Mavzu: Tekislikda ikkinchi tartib chiziqlar tenglamalari

1. Ellips ta’rifi va tenglamasi.
2. Ellips tenglamasini tekshirish.
3. Ellips ekssentrisiteti va direktrisalari.
4. Giperbolaning kanonik tenglamasi.
5. Giperbola asimptotalari.
7. Parabola kanonik tenglamasi.

Darsning maqsad va vazifalari

O'quv mashg'ulotining maqsadi: Ellips ta'rifi va tenglamasi, yellips tenglamasini tekshirish, yellips ekssentrisiteti va direktrisalari, giperbolaning kanonik tenglamasi, giperbola tenglamasini tekshirish, giperbola asimptotalari, teng tomonli va qo'shma giperbolalar hakidagi bilimlarni hamda to'liq tasavvurni shakllantirish.

1. Ellips ta'rifi, kanonik tenglamasi.

T a ' r i f. Ellips deb tekislikdagi shunday nuqtalarning geometrik o`rniga aytiladiki, bu nuqtalarning har biridan fokuslar deb ataluvchi $F_1$ va $F_2$ nuqtalargacha bo`lgan masofalari yig'indisi berilgan PQ kesma uzunligiga teng bo'ladi. Bu yerda $PQ > F_1F_2$.

Fokuslar orasidagi masofani $F_1F_2=2c$, $PQ=2a$ deb olamiz. Ta'rifga asosan $a>c$ bo`ladi.

Agar F_1 va F_2 nuqtalar ustma-ust tushsa, u holda ta'rifga ko`ra ellips radiusi $a$ ga teng aylana bo`ladi. Bu holda ellipsning fokuslari aylana markazi bilan ustma-ust tushadi. Shunday qilib, aylana ellipsning xususiy holidir.

Ellipsning F_1, F_2 fokuslari orasidagi masofani ellipsning fokal masofasi deyiladi. M nuqta ellips nuqtasi bo`lsin, u holda F_1M va F_2M kesmalarni M nuqtaning fokal radiuslari deyiladi. Fokal radiuslarni $r_1=F_1M$, $r_2=F_2M$ bilan belgilaymiz (79-chizma).

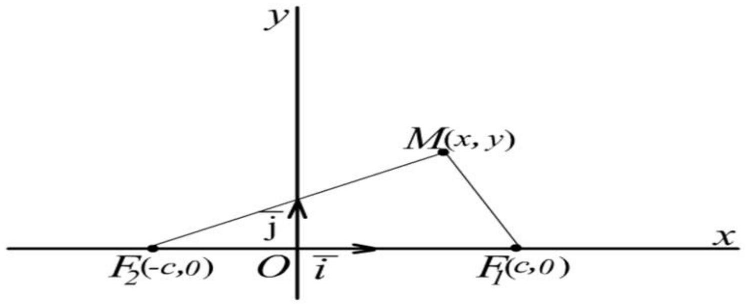
Ixtiyoriy M nuqta ellipsda yotsa, ta'rifga ko`ra

$$F_1M + F_2M = 2a,$$

yoki

$$r_1+r_2=2a \qquad (1)$$

(40.1) ellipsning ta'rifidan bevosita kelib chiqqan tenglamasidir.



79-chizma

Ellipsning to`g’ri burchakli dekart koordinatalar sistemasidagi tenglamasini topaylik.

Buning uchun dekart koordinatalar sistemasini quyidagicha tanlab olamiz. F_1F_2 to’g’ri chiziq bilan Ox absissa o`qi ustma-ust tushsin. $F_1F_2$ – kesmani o`rtasi O nuqta bo`lsin.

U holda fokuslar $F_1(c,0)$ va $F_2(-c,0)$ koordinatalarga $M(x,y)$ koordinatalarga ega bo`ladi Tekislikdagi ixtiyoriy M nuqtaning fokal radiuslari quyidagilarga teng:

$$r_1=F_1M=\sqrt{(x-c)^2+y^2}\ ,\ r_2=F_2M=\sqrt{(x+c)^2+y^2} \qquad (2)$$

Topilgan qiymatlarni (1) tenglikka qo`yib

$$\sqrt{(x-c)^2+y^2}+\sqrt{(x+c)^2+y^2}=2a$$

ni hosil qilamiz. Bu tenglamani

$$\sqrt{(x-c)^2+y^2}=2a-\sqrt{(x+c)^2+y^2}$$

ko`rinishda yozib olib, tenglikni ikkala tomonini kvadratga ko`tarib ixchamlab quyidagini hosil qilamiz,

