ta buyum ichida rosa m ta standart buyum borligini ehtimolini toping.

13-variant

1) a) Ifodani soddalashtiring: $\frac{3}{2(2n-1)}C_n^{2n-3}$.

b) Kitob javonida 9 ta matematikadan, 4 ta chet tilidan va 6 ta ona tilidan kitob turibdi. Javondan bitta kitobni nechta usulda tanlash mumkin?

2) M={1,2,3,4,5,6,7,8,9,0} to'plam elementlaridan foydalanib 3 raqamli yengil avtomashina nomerlari soni toping.

3) Yashikda 15 ta detal bo'lib, ulardan 10 tasi bo'yalgan. Yig'uvchi tavakkaliga 3 ta detal oladi. Olingan detallarning bo'yalgan bo'lishi ehtimolini toping.

14-variant

1) a) Musobaqada 12 ta jamoa ishtirok etadi. Uchta turli medalni necha xil usul bilan taqsimlash mumkin?

b) Sehrli mamlakatda uchta shahar bor: A, B va C. A shahardan B shaharga 6 ta yo'l boradi, B shahardan C shaharga esa – 4 ta yo'l. A shahardan C shaharga nechta usulda borsa bo'ladi?

2) 100 dan 10000 gacha bo'lgan sonlar orasida uchta bir xil raqam uchraydigan nechta son mavjud?

3) Xaltachada 5 ta bir xil kub bor. Har bir kubning barcha tomonlariga quyidagi harflardan biri yozilgan: o, p, r, s, t. Bittalab olingan va "bir qator qilib" terilgan kublarda "sport" so'zini o'qish mumkinligi ehtimolini toping.

15-variant

1) a) Gruppada 30 ta o'quvchi bor. Ularning ichidan 3 kishini kompyuterda ishlash uchun ajratish kerak. Buni necha usul bilan bajarish mumkin?

b) Dukonda 7 ta tur pidjak, 5 ta tur shim va 4 ta tur galstuk sotilmoqda. Pidjak, shim va galstukdan iborat komplektni nechta usul bilan sotib olsa bo'ladi?

2) Ikkita o'yin soqqasi tashlanadi. Chiqqan ochkolar yig'indisining 7 ga teng bo'lishi ehtimolini toping.

3) Oltita bir xil kartochkaning har biriga quyidagi harflardan biri yozilgan –

a, t, m, r, s, o. Kartochkalar yaxshilab aralashtirilgan. Bittalab olingan va

"bir qator qilib" terilgan to'rtta kartochkada "soat" so'zini o'qish mumkinligi ehtimolini toping.

16-variant

1) a) Turli rangdagi 5 to'p mato bor. Bu matolardan har bir mato faqat bitta polosani egallaydigan qilib nechta turli besh rangli bayroqlar tayyorlash mumkin?

b) Agar oltita turli rangli mato bor bo'lsa, bir xil kenglikdagi gorizontal polosali uch rangli bayroq nechta usul bilan tikilsa bo'ladi?

2) M={1,2,3,4,5,6,7,8,9,0} to'plam elementlaridan foydalanib 3 raqamli yuk avtomashina nomerlari soni toping.

3) Hamma tomoni bo'yalgan kub mingta bir xil o'lchamli kubchalarga bo'lingan va yaxshilab aralashtirilgan. Tavakkaliga olingan kubchaning

a) bitta; b) ikkita; c) uchta tomoni bo'yalgan bo'lish ehtimolini toping.

17-variant

1) a) Tenglamani yeching: $\frac{P_{x+2}}{P_x}=72;$

b) "Matbuot tarqatuvchi" dukonida 5 ta tur konvert va 4 ta tur marka sotilmoqda. Konvert bilan markani nechta usulda sotib olishimiz mumkin?

2) O'n birta odamdan iborat futbol komandasida necha xil usul bilan kapitan va unga yordamchi tanlash mumkin?

3) Aralashtirilgan 36 talik kartalar dastasidan tavakkaliga bittasi olinadi. Olingan kartaning a) "tuz" bo'lishini b) rasmli (ya'ni "korol", "dama" yoki "valet") bo'lishini ehtimoli qanday?

18-variant

1) a) Tenglamani yeching $A_x^4=A_{x-2}^2$,

b) 25 nafar o'quvchidan tashkil topgan sinfda sinf sardori, sinf sardorining yordamchisi va yoshlar ittifoqi etakchisini saylash zarur. Har bir o'quvchi bu vazifalardan faqat bittasini bajaradi deb hisoblansa, saylov natijalari uchun qancha imkoniyat mavjud?

2) M={1,2,3,4,5,6,7,8,9,0} to'plam elementlaridan foydalanib 4 raqamli yengil va yuk avtomashina nomerlari soni toping.

3) Qutida m ta oq va n ta qora sharlar bor. Qutidan tavakkaliga bitta shar olinadi. Olingan sharning oq bo'lishi ehtimolini toping.

19-variant

1) a) 20 ta belgidan tashkil topgan alfavit berilgan bo'lsin.

Uzunligi 3 ga teng bo'lgan va undagi belgilar takrorlanmaydigan so'zlar soni nechaga teng?

b) Sinfda 10 ta fan o'qitiladi va har kuni turli 6 xil dars o'tiladi. Kunlik dars jadvali necha turli usul bilan tuzilishi mumkin? Darslar takrorlanishi mumkin bo'lsa-chi?

2) Necha xil usul bilan qatorga qizil, qora, ko'k va yashil rangli koptoklarni qo'yish mumkin.

3) Bitta shashqoltosh (kubik, o'yin soqqasi) tashlangan. Quyidagi ehtimollarni toping.

a. juft ochko tushishi;

b. 5 ochkodan kam bo'lmagan ochko tushishi.

20-variant

1) a) 20 ta belgidan tashkil topgan alfavit berilgan bo'lsin.

