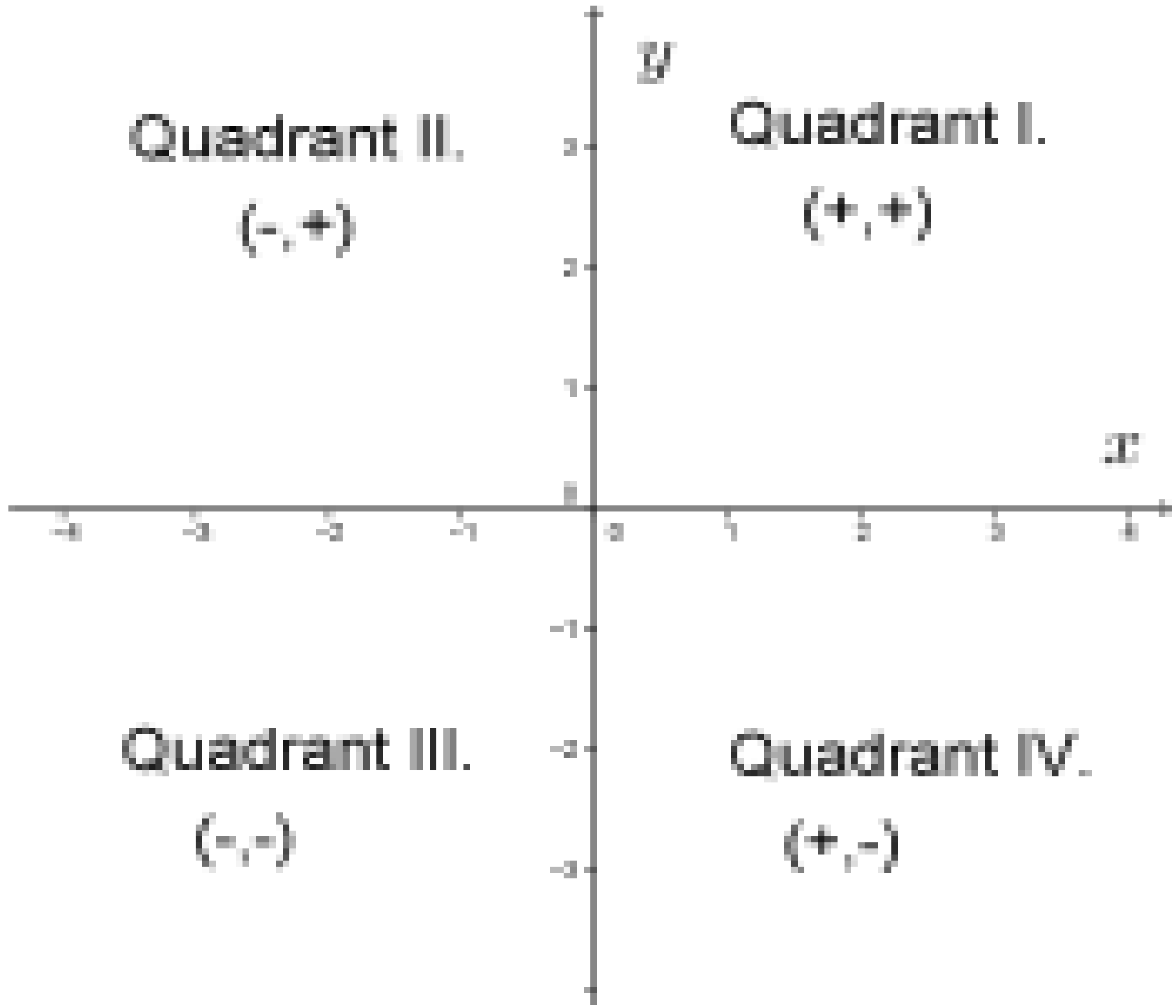
10-mavzu. Tekislikda koordinatalar sistemasi