$$a\sqrt{(x-c)^2+y^2}=a^2-cx\ ,$$

Yana kvadratga ko`tarib ixchamlasak

$$(a^2-c^2)x^2+a^2y^2=a^2(a^2-c^2) \qquad (3)$$

$a>c \Rightarrow a^2>c^2$, demak $a^2-c^2>0$ bu sonni

$$b^2=a^2-c^2 \qquad (4)$$

kabi belgilab olsak (40.3) tenglama

$$b^2x^2+ay^2=a^2b^2 \qquad (5)$$

ko`rinishga keladi. (40.5) ni $a^2b^2$ ga bo`lib ushbu tenglamaga ega bo`lamiz:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (6)$$

Shunday qilib, γ ellipsning ixtiyoriy nuqtasining koordinatalari (6) tenglaman qanoatlantirishi isbotlandi.

Endi teskari jumlani isbotlaylik. Koordinatalari (6) tenglaman qanoatlantiruvchi ixtiyoriy M nuqtani ellipsda yotishini isbotlaymiz.

(40.6) tenglikdan $y^2 = b^2 (1 - \frac{x^2}{a^2})$ qiymatini (2) ga qo`yib, (4) ni hisobga olsak ushbuga ega bo`lamiz:

$$r_1=F_1M=\sqrt{(a-\frac{c}{a}x)^2}=\left|a-\frac{c}{a}x\right|,$$

$$r_2=F_2M=\sqrt{(a+\frac{c}{a}x)^2}=\left|a+\frac{c}{a}x\right|.$$

(40.6) tenglamadan $|x|\leq a$, $a>c$ bo'lgani uchun $0<\frac{c}{a}<1$ bo`ladi, u holda $\frac{c}{a}|x|<a$

bundan esa $a-\frac{c}{a}x>0$, $a+\frac{c}{a}x>0$. Shuning uchun

$$r_1=F_1N=a-\frac{c}{a}x,\quad r_2=F_2N=a+\frac{c}{a}x \qquad (7)$$

Demak, $r_1+r_2=2a$. Ya'ni koordinatalari (6) tenglamani qanoatlantiruvchi M nuqta ellipsda yotadi.

(6) tenglama ellipsning kanonik tenglamasi deyiladi.

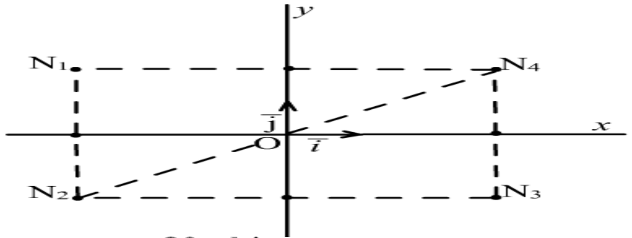
Ellipsning xossalari.

Bu yerda ellipsning xossalarini o`rganib, uning shaklini chizamiz.

1°. (6) tenglamadan ko'rinadiki, ellips ikkinchi tartibli chiziqdir.

2°. Agar $N(x,y) \in \gamma$ bo`lsa, u holda x,y koordinatalar (6) tenglamani qanoatlantiradi, shuning uchun $x^2 \leq a^2$, $y^2 \leq b^2$, demak

$$-a \leq x \leq a,\ -b \leq x \leq b$$



80-chizma

Ya'ni ellipsning hamma nuqtalari tomonlari $2a$ va $2b$ dan iborat bo`lgan $N_1N_2N_3N_4$ to`g'ri to'rtburchak ichida joylashgan (80-chizma).

3°. Agar $N(x,y) \in \gamma$ bo`lsa, u holda $N'(-x,-y) \in \gamma$ shuning uchun $O$ nuqta ellipsning yagona simmetriya markazi bo`ladi

Agar $N(x,y) \in \gamma$, u holda $N'(-x,y)$ va $N'(x,-y)$ nuqtalar ham ellipsda yotadi. Chunki ellips ikkinchi tartibli chiziq. Demak, Ox va Oy o`qlari ellipsning simmetriya o`qlari bo`ladi. Ellips aylanadan farqli o`laroq boshqa simmetriya o`qlarga ega emas.