Uzunligi 3 ga teng bo'lgan barcha so'zlar soni nechaga teng?

b) Tezyurar poezdda 17 ta vagon bor. 17 nafar hizmatchini shu vagonlar orasida nechta usulda taqsimlash mumkin? Bunda har bir

vagonga 1 nafar hizmatchini tayinlash mumkin

2) Mamlakatda 20 ta shahar bo'lib, har bir shahar avia yo'llar bilan bog'langan. Bu mamlakatda nechta avia yo'llar bor?

3) Ikkita tanga tashlangan. Agar A – tangalar bir xil tomonlar bilan tushishi hodisasi, B – turli tomonlar bilan tushishi hodisasi bo'lsa, qaysi hodisaning ehtimoli kattaroq?

21-variant

1) a) 2,7,8 raqamlaridan nechta uch xonali son hosil qilish mumkin?

b) 30 nafar o'quvchisi bor sinfda fan olimpiadasida qatnashish uchun 3 nafar o'quvchidan tarkib topgan jamoani tanlab olishimiz kerak. Buni nechta usulda amalga oshirish mumkin?

2) Komissiya tarkibi 3 ta odamdan iborat bo'lsa, 20 ta odamdan necha xil usul bilan komissiya tuzish mumkin?

3) Uchta tanga tashlangan. Ikki marta "gerb" tomoni bilan tushishi ehtimolini toping.

22-variant

1) a) Tenglamani yeching $A_x^4 = A_{x-2}^2$.

b) 25 nafar o'quvchidan tashkil topgan sinfda sinf sardori, sinf sardorining yordamchisi va yoshlar ittifoqi etakchisini saylash zarur. Har bir o'quvchi bu vazifalardan faqat bittasini bajaradi deb hisoblansa, saylov natijalari uchun qancha imkoniyat mavjud?

2) Uchburchakning tomonlarida 20 ta , 6 ta va 4 ta nuqtalar belgilandi(Bu belgilangan nuqtalarning hech qaysi uchburchakning uchlari bo'lmaydi). Uchlari belgilangan nuqtalarda bo'lgan nechta uchburchak mavjud?

3) 52 talik kartalar dastasidan tavakkaliga uchtasi olinadi. Ularning "3", "7" va "tuz" karta bo'lishi ehtimoli qanday?

23-variant

1) a) 20 ta belgidan tashkil topgan alfavit berilgan bo'lsin.

Uzunligi 3 ga teng bo'lgan va undagi belgilar takrorlanmaydigan so'zlar soni nechaga teng?

b) Sinfda 10 ta fan o'qitiladi va har kuni turli 6 xil dars o'tiladi. Kunlik dars jadvali necha turli usul bilan tuzilishi mumkin? Darslar takrorlanishi mumkin bo'lsa-chi?

2) Avtomobilning nomeri bir, ikki yoki uchta harfdan va to'rta raqamdan iborat bo'lishi mumkin. Agar o'zbek alifbosida 29 ta harf bo'lsa, nechta avtomobil nomeri mavjud?

3) Telefon raqami 6 ta raqamdan iborat. Telefon raqamining: a) raqamlari turli xil bo'lishi; b) raqamlari 3 ga karrali bo'lishi ehtimollarini toping.

24-variant

1) a) 20 ta belgidan tashkil topgan alfavit berilgan bo'lsin.

Uzunligi 3 ga teng bo'lgan barcha so'zlar soni nechaga teng?

b) Tezyurar poezdda 17 ta vagon bor. 17 nafar hizmatchini shu vagonlar orasida nechta usulda taqsimlash mumkin? Bunda har bir vagonga 1 nafar hizmatchini tayinlash mumkin

2) Karta qutisida 52 ta karta bo'lib, bu qutidan 10 ta karta olindi. Olingan kartalar orasida hech bo'lmaganda bitta tuz bo'lishi uchun bu hodisani necha marta takrorlash kerak?

3) Qutida faqat ranglari bilan farqlanuvchi 22 ta shar bor: 9 ta ko'k, 5 ta sariq va 8 ta oq. Qaysi hodisaning ehtimoli kattaroq: qutidan sariq sharning chiqishimi yoki shashqoltosh tashlanganda 5 ochko tushishimi?

25-variant

1) a) 2,7,8 raqamlaridan nechta uch xonali son hosil qilish mumkin?

b) 30 nafar o'quvchisi bor sinfda fan olimpiadasida qatnashish uchun 3 nafar o'quvchidan tarkib topgan jamoani tanlab olishimiz kerak. Buni nechta usulda amalga oshirish mumkin?

2) Har kuni navbatchilikka 2 tadan odam qo'yib 6 kunga 12 ta odamni taqsimlab, necha xil usul bilan navbatchilik ro'yxatini tuzish mumkin? (Har bir odam bir marta navbatchilik qiladi)

3) O'nta biletdan ikkitasi yutuqli. Tavakkaliga olingan 5 ta bilet orasida bittasi yutuqli bo'lish ehtimolini toping.

21-mavzu: To'la ehtimollik va Bayes formulalari. Bog'liq bo'lmagan tajribalar ketma- ketligi.

Ehtimollarni qo'shish va ko'paytirish teoremalari.

1-teorema. Ikkita birgalikda bo'lmagan hodisadan istalgan birining ro'y berish ehtimoli bu hodisalar ehtimollarining yig'indisiga teng:

$$P(A+B)=P(A)+P(B).$$

Natija. Har ikkitasi birgalikda bo'lmagan bir nechta hodisalardan istalgan birining ro'y berishi ehtimoli bu hodisalar ehtimollarining yig'indisiga teng:

$$P(A_1+A_2+...+A_n)=P(A_1)+P(A_2)+...+P(A_n).$$

Misol. Sexda bir necha stanok ishlaydi. Smena davomida bitta stanok sozlashni talab etish ehtimoli 0,2 ga teng, ikkita stanokni sozlashni talab etish ehtimoli 0,13 ga teng. Smena davomida ikkitadan ortiq stanokni sozlashni talab etish ehtimoli esa 0,07 ga teng. Smena davomida stanoklarni sozlashni talab etilishini ehtimolini toping.