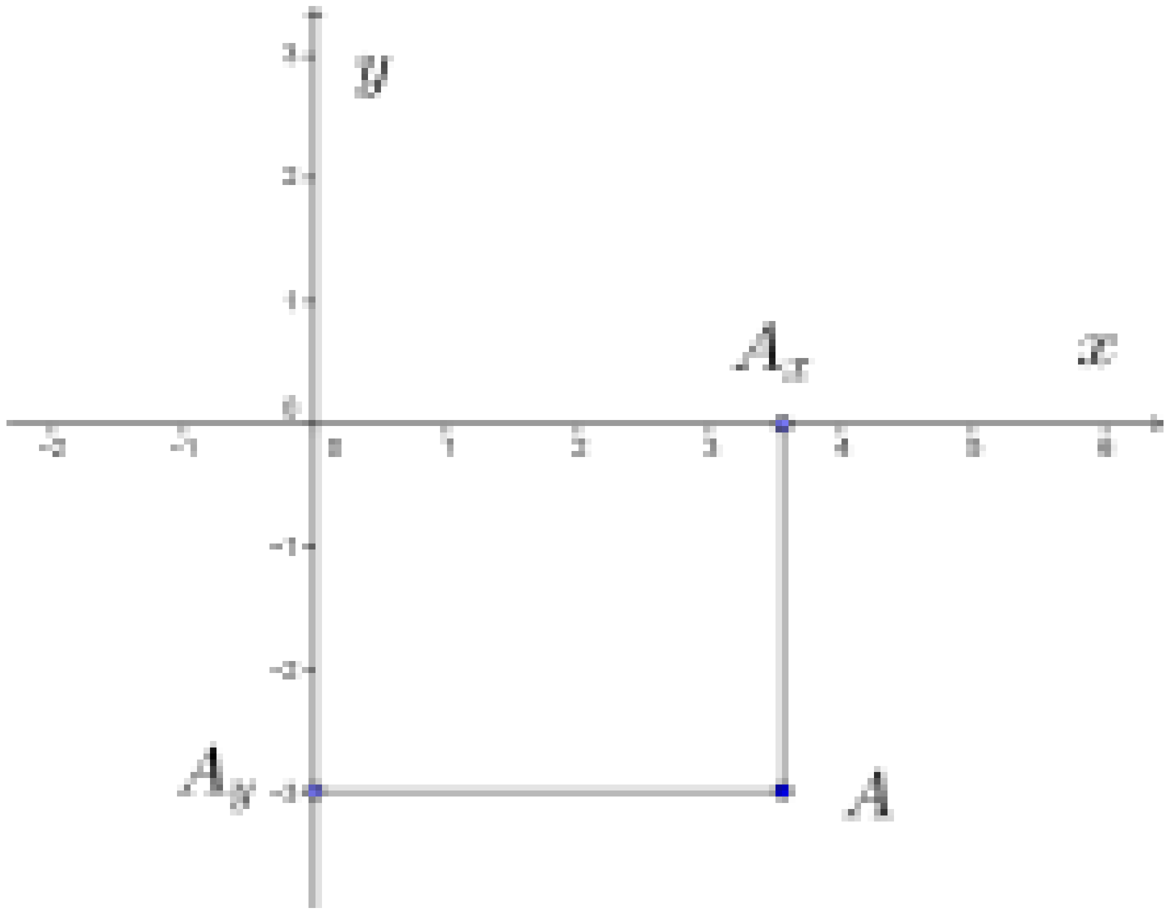
Adabiyotlar.

* Klaus Helft Mathematical preparation course before studying physics. Institute of Theoretical Physics University of Heidelberg. Please send error messages to k.helft @thphys.uni- heidelberg.de November 11, 2013.
* Herbert Gintis , Mathematical Literacy for Humanists, Printed in the United States of America, 2010


y
Quadrant II.
(-,+)
Quadrant I.
(+,+)
x
Quadrant III.
(-,-)
Quadrant IV.
(+,-)

Faraz qilaylik dekart koordinatalar sistemasida $x > 0$ va $y < 0$ sonlari berilgan bo'lsin.Absissa o'qining musbat yo'nalishida koordinatalar boshidan x masofada yotgan nuqtani A_x orqali, Ordinata o'qining manfiy yo'nalishida koordinatalar boshidan $|y|$ masofada yotgan nuqtani A_y orqali belgilaymiz. A_x va A_y nuqtalardan y va x o'qlariga o'tkazilgan parallel to'g'ri chiziqlar A nuqtada kesishsin. Natijada bu nuqta x absissali va y ordinatali nuqta deyiladi. Xuddi shunga o'xshash ixtiyoriy ishorali koordinatalarni aniqlash mumkin.