4°. Ellipcning koordinata o`qlari bilan kesishgan nuqtalarini topaylik:

a) $y=0$, (6) $\Rightarrow x^2=a^2,\ x=\pm a$ demak, tllips Ox o`qni $A_1(a;0)$ va $A_2(-a;0)$ nuqtalarda kesadi

b) $x=0$, (6) $\Rightarrow y^2=b^2,\ y=\pm b$. ellips Oy o'qni $B_1(0,b)$ va $B_2(0,-b)$ nuqtalarda kesadi. Bu nuqtalarni ellipsning uchlari deyiladi. A_1A_2 va B_1B_2 kesmalar mos ravishda ellipsning katta va kichik o`qlari deyiladi. Bu kesmalar $O$ nuqtada teng ikkiga bo`linadi. $OA_1=OA_2=a$, $OB_1=OB_2=b$ bu kesmalarni mos ravishda ellipsning katta va kichik yarim o`qlari deyiladi.

Ellipsning ekstsentrisiteti va direktrisalari

T a ‘ r i f. Ellipsning fokuslari orasidagi masofaning ellipsning katta o`qi uzunligiga nisbati shu ellipsning ekstsentrisiteti deb ataladi. Ekstsentrisitet e harfi bilan belgilanadi.

$$e=\frac{2c}{2a},\ e=\frac{c}{a} \qquad (1)$$

c-fokal masofa, a- katta yarim o`q.

Shuning uchun $0<e<1$ har bir ellipsning ekstsentrisitenti birdan kichik.

40-§, (40.7) tenglikni e’tiborga olsak, u holda ellipsning fokal radiuslarini ekstsentrisitent orqali

$$r_1=a-ex,\ r_2=a+ex \qquad (2)$$

ko’rinishda yozish mumkin.

Ekstsentrisitet nolga teng bo`lishi uchun $c=0$ bo`lishi zarur va yetarlidir.

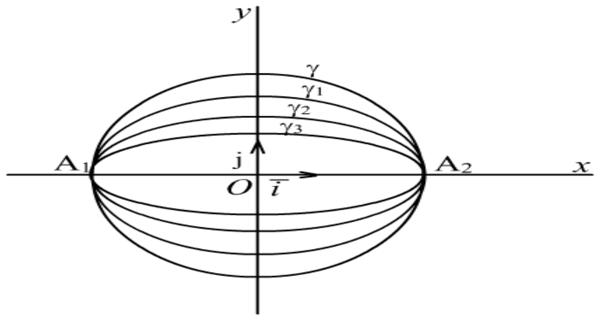
Bunda ellips aylana bo`lib qoladi.

$c^2=a^2-b^2$ ekanligini e’tiborga olsak , u holda

$$e^2=\frac{c^2}{a^2}=\frac{a^2-b^2}{a^2}=1-\left(\frac{b}{a}\right)^2;$$

bundan

$$e=\sqrt{1-\left(\frac{b}{a}\right)^2}\ \text{va}\ \frac{b}{a}=\sqrt{1-e^2};$$



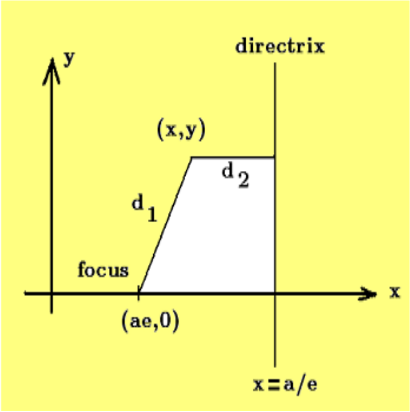
82-chizma

Ellipsning ekstsentrisiti e $0 < e < 1$ va ixtiyoriy musbat a soni quyidagi

$$ae < \frac{a}{e},$$

tengsizlikni qanoatlantiradi.

Demak, agar ellipsning fokus nuqtasi deb $(ae, 0)$ nuqtani, ellipsning direktrisasi deb $x = \frac{a}{e}$ chiziqni tanlab olsak, u holda tanlangan nuqta, tanlangan to'g'ri chiziq va umimiy nuqta (x, y) uchun quydagi tenglik o'rinli.