Yechish: Quyidagi hodisalarni qaraymiz.

A – Smena davomida bitta stanokni sozlash talab etiladi.

B – Smena davomida ikkita stanokni sozlash talab etiladi.

C – Smena davomida ikkitadan ortiq stanokni sozlash talab etiladi.

A, B va C hodisalar o'zaro birgalikda emas. Bizni quyidagi hodisa qiziqtiradi: (A+B+C) – smena davomida sozlash uchun zarur bo'ladigan stanoklar:

$$P(A+B+C)=P(A)+P(B)+P(C)=0,2+0,13+0,07=0,4.$$

Misol. Yashikda 10 ta qizil va 6 ta ko'k shar bor. Tavakkaliga 2 ta shar olinadi. Olingan ikkala sharning bir xil rangli bo'lish ehtimolini toping.

Yechish: A hodisa olingan ikkala shar qizil bo'lishi, B hodisa esa olingan ikkala sharning ko'k bo'lish hodisasi bo'lsin. Ko'rinib turibdiki, A va B hodisalar birgalikda bo'lmagan hodisalar. Demak,

$$P(A+B)=P(A)+P(B).$$

A hodisaning ro'y berishiga C_{10}^2 ta natija imkoniyat yaratadi. B hodisaning ro'y berishiga esa C_6^2 ta natija imkoniyat yaratadi. Umumiy ro'y berishi mumkin bo'lgan natijalar soni esa C_{16}^2 ga teng.

U holda:

$$P(A+B)=\frac{C_{10}^2+C_6^2}{C_{16}^2}=\frac{\frac{10\cdot 9}{2}+\frac{6\cdot 5}{2}}{\frac{16\cdot 15}{2}}=\frac{60}{120}=\frac{1}{2}.$$

Misol. Ikki ovchi bo'riga qarata bittadan o'q uzishdi. Birinchi ovchining bo'riga tekkizish ehtimoli 0,7 ga, ikkinchisiniki 0,8 ga teng. Hech bo'lmaganda bitta o'qning bo'riga tegish ehtimolini toping.

Yechish. A hodisa birinchi ovchining bo'riga o'qni tekkizishi, B hodisa esa ikkinchi ovchining bo'riga o'qni tekkizishi bo'lsin. Ko'rinib turibdiki, A va B hodisalar birgalikda bo'lgan, ammo bir-biriga bog'liq bo'lmagan hodisalar. U holda $P(A+B)=P(A)+P(B)-P(AB)=P(A)+P(B)-P(A)\cdot P(B)=$

$$=0,7+0,8-0,7\cdot 0,8=0,94.$$

2-teorema. Ikkita erkli hodisalarning birgalikda ro'y berish ehtimoli, bu hodisalar ehtimollarining ko'paytmasiga teng:

$$P(AB)=P(A)\cdot P(B).$$

Natija. Bir nechta erkli hodisalarning birgalikda ro'y berish ehtimoli, bu hodisalar ehtimollarini ko'paytmasiga teng:

$$P(A_1\cdot A_2\cdot ...\cdot A_n)=P(A_1)\cdot P(A_2)\cdot ...\cdot P(A_n).$$

3-teorema. Ikkita bog'liq hodisalarning birgalikda ro'y berish ehtimoli ulardan birining ehtimolini ikkinchisining shartli ehtimoliga ko'paytmasiga teng.

$$P(AB)=P(A)\cdot P(B/A)=P(B)\cdot P(A/B).$$

Natija. Bir nechta bog'liq hodisalarning birgalikda ro'y berish ehtimoli ulardan birining ehtimolini qolganlarining shartli ehtimollariga ko'paytirilganligiga teng, shu bilan birga, har bir keyingi hodisaning ehtimoli oldingi hamma hodisalar ro'y berdi degan farazda hisoblanadi:

$$P(A_1 \cdot A_2 \cdot ... \cdot A_n) = P(A_1) \cdot P(A_2 / A_1) \cdot P(A_3 / A_1A_2) \cdot ... \cdot P(A_n / A_1A_2...A_{n-1}).$$

4-teorema. Ikkita birgalikda bo'lgan hodisadan kamida bittasining ro'y berish ehtimoli bu hodisalarning ehtimollari yig'indisidan ularning birgalikda ro'y berish ehtimolining ayirmasiga teng:

$$P(A+B) = P(A) + P(B) - P(AB).$$

Agar A va B hodisalar bog'liq bo'lsa, $P(A+B) = P(A) + P(B) - P(B)P(A/B)$, bog'liq bo'lmasa $P(A+B) = P(A) + P(B) - P(A) \cdot P(B)$. formulalaridan foydalanamiz.

5-teorema. Birgalikda bog'liq bo'lmagan $A_1, A_2, ..., A_n$ hodisalaridan kamida bittasining ro'y berishidan iborat A hodisaning ehtimoli 1dan $\overline{A_1}, \overline{A_2}, ..., \overline{A_n}$ qarama-qarshi hodisalar ehtimollari ko'paytmasining ayirmasiga teng:

$$P(A) = 1 - P(\overline{A_1})P(\overline{A_2})...P(\overline{A_n})$$

Misol. Tanga va kubik bir vaqtda tashlangan. "Gerb tushishi" va "3" ochko tushishi hodisalarining birgalikda ro'y berish ehtimolini toping.