Dekart koordinatalar sistemasidagi A nuqtaning abscissa va ordinatalarini topish uchun quyidagi ishni amalga oshiramiz:



15.4 chizma

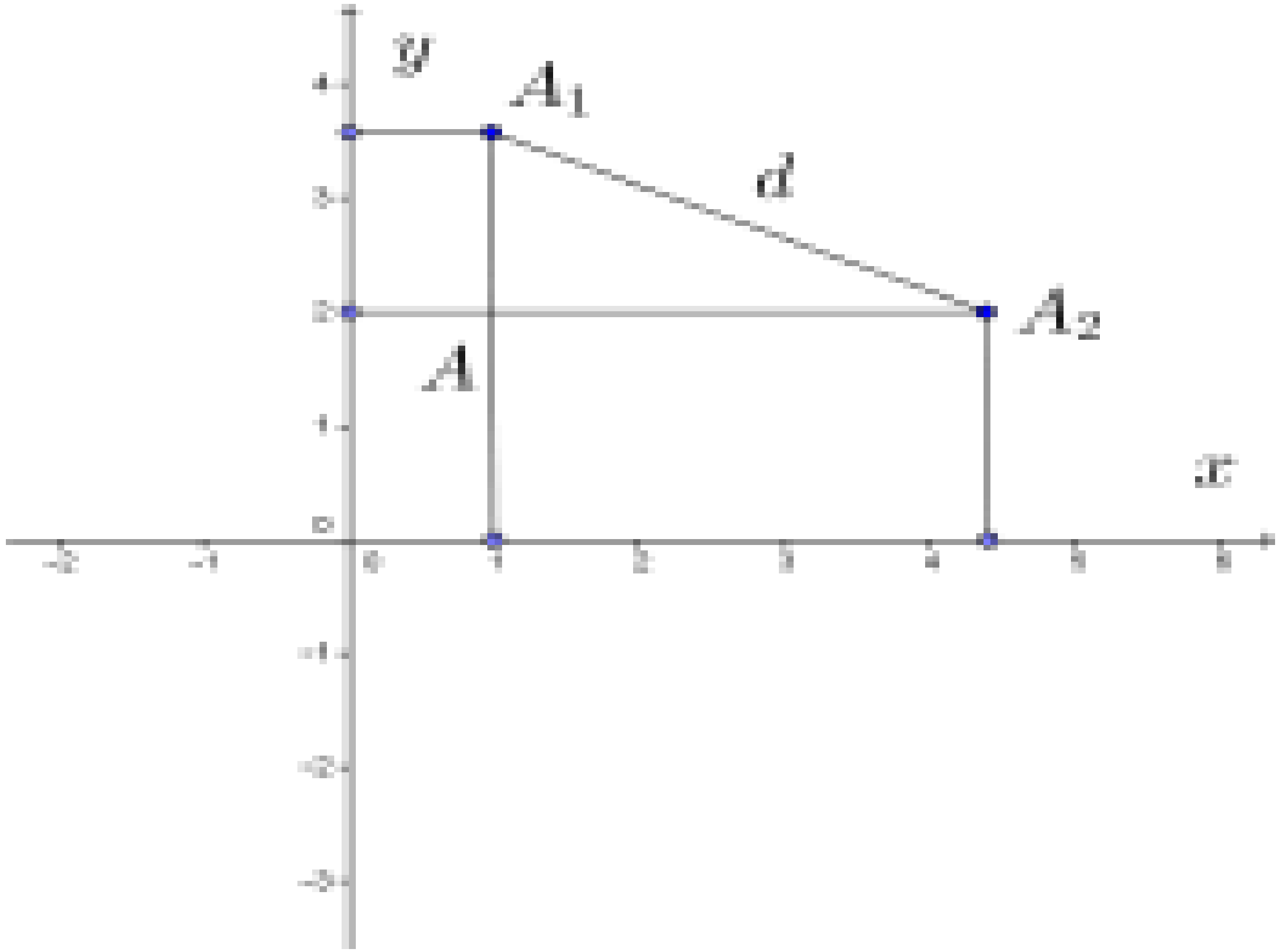
Ikki nuqta orasidagi masofa.