$d_1 =$ distance of (x, y) to focus $= \sqrt{(x-ae)^2 + y^2}$

$d_2 = \perp$ distance of (x, y) to directrix $= |x - a/e|$

Yuqoridagilardan shuni aytish mumkinki, ellips quydagi tenglikni qanoatlantiruvchi (x, y) nuqtalarga aytiladi:

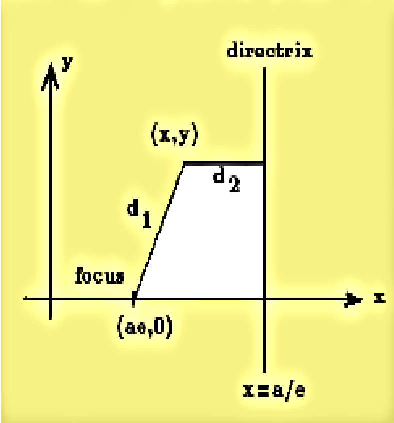
$$E_1 = \{\, (x, y) \mid \sqrt{(x-ae)^2 + y^2} = e|x - a/e|,\ 0 < e < 1 \,\}$$

Ellipse

The eccentricity e of an ellipse satisfies $0 < e < 1$ so that for any given positive number a one can state that

$$ae < \frac{a}{e}, \qquad 0 < e < 1 \tag{1.68}$$

Consequently, if the point $(ae, 0)$ is selected as the focus of an ellipse and the line $x = a/e$ is selected as the directrix of the ellipse, then in relation to this fixed focus and fixed line a general point (x, y) will satisfy



$d_1 =$ distance of (x, y) to focus $= \sqrt{(x-ae)^2 + y^2}$

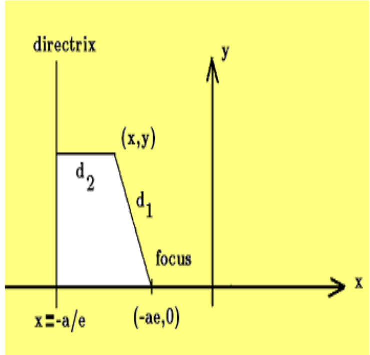
$d_2 = \perp$ distance of (x, y) to directrix $= |x - a/e|$

The ellipse can then be defined as the set of points (x, y) satisfying the constraint condition $d_1 = e d_2$ which can be expressed as the set of points

$$E_1 = \{\, (x, y) \mid \sqrt{(x|-ae)^2 + y^2} = e|x - a/e|,\ 0 < e < 1 \,\} \tag{1.69}$$

$$\frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1 \quad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\quad b^2=a^2(1-e^2)$$

Agar ellipsning fokus nuqtasi deb $(-ae,0)$ nuqtani, ellipsning direktrisasi deb $x=-\frac{a}{e}$ chiziqni tanlab olsak, u holda tanlangan nuqta, tanlangan to'g'ri chiziq va umimiy nuqta (x,y) uchun quydagi tenglik o'rinli.

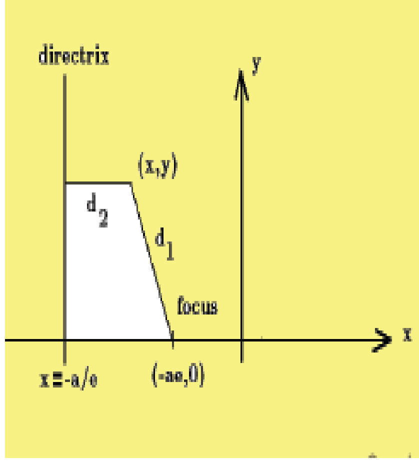


$d_1=$distance of (x,y) to focus $=\sqrt{(x+ae)^2+y^2}$

$d_2=\perp$ distance of (x,y) to directrix $=|x+a/e|$

$$\frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1 \quad \text{or} \quad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\quad b^2=a^2(1-e^2) \qquad (1.70)$$

where the eccentricity satisfies $0<e<1$. In the case where the focus is selected as $(-ae,0)$ and the directrix is selected as the line $x=-a/e$, there results the following situation



$d_1=$distance of (x,y) to focus $=\sqrt{(x+ae)^2+y^2}$

$d_2=\perp$ distance of (x,y) to directrix $=|x+a/e|$

Bu hol uchun quyidagi to'plamni qaraymiz.

$$E_2 = \{ (x, y) \mid \sqrt{(x + ae)^2 + y^2} = e|x + a/e|, \ 0 < e < 1 \}$$

E_2 to'plamdagi tenglikni shakl almashtirishlar yordamida quyidagi sodda ko'rinishga keltirish mumkin.