Yechish: A hodisa tanganing "gerb" tushishi, B hodisa esa kubik tashlanganda "3" ochko tushishi bo'lsin. A va B hodisalar bog'liq bo'lmagan hodisalar. U holda:

$$P(A \cdot B) = P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}.$$

Misol. Sexda 7 ta erkak va 3 ta ayol ishchi ishlaydi. Tabel raqamlari bo'yicha tavakkaliga 3 kishi ajratildi. Barcha ajratib olingan kishilar erkaklar bo'lish ehtimolini toping.

Yechish: Hodisalarni quyidagicha belgilaylik: A hodisa birinchi ajratilgan erkak kishi, B ikkinchi ajratilgan erkak kishi, C uchinchi ajratilgan erkak kishi.

Birinchi ajratilgan kishining erkak bo'lishi ehtimoli:

$$P(A) = \frac{7}{10}.$$

Birinchi ajratilgan kishining erkak kishi bo'lganligi shartida ikkinchi kishining erkak bo'lishi ehtimoli, ya'ni B hodisaning shartli ehtimoli:

$$P(B / A) = \frac{6}{9} = \frac{2}{3}.$$

Oldin ikki erkak kishi ajratib olinganligi shartida uchinchi ajratilgan kishi erkak bo'lishi ehtimoli, ya'ni C hodisaning shartli ehtimoli:

$$P(C / AB) = \frac{5}{8}.$$

Ajratib olingan kishilarning hammasi erkak ishchilar bo'lishi ehtimoli:

$$P(ABC) = P(A) \cdot P(B/A) \cdot P(C/AB) = \frac{7}{10} \cdot \frac{2}{3} \cdot \frac{5}{8} = \frac{7}{24}.$$

Misol. Ko'prik yakson bo'lishi uchun bitta aviatsion bombaning kelib tushishi kifoya. Agar ko'prikka tushish ehtimollari mos ravishda 0,3; 0,4; 0,6; 0,7 bo'lgan 4 ta bomba tashlansa, ko'prikni yakson bo'lish ehtimolini toping.

Yeshish: Demak, kamida bitta bombaning ko'prikka tushishi, uni yakson bo'lishi uchun yetarli (A hodisa). U holda, izlanayotgan ehtimollik quyidagicha bo'ladi:

$$P(A) = 1 - 0,7 \cdot 0,6 \cdot 0,4 \cdot 0,3 \approx 0,95.$$

To'la ehtimollik formulasi. Bayes formulasi. Biror A hodisa hodisalarning to'la guruhini tashkil etadigan $B_1, B_2, ..., B_n$ hodisalarning (ular gipotezalar deb ataladi) biri bilan ro'y berishi mumkin bo'lsin. Bu gipotezalarning ehtimollari ma'lum, ya'ni $P(B_1), P(B_2), ..., P(B_n)$ berilgan. Bu gipotezalarning har biri amalga oshganida A hodisaning ro'y berish shartli ehtimollari ham ma'lum, ya'ni $P(A/B_1), P(A/B_2), ..., P(A/B_n)$ ehtimollar berilgan. U holda A hodisaning ehtimoli "to'la ehtimol" formulasi deb ataluvchi quyidagi formula bilan aniqlanadi:

$$P(A) = P(B_1)P(A/B_1) + P(B_2)P(A/B_2) + ... + P(B_n)P(A/B_n) = \sum_{k=1}^{n} P(B_k)P(A/B_k).$$

Misol. Birinchi qutida 2 ta oq , 6 ta qora, ikkinchi qutida esa 4 ta oq, 2 ta qora shar bor. Birinchi qutidan tavakkaliga 2 ta shar olib, ikkinchi qutiga solindi, shundan keyin ikkinchi qutidan tavakkaliga bitta shar olindi. Olingan sharning oq bo'lish ehtimolini toping.

Yechish: Quyidagi belgilashlarni kiritamiz:

A – ikkinchi qutidan olingan shar oq.

B_1 – birinchi qutidan ikkinchi qutiga 2 ta oq shar solingan.

B_2 – birinchi qutidan ikkinchi qutiga 2 ta turli rangdagi shar solingan.

B_3 – birinchi qutidan ikkinchi qutiga 2 ta qora shar solingan.

B_1, B_2, B_3 – hodisalar hodisalarning to'la guruhini tashkil etadi.

U holda, to'la ehtimol formulasiga ko'ra:

$$P(A)=P(B_1)P(A/B_1)+P(B_2)P(A/B_2)+P(B_3)P(A/B_3).$$

Bunda:

$$P(B_1)=\frac{C_2^2}{C_8^2}=\frac{1}{28};\quad P(B_2)=\frac{C_2^1C_6^1}{C_8^2}=\frac{12}{28};\quad P(B_3)=\frac{C_6^2}{C_8^2}=\frac{15}{28};\quad P(A/B_1)=\frac{3}{4};$$

$$P(A/B_2)=\frac{5}{8};\quad P(A/B_3)=\frac{1}{2}.$$

U holda:

$$P(A)=\frac{1}{28}\cdot\frac{3}{4}+\frac{12}{28}\cdot\frac{5}{8}+\frac{15}{28}\cdot\frac{1}{2}=\frac{9}{16}.$$

Birgalikda bo'lmagan, hodisalarning to'la guruhini tashkil etadigan B_1, B_2, ... B_n hodisalar berilgan va ularning $P(B_1), P(B_2),..., P(B_n)$ ehtimollari ma'lum bo'lsin. Tajriba o'tkaziladi va uning natijasida A hodisa ro'y beradi deylik, bu hodisaning har bir gipoteza bo'yicha shartli ehtimoli, ya'ni $P(A/B_1), P(A/B_2),..., P(A/B_n)$ ma'lum. A hodisa ro'y berishi munosabati bilan gipotezalarning ehtimollarini qayta baholash uchun, boshqacha aytganda, $P(B_1/A), P(B_2/A),..., P(B_n/A)$ shartli ehtimolini topish uchun

$$P(B_i/A)=\frac{P(B_i)\cdot P(A/B_i)}{\sum_{k=1}^{n}P(B_k)\cdot P(A/B_k)}\quad (i=1,2,...n),$$