Bizga Oxy tekislikda $A_1(x_1;y_1)$ va $A_2(x_2;y_2)$ nuqtalar berilgan bo'lsin. Bu A_1 va A_2 nuqtalar orasidagi masofa ularning koordinatalariga bog'liqdir.

Aytaylik $x_1 \neq y_1$ va $x_2 \neq y_2$ bo'lsin. A_1 va A_2 nuqtalardan koordinata o'qlariga parallel to'g'ri chiziqlar o'tkazamiz (o'tkazilgan parallel to'g'ri chiziqlar A nuqtada kesishsin). U holda A va A_1 nuqtalar orasidagi masofa $|y - y_1|$ ga, A va A_2 nuqtalar orasidagi masofa esa $|x - x_1|$ ga teng bo'ladi. Hosil bo'lgan A_1AA_2 uchburchak to'g'ri burchakli ekanidan Pifagor teoremasiga ko'ra:

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 = d^2 \qquad (*)$$

(*) formula tekislikda ikkita nuqta orasidagi masofani aniqlaydi.


y
A1
d
A2
A
x

Kesmani berilgan nisbatda bo'lish.

Bizga Oxy tekislikda ikkita turli $A_1(x_1;y_1)$ va $A_2(x_2;y_2)$ nuqtalar berilgan bo'lsin. A_1A_2 kesmani $\lambda_1:\lambda_2$ nisbatda bo'luvchi A nuqtaning x va y koordinatalarini topaylik.

Aytaylik A_1A_2 kesma x o'qiga parallel bo'lmasin. A_1, A, A_2 nuqtalarning y o'qdagi proyeksiyalari mos ravishda $\overline{A}_1, \overline{A}, \overline{A}_2$ bo'lsin. U holda

$$\frac{A_1A}{AA_2} = \frac{\overline{A}_1\overline{A}}{\overline{A}\overline{A}_2} = \frac{\lambda_1}{\lambda_2}$$

o'rinliligidan va $\overline{A}_1\overline{A} = |y_1 - y|, \overline{A}\overline{A}_2 = |y - y_2|$ ekanidan quyidagiga ega bo'lamiz.

$$\frac{|y_1 - y|}{|y - y_2|} = \frac{\lambda_1}{\lambda_2}$$

$\overline{A}$ nuqta $\overline{A}_1$ va $\overline{A}_2$ nuqtalar orasida yotganidan $y_1 - y$ va $y - y_2$ ifodalar bir xil ishorali bo'ladi. Demak $\dfrac{|y_1 - y|}{|y - y_2|} = \dfrac{y_1 - y}{y - y_2} = \dfrac{\lambda_1}{\lambda_2}$

Bundan y ni topsak: $y = \dfrac{\lambda_2 y_1 + \lambda_1 y_2}{\lambda_2 + \lambda_1}$

Xuddi shunga o'xshash $x = \dfrac{\lambda_2 x_1 + \lambda_1 x_2}{\lambda_2 + \lambda_1}$

Qisqalik uchun $\dfrac{\lambda_1}{\lambda_2 + \lambda_1} = t$ u holda $\dfrac{\lambda_2}{\lambda_2 + \lambda_1} = 1 - t$

Yuqoridagi belgilashlarga ko'ra $x = (1-t)x_1 + tx_2$, $y = (1-t)y_1 + ty_2$ $0 \le t \le 1$

Bibliography

Csaba Vincze and Laszlo Kozma "College Geometry" March 27,2014 pp.161-170