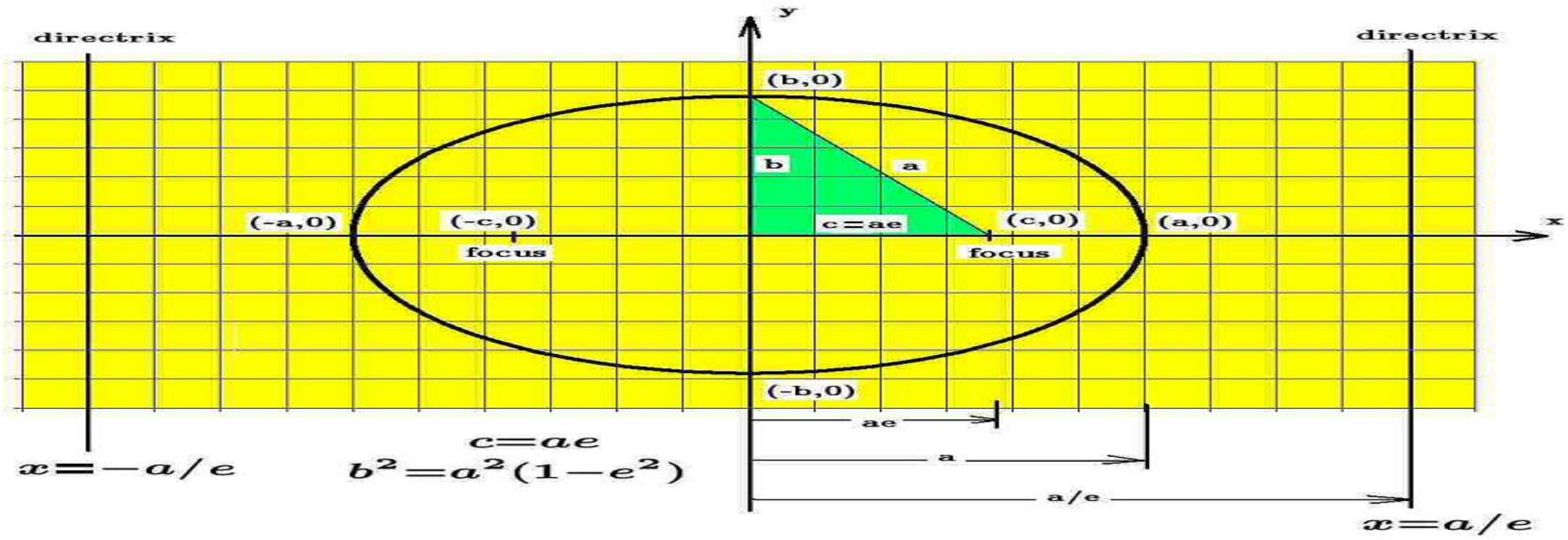


Figure 1-41. The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Bunda va $c = ae$ $\qquad b^2 = a^2(1 - e^2) = a^2 - c^2$

Natijada

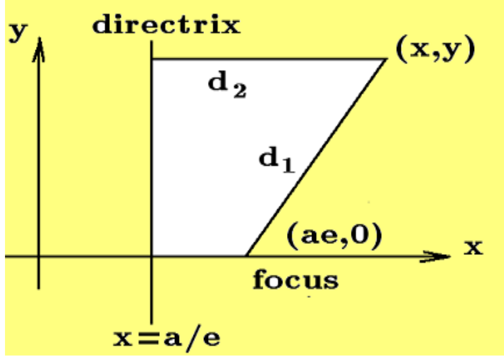
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad 0 < e < 1, \quad b^2 = a^2(1 - e^2), \quad c = ae$$

tengliklarga ega bo'lamiz. Shunday qilib ellipsning fokusi $(ae, 0)$, $(-ae, 0)$ nuqtalar

uning direktrissalari esa $\begin{matrix} d_1: & x - \frac{a}{e} = 0 \\ d_2: & x + \frac{a}{e} = 0 \end{matrix}$ to'g'ri chiziqlardir.

Giperbola ta’rifi, kanonik tenglamasi.