Bayes formulalaridan foydalaniladi.

b) $P(B_1/A)$ ehtimolni Bayes formulasidan foydalanib topamiz:

$$P(B_1/A)=\frac{P(B_1)P(A/B_1)}{P(A)}=\frac{\frac{1}{28}\cdot\frac{3}{4}}{\frac{9}{16}}=\frac{1}{21}.$$

Misol. Ikkita avtomat bir xil detallar ishlab chiqaradi, bu detallar keyin umumiy konveyerga o'tadi. Birinchi avtomatning unumdorligi ikkinchi avtomatning unumdorligidan ikki marta ko'p. Birinchi avtomat o'rta hisobda detallarning 60 foizini, ikkinchi avtomat esa o'rtacha hisobda detallarning 84 foizini a'lo sifat bilan ishlab chiqaradi. Konveyerda tavakkaliga olingan detal a'lo sifatli bo'lib chiqdi. Bu detalni birinchi avtomat ishlab chiqargan bo'lish ehtimolini toping.

Yechish: A – detal a'lo sifatli bo'lish hodisasi bo'lsin. Bu yerda ikkita taxmin (gipoteza) qilish mumkin: B_1 – detalni birinchi avtomat ishlab chiqargan, shu bilan birga:

$$P(B_1)=\frac{2}{3}\ \text{gat eng.}$$

Chunki birinchi avtomat ikkinchi avtomatga qaraganda ikki marta ko'p detal ishlab chiqaradi.

B_2 – detalni ikkinchi avtomat ishlab chiqargan, shu bilan birga:

$$P(B_2)=\frac{1}{3}.$$

Agar detalni birinchi avtomat ishlab chiqargan bo'lsa, detal a'lo sifatli bo'lishining shartli ehtimoli

$$P(A/B_1)=0,6\ \text{gat eng.}$$

Xuddi shunga o'xshash:

$P(A/B_2)=0,84$ ga teng.

Tavakkaliga olingan detalning a'lo sifatli bo'lish ehtimoli to'la ehtimol formulasiga ko'ra quyidagiga teng:

$$P(A)=P(B_1)P(A/B_1)+P(B_2)\cdot P(A/B_2)=\frac{2}{3}\cdot 0,6+\frac{1}{3}\cdot 0,84=0,68.$$

Olingan a'lo sifatli detalni birinchi avtomat ishlab chiqargan bo'lish ehtimoli Bayes formulasiga ko'ra quyidagiga teng:

$$P(B_1/A)=\frac{P(B_1)P(A/B_1)}{P(A)}=\frac{\frac{2}{3}\cdot 0.6}{0.68}=\frac{10}{17}.$$

Mustaqil yechish uchun mashqlar

1-variant

1) Yashikda 6 ta yashil va 5 ta qizil tugmalar bor. Tavakkaliga 2 ta tugma olinadi. Olingan ikkala tugmaning ham bir xil rangli bo'lish ehtimolini toping.

2) Yashikda 1-zavodda tayyorlangan 12 ta detal, 2-zavodda tayyorlangan 20 ta detal va 3-zavodda tayyorlangan 18 ta detal bor. 1-zavodda tayyorlangan detalning a'lo sifatli bo'lishi ehtimoli 0,9ga teng, 2-zavodda va 3-zavodda mos ravishda 0,6 va 0,9 ga teng. Tavakkaliga olingan detalning a'lo sifatli bo'lishi ehtimolini toping.

2-variant

1) Tanga va o'yin soqqasi bir vaqtda tashlanadi. "Raqam tushish" va "4" ochko tushishi hodisalarining birgalikda ro'y berish ehtimolini toping.

2) Yashikda 1-zavodda tayyorlangan 12 ta detal, 2-zavodda tayyorlangan 20 ta detal va 3-zavodda tayyorlangan 18 ta detal bor. 1-zavodda tayyorlangan detalning a'lo sifatli bo'lishi ehtimoli 0,9ga teng, 2-zavodda va 3-zavodda mos ravishda 0,6 va 0,9 ga teng. Tavakkaliga

olingan detalning a'lo sifatli bo'lishi ehtimolini toping.

3-variant

1) Qutida 3 ta oq va 8 ta qizil shar bor. Qutidan tavakkaliga bitta shar, keyin yana bitta shar olindi. Olingan sharlardan birinchisi oq, ikkinchisi qizil bo'lish ehtimolini toping.

2) Birinchi idishda 10 ta shar bo'lib, ularning 8 tasi oq, ikkinchi idishda 20 ta shar bo'lib, ularning 4 tasi oq. Har bir idishdan tavakkaliga bittadan shar olinib, keyin bu ikki shardan yana bitta shar tavakkaliga olindi. Oq shar olinganlik ehtimolini toping.

4-variant

1) Birinchi yashikda 6 ta oq va 14 ta qizil shar bor. Ikkinchi yashikda esa 4 ta oq va 6 ta qizil shar bor. Agar har bir yashikdan bittadan shar olinsa, hech bo'lmaganda bitta sharning oq bo'lish ehtimolini toping.

2) Birinchi idishda 10 ta shar bo'lib, ularning 8 tasi oq, ikkinchi idishda 20 ta shar bo'lib, ularning 4 tasi oq. Har bir idishdan tavakkaliga bittadan shar olinib, keyin bu ikki shardan yana bitta shar tavakkaliga olindi. Oq shar olinganlik ehtimolini toping.