Giperbolaning ekstsentrisitetini e harf bilan belgilaymiz. Giperbolaning fokus nuqtasi deb $(ae,0)$ nuqtani, uning direktrisasi deb $x=\frac{a}{e}$ chiziqni tanlab olsak, u holda tanlangan nuqta, tanlangan to’g’ri chiziq va umimiy nuqta (x,y) uchun quydagi tenglik o’rinli.



$$H_1 = \{ (x,y) \mid \sqrt{(x-ae)^2+y^2} = e|x-a/e|,\ e>1 \}$$

$$\sqrt{(x-ae)^2+y^2} = e|x-a/e|$$

tenglikni elementar almashtirishlar yordamida

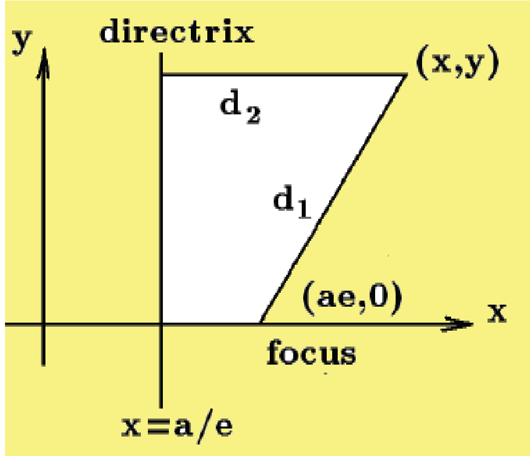
$$\frac{x^2}{a^2} - \frac{y^2}{a^2(e^2-1)} = 1$$

tenglikka ega bo’lamiz.

Giperbolaning fokus nuqtasi deb $(-ae,0)$ nuqtani, uning direktrisasi deb $x=-\frac{a}{e}$ chiziqni tanlab olsak, u holda tanlangan nuqta, tanlangan to’g’ri chiziq va umimiy nuqta (x,y) uchun quydagi tenglik o’rinli.

Hyperbola

Let $e>1$ denote the eccentricity of a hyperbola. Again let $(ae,0)$ denote the focus of the hyperbola and let the line $x=a/e$ denote the directrix of the hyperbola. The hyperbola is defined such that points (x,y) on the hyperbola satisfy $d_1 = ed_2$ where d_1 is the distance from (x,y) to the focus and d_2 is the perpendicular distance from the point (x,y) to the directrix. The hyperbola can then be represented by the set of points



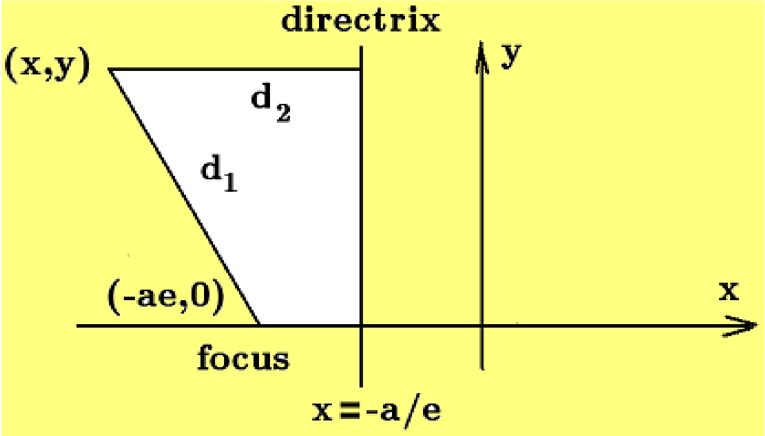
$$H_1 = \{ (x,y) \mid \sqrt{(x-ae)^2+y^2} = e|x-a/e|,\ e>1 \}$$

A simplification of the constraint condition on the set of points (x,y) produces the alternative representation of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{a^2(e^2-1)} = 1, \qquad e>1 \tag{1.78}$$

Placing the focus at the point $(-ae,0)$ and using as the directrix the line $x=-a/e$, one can verify that the hyperbola is represented by the set of points