5-variant

1) Uchta to'pdan otishda nishonga tekkizish ehtimoli mos ravishda $P_1=0,9$; $P_2=0,7$; $P_3=0,8$. Nishon yakson qilinishi uchun bitta o'qning nishonga tegishi kifoya qilsa, uchala to'pdan bir vaqtda nishonning yakson qilinish ehtimolini toping.

2) Uchta idishning har birida 6 tadan qora shar va 4 tadan oq shar bor. Birinchi idishdan tavakkaliga bitta shar olinib, uchinchi idishga solindi. Uchinchi idishdan tavakkaliga olingan sharning oq bo'lish ehtimolini toping.

6-variant

1) Merganni bitta o'q uzishda nishonga tekkizish ehtimoli P=0,8. Mergan uchta o'q uzdi. Uchala o'qning ham nishonga tegish ehtimolini toping.

2) Uchta idishning har birida 6 tadan qora shar va 4 tadan oq shar bor. Birinchi idishdan tavakkaliga bitta shar olinib, uchinchi idishga solindi. Uchinchi idishdan tavakkaliga olingan sharning oq bo'lish ehtimolini toping.

7-variant

1) Yashikda 7 ta oq, 4 ta qora va 4 ta ko'k shar bor. Har bir tajriba qutidan 1 ta shar olishdan iborat. Olingan shar qaytib qo'yilmaydi. Birinchi sinashda oq shar A, ikkinchisida qora B, uchinchisida ko'k shar chiqish ehtimolini toping.

2) Uchta idishning har birida 6 tadan qora shar va 4 tadan oq shar bor. Birinchi idishdan tavakkaliga bitta shar olinib, uchinchi idishga solindi. Uchinchi idishdan tavakkaliga olingan sharning oq bo'lish ehtimolini toping.

8-variant

1) Qutida 5 ta oq va 5 ta qora shar bor. Tavakkaliga 3 ta shar olinadi. Olingan uchala sharning ham bir xil rangli bo'lish ehtimolini toping.

2) Uchta idishning har birida 6 tadan qora shar va 4 tadan oq shar bor. Birinchi idishdan tavakkaliga bitta shar olinib, uchinchi idishga solindi. Uchinchi idishdan tavakkaliga olingan sharning oq bo'lish ehtimolini toping.

9-variant

1) Uchta merganning nishonga tekkizish ehtimoli mos ravishda 0,6; 0,8 va 0,9 ga teng. Uchta mergan baravariga o'q uzganda nishonga hech

bo'lmaganda bitta o'qning tegishi ehtimolini toping.

2) Elektron raqamli mashinaning ishlash vaqtida arifmetik qurilmada, operativ xotira qurilmasida, qolgan qurilmalarda buzilish yuz berish ehtimollari 3:2:5 kabi nisbatda. Arifmetik qurilmada, operativ xotira qurilmasida va boshqa qurilmalardagi buzilishning topilish ehtimoli mos ravishda 0,8; 0,9; 0,9 ga teng. Mashinada yuz bergan buzilishning topilishi ehtimolini toping.

10-variant

1) Birinchi qutida 3 ta oq va 7 ta qora shar bor. Ikkinchi qutida esa 6 ta oq va 4 ta qora shar bor. Agar har bir qutidan bittadan shar olinsa, hech bo'lmaganda bitta sharning oq bo'lish ehtimolini toping.

2) Elektron raqamli mashinaning ishlash vaqtida arifmetik qurilmada, operativ xotira qurilmasida, qolgan qurilmalarda buzilish yuz berish ehtimollari 3:2:5 kabi nisbatda. Arifmetik qurilmada, operativ xotira qurilmasida va boshqa qurilmalardagi buzilishning topilish ehtimoli mos ravishda 0,8; 0,9; 0,9 ga teng. Mashinada yuz bergan buzilishning topilishi ehtimolini toping.

11-variant

1) Texnik nazorat bo'limi buyumlarning yaroqliligini tekshiradi. Buyumning yaroqli bo'lish ehtimoli 0,9 ga teng. Tekshirilgan ikkita buyumdan faqat bittasi yaroqli bo'lish ehtimolini toping

2) Benzokolonka joylashgan shossedan o'tadigan yuk mashinalari sonining o'sha shossedan o'tadigan yengil mashinalar soniga nisbati 3:2 kabi. Yuk mashinaning benzin olish ehtimoli 0,1 ga teng, yengil mashina uchun bu ehtimol 0,2 teng. Benzokolonka yoniga benzin olish uchun mashina kelib to'xtadi. Uning yuk mashina bo'lish ehtimolini toping.

12-variant

1) Talabaga kerakli formulani uchta qo'llanmada bo'lish ehtimoli mos

ravishda 0,6; 0,7; 0,8 ga teng. Formula: a) faqat bitta qo'llanmada; b) faqat ikkita qo'llanmada; c) formula uchala qo'llanmada bo'lish ehtimolini toping.

2) Benzokolonka joylashgan shossedan o'tadigan yuk mashinalari sonining o'sha shossedan o'tadigan yengil mashinalar soniga nisbati 3:2 kabi. Yuk mashinaning benzin olish ehtimoli 0,1 ga teng, yengil mashina uchun bu ehtimol 0,2 teng. Benzokolonka yoniga benzin olish uchun mashina kelib to'xtadi. Uning yuk mashina bo'lish ehtimolini toping.

13-variant

1) Talaba programmadagi 25 ta savoldan 20 tasini biladi. Talabaning imtihon oluvchi taklif etgan uchta savolni bilish ehtimolini toping.

2) Ixtisoslashtirilgan kasalxonaga bemorlarning o'rta hisobda 30 foizi K kasallik bilan, 50 foizi L kasallik bilan, 20 foizi M kasallik bilan qabul qilindi. K kasallikni to'liq davolash ehtimoli 0,7 ga teng, L va M kasalliklar uchun bu ehtimol mos ravishda 0,8 ga va 0,9 ga teng. Kasallikka qabul qilingan bemor butunlay sog'ayib ketdi. Bu bemor K kasallik bilan og'rigan bo'lish ehtimolini toping.