$$H_2 = \{ (x,y) \mid \sqrt{(x+ae)^2+y^2} = e|x+a/e|, \ e>1 \}$$



$$\sqrt{(x+ae)^2+y^2} = e|x+a/e|$$

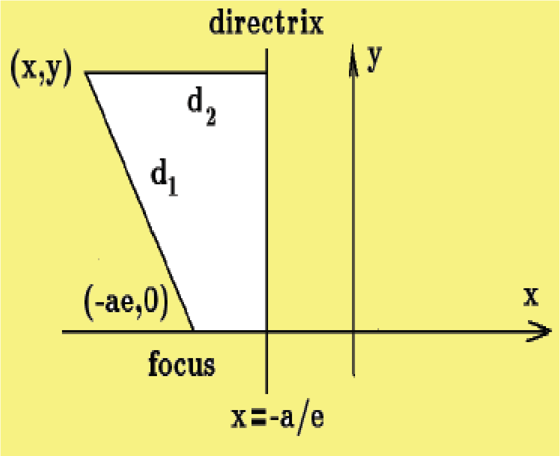
tenglikni soddalashtirishda $c = ae$ and $b^2 = a^2(e^2-1) = c^2 - a^2 > 0$ belgalashni hisobga olsak, u holda

$$\sqrt{(x+ae)^2+y^2} = e|x+a/e|$$

tenglikni

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

ko'rinishdagi tenglikka olib kelish mumkin. Shunday qilib giperbolning fokuslari $(-ae,0)$ va $(ae,0)$ nuqtalar uning direktrissalari esa $d_1: \quad x - \frac{a}{e} = 0$, $d_2: \quad x + \frac{a}{e} = 0$ to'g'ri chiziqlardir.



$$H_2 = \{ (x,y) \mid \sqrt{(x+ae)^2+y^2} = e|x+a/e|, \ e>1 \}$$

and it can be verified that the constraint condition on the points (x,y) also simplifies to the equation (1.78).

Define $c = ae$ and $b^2 = a^2(e^2-1) = c^2 - a^2 > 0$ and note that for an eccentricity $e > 1$ there results the inequality $c > a$. The hyperbola can then be described as having the foci $(c,0)$ and $(-c,0)$ and directrices $x = a/e$ and $x = -a/e$. The hyperbola represented by

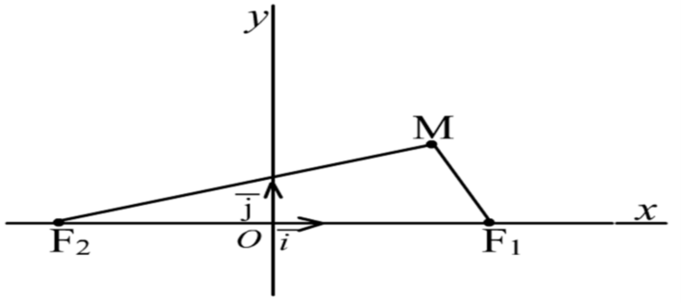
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \qquad b^2 = a^2(e^2-1) = c^2 - a^2 \tag{1.79}$$

is illustrated in the figure 1-43.

Giperbola to`g'ri burchakli dekart koordinatalar sistemasidagi tenglamasini chiqarish uchun, koordinatalar sistemasini ellips bilan ish ko`rgandek qilib tanlaymiz.

$F_1F_2=2c$ bo`lgani uchun olingan koordinatalar sistemasida $F_1(c,0)$, $F_2(-c,0)$, $M(x,y)$ kordinatalarga ega bo`ladi (90-chizma).

U holda $r_1=F_1M=\sqrt{(x-c)^2+y^2}$, $r_2=F_2M=\sqrt{(x+c)^2+y^2}$ (45.3)



90-chizma

Giperbola ta'rifiga ko`ra ya'ni (45.2) formaulaga asosan

$$\left|\sqrt{(x-c)^2+y^2}-\sqrt{(x+c)^2+y^2}\right|=2a$$ ni hosil qilamiz

bu tenglamani quyidagicha yozib olamiz

$$\sqrt{(x-c)^2+y^2}=\sqrt{(x+c)^2+y^2}\pm 2a$$

bu tenglamani kvadratga oshirib quyidagiga ega bo`lamiz

$$\pm a\sqrt{(x-c)^2+y^2}=a^2-cx$$

yana kvadratga oshirib ba'zi bir almashtirishlarni bajarib, quyidagilarni yozamiz

$$(c^2-a^2)x^2-a^2y^2=a^2(c^2-a^2) \qquad (45.4)$$

$$b^2=c^2-a^2>0 \qquad (45.5)$$

belgilab, bu belgilanishlarni e'tiborga olsak

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

Agar $N(x,y)$ nuqta giperbolada yotsa, (45.6) tenglamadan: $/x/\geq a$. demak $x=\pm a$ to`g'ri chiziqlar bilan chegaralangan tasmada (polosa) da giperbolaning birorta ham nuqtasi yo`q (86-chizma).