14-variant

1) Yashikda 1 dan 10 gacha raqamlangan 10 ta bir xil kubik bor. Tavakkaliga bittadan 3 ta kubik olinadi. Birin-ketin 1,2,3 raqamli kubiklar chiqish ehtimolini quyidagi hollarda toping:

a) kubiklar olingach, yashikka qaytarib solinmaydi;

b) olingan kubik yashikka qaytarib solinadi.

2) Ixtisoslashtirilgan kasalxonaga bemorlarning o'rta hisobda 30 foizi K kasallik bilan, 50 foizi L kasallik bilan, 20 foizi M kasallik bilan qabul qilindi. K kasallikni to'liq davolash ehtimoli 0,7 ga teng, L va M kasalliklar uchun bu ehtimol mos ravishda 0,8 ga va 0,9 ga teng. Kasallikka qabul qilingan bemor butunlay sog'ayib ketdi. Bu bemor K kasallik bilan og'rigan bo'lish ehtimolini toping.

15-variant

1) Biror joy uchun iyul oyida bulutli kunlarning o'rtacha soni oltiga teng. Birinchi va ikkinchi iyulda havo ochiq bo'lish ehtimolini toping.

2) Sharlar solingan 2 ta bir xil yashik bor. Birinchi yashikda 2 ta oq va 1 ta qora shar, ikkinchi yashikda esa 1 ta oq va 4 ta qora shar bor. Tavakkaliga bitta yashik tanlanadi va undan bitta shar olinadi. Olingan sharning oq bo'lish ehtimolini toping.

16-variant

1) Guruhda 10 ta talaba bo'lib, ularning 7 nafari a'lochilar. 4 ta talaba dekanatga chaqirtirildi. Ularning barchasi a'lochi bo'lish ehtimolini toping.

2) Sharlar solingan 2 ta bir xil yashik bor. Birinchi yashikda 2 ta oq va 1 ta qora shar, ikkinchi yashikda esa 1 ta oq va 4 ta qora shar bor. Tavakkaliga bitta yashik tanlanadi va undan bitta shar olinadi. Olingan sharning oq bo'lish ehtimolini toping.

17-variant

1) Buyumlar partiyasidan tovarshunos oliy nav buyumlarni ajratmoqda. Tavakkaliga olingan buyumning oliy nav bo'lish ehtimoli 0,8 ga teng. Tekshirilgan uchta buyumdan faqat ikkitasi oliy nav bo'lish ehtimolini toping.

2) Qutidagi 20 ta sharni (12 ta oq va 8 ta qora) aralashtirish jarayonida bitta shar yo'qotib qo'yildi. Qolgan 19 ta shardan tavakkaliga bitta shar olindi. Olingan sharning oq bo'lish ehtimolini toping.

18-variant

1) Birinchi yashikda 4 ta oq va 8 ta qora shar bor. Ikkinchi yashikda 10 ta oq va 6 ta qora shar bor. Har qaysi yashikdan bittadan shar olinadi. Ikkala sharning ham oq chiqish ehtimolini toping.

2) Qutidagi 20 ta sharni (12 ta oq va 8 ta qora) aralashtirish jarayonida

bitta shar yo'qotib qo'yildi. Qolgan 19 ta shardan tavakkaliga bitta shar olindi. Olingan sharning oq bo'lish ehtimolini toping.

19-variant

1) Birinchi yashikda 5 ta oq va 10 ta qizil shar bor. Ikkinchi yashikda esa 10 ta oq va 5 ta qizil shar bor. Agar har bir yashikdan bittadan shar olinsa, hech bo'lmaganda bitta sharning oq bo'lish ehtimolini toping.

2) Sharlar solingan 2 ta bir xil yashik bor. Birinchi yashikda 3 ta oq va 2 ta qora, ikkinchi yashikda esa 4 ta oq va 4 ta qora shar bor. Birinchi yashikdan ikkinchi yashikka 2 ta shar tashlandi. Shundan keyin ikkinchi yashikdan bitta shar olindi. Olingan sharning oq bo'lish ehtimolini toping.

20-variant

1) Bitta smenada stanokning ishlamay qolishi ehtimoli 0,05 ga teng. Uchta smenada stanokning ishlab turish ehtimolini toping.

2) Sharlar solingan 2 ta bir xil yashik bor. Birinchi yashikda 3 ta oq va 2 ta qora, ikkinchi yashikda esa 4 ta oq va 4 ta qora shar bor. Birinchi yashikdan ikkinchi yashikka 2 ta shar tashlandi. Shundan keyin ikkinchi yashikdan bitta shar olindi. Olingan sharning oq bo'lish ehtimolini toping.

21-variant

1) Tanga birinchi marta "gerb" tomoni bilan tushguncha tashlanadi. Tashlashlar sonining juft son bo'lish ehtimolini toping.

2) Ikki mergan bir-biriga bog'liqmas ravishda, nishonga qarata bittadan o'q uzishdi. Birinchi merganning nishonga o'q tekkizish ehtimoli 0,8 ga teng, ikkinchi merganniki esa 0,4 ga teng. O'qlar otilgandan keyin bitta o'qning nishonga tekkani ma'lum bo'ldi. O'qni birinchi mergan nishonga tekkizgan bo'lishi ehtimolini toping.

22-variant

1) Otilgan torpedoning kemani cho'ktirib yuborish ehtimoli 0,5 ga teng. Agar kemani cho'ktirib yuborish uchun bitta torpedoning mo'ljalga tegishi yetarli bo'lsa,

4 ta torpedoning kemani cho'ktirib yuborish ehtimolini toping.