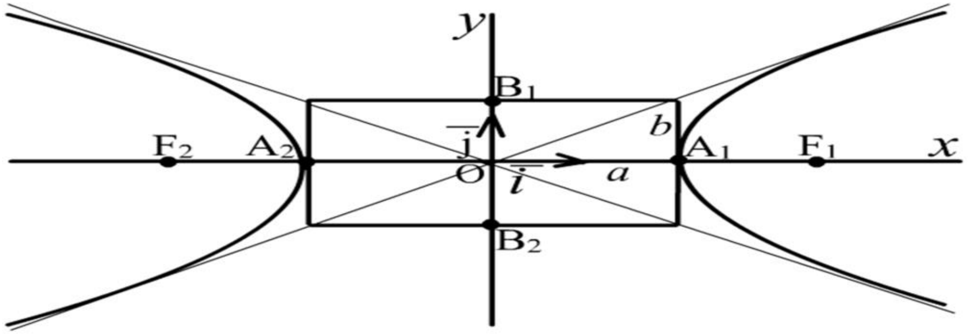
Giperbola tenglamasini y ga nisbatan echaylik

$$y=\pm\frac{b}{a}\sqrt{x^2-a^2} \qquad (46.1)$$

bu tenlamaga e'tibor bersak x o`zgaruvchi $a$ dan $+\infty$ gacha o`sib borganda va $-a$ dan $-\infty$ gacha kamayganda, y miqdor $-\infty<y<+\infty$ oraliqda o`zgaradi. Demak, giperbola ikki qismdan iborat bo`lib, 91-chizmada tasvirlangan.

Ularni giperbolaning tarmoqlari deyiladi.

Giperbolaning o`ng tarmog'i $x\geq a$ yarim tekislikda, chap yarim tarmogi $x<-a$ yarim tekislikda yotadi.



91-chizma

Giperbolaning ekstsentrisiteti, asimptotalari va direktrisalari.

Giperbolaning shaklini aniq tasvirlash uchun yassi chiziqning asimptotasi tushunchasini kiritamiz. Bizga λ chiziqni kesmaydigan d to`gri chiziq berilgan bo`lsin.

Ta'rif. Agar $N\in\lambda$ nuqta shu λ chiziq bo`yicha harakat qilganda uning $d$ to`g'ri chiziqqacha bo`lgan masofasi nolga intilsa, to`g'ri chiziq λ chizining asimptotasi deyiladi.

Giperbola markazidan o`tuvchi $d$ to`g'ri chiziq

$$x=a_1 t$$

$$y=a_2 t \qquad (47.1)$$

parametrik tenglamasi bilan berilgan. (45.6) va (47.1) tenglamalarni sistema qilib echamiz

$$\left(\frac{a_1^2}{a^2}-\frac{a_2^2}{b^2}\right)t^2=1 \qquad (47.2)$$

1) agar $\frac{a_1^2}{a^2}-\frac{a_2^2}{b^2}>0$ bo`lsa, (47.2) tenglama $t_{1,2}=\pm\frac{ab}{\sqrt{a_1^2b^2-a_2^2a^2}}$ demak , $d$ to`g'ri chiziq giperbola bilan ikkita $N_1(a_1t,\ a_2t)$ va $N_2(a_1t_2,\ -a_2t_2)$ nuqtalarda kesishadi.

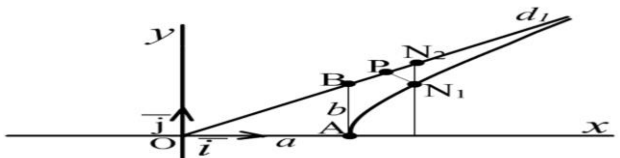
2. Agar $\frac{a_1^2}{a^2}-\frac{a_2^2}{b^2}<0$ bo`lsa, u holda $d$ to`g'ri chiziq giperbolani kesmaydi.

Xususan, $\frac{a_1^2}{a^2}-\frac{a_2^2}{b^2}=0$, u holda $\frac{a_2}{a_1}=\pm\frac{b}{a}$. d_1: $y=\frac{b}{a}x$, d_2: $y=-\frac{b}{a}x$ tenglama bilan aniqlangan d_1, d_2 to`g'ri chiziqlar giperbola assimptotalari deyiladi.

Giperbola koordinatalar o