2) Ikki mergan bir-biriga bog'liqmas ravishda, nishonga qarata bittadan o'q uzishdi. Birinchi merganning nishonga o'q tekkizish ehtimoli 0,8 ga teng, ikkinchi merganniki esa 0,4 ga teng. O'qlar otilgandan keyin bitta o'qning nishonga tekkani ma'lum bo'ldi. O'qni birinchi mergan nishonga tekkizgan bo'lishi ehtimolini toping.

23-variant

1) Elektr zanjiriga erkli ishlaydigan 3 ta element ketma-ket ulangan. Birinchi, ikkinchi va uchinchi elementlarning buzilish ehtimollari mos ravishda quyidagiga teng: $P_1=0,1$; $P_2=0,15$; $P_3=0,2$. Zanjirda tok bo'lmasligi ehtimolini toping.

2) Uchta zavod soat ishlab chiqaradi va magazinga jo'natadi. Birinchi zavod butun mahsulotning 40 foizini, ikkinchi zavod 45 foizini, uchinchi zavod esa 15 foizini tayyorlaydi. Birinchi zavod chiqargan soatlarning 80 foizi, ikkinchi zavod chiqargan soatlarning 70 foizi, uchinchi zavod chiqargan soatlarning 90 foizi ilgarilab ketadi. Sotib olingan soatning ilgarilab ketishi ehtimolini toping.

24-variant

1) Ikki sportchidan har birining mashqni muvaffaqiyatli bajarish ehtimoli 0,5 ga teng. Sportchilar mashqni navbat bilan bajaradilar, bunda har bir sportchi o'z kuchini ikki marta sinab ko'radi. Mashqni birinchi bo'lib bajargan sportchi mukofot oladi. Sportchilarning mukofotni olishlari ehtimolini toping.

2) Uchta zavod soat ishlab chiqaradi va magazinga jo'natadi. Birinchi zavod butun mahsulotning 40 foizini, ikkinchi zavod 45 foizini, uchinchi zavod esa 15 foizini tayyorlaydi. Birinchi zavod chiqargan soatlarning 80 foizi, ikkinchi zavod chiqargan soatlarning 70 foizi, uchinchi zavod chiqargan soatlarning 90 foizi ilgarilab ketadi. Sotib olingan soatning ilgarilab ketishi ehtimolini toping.

25-variant

1) Merganning uchta o'q uzishda kamida bitta o'qni nishonga tekkizish

ehtimoli 0,875 ga teng. Uning bitta o'q uzishda nishonga tekkizish ehtimolini toping.

2) Samolyotga qarata uchta o'q otildi. Birinchi o'qning nishonga tegish ehtimoli 0,5 ga, ikkinchisiniki 0,6 ga, uchinchisiniki esa 0,8 ga teng. Bitta o'q tekkanda samolyotning urib tushirilish ehtimoli 0,3 ga, ikkita o'q tekkanda 0,6 ga teng. Uchta o'q tekkanda, samolyot urib tushiriladi. Samolyotning urib tushirilish ehtimolini toping.

XULOSA

Ushbu o'quv qo'llanma pedagogika oliy ta'lim muassasalari tabiiy fanlar va texnika yo'nalishlari 1-bosqich talabalari uchun mo'ljallangan bo'lib, tabiiy hamda texnika fanlar yo'nalish bakalavr bosqich talabalariga yo'nalishi uchun oliy matematika fanida kerakli mavzu va bo'limlarni kasbga yo'naltirib o'qitish ishlarini tashkil qilishga qaratilgan bo'lib, o'quv qo'llanma orqali oliy ta'lim muassasalari fan o'qituvchilari va talabalari quyidagi natijalarga erishishlari ko'zda tutilgan.

chiziqli algebra elementlari va vektorli algebra elementlari mavzularini o'qitishda yangi pedagogik texnologiyalar yordamida o'qitishni takomillashtirish metodlari asosida bilim olishlari va olingan bilimlarini fanlararo integratsiyasini amaliy mashg'ulotlarda qo'llay olishlari

analitik geometriya elementlari bilan tanishtirishda ishlab chiqilgan metodikalari asosida analitik geometriyua elementlari mavzularini o'zlashtirishlari va olingan bilimlarini kasbiy faoliyatga bog'lab amaliy mashg'ulotlarda qo'llay olishlari lozim

matematik analizga kirish, bir o'zgaruvchi funksiyasining differensial hisobi, bir o'zgaruvchi funksiyasining integral hisobi mavzularini fanlararo bog'liqlikda o'qitishni tashkil qilish orqali funksiya hosilasi va integrallari mavzularini zamonaviy kasbiy faoliyatda qo'llay olish ko'nikmasiga ega bo'lishlari

oddiy differensial tenglamalar bo'limi mavzularini o'qitishda kasbga yo'naltirishda bir qatorak texnik, kimyoviy hamda biologic masalalarning yechimlarini differensial tenglamalar tuzish hamda yechimi orqali masala javobini topishni o'rgatish va talabalarning o'rganishlarini nazorat qilish

extimollar nazariyasi iqtisodiy va texnika masalalariga tadbiq qilingan holda o'qitishni takomillashtirish uchun yetarlicha ma'lumotlar keltirilgan bo'lib, qo'llanmaning asosiy vazifasi sifatida oliy ta'lim muassasalarida tabiiy hamda texnika fanlar yo'nalishlarida fanlararo bog'liqlikda va kasbiy faoliyatga yo'naltirib darslarni tashkil qilish bo'lib, kasbga yo'naltirish uchun bir qator misol va masalalar berilgan.

Bulardan tashqari ushbu o'quv qo'llanma o'quv dasturining kredik moodle dasturidagi asosiy vazifalardan, talabalarning mustaqil ta'limini tashkillashtirish vazifalari asosida shakllantirilgan.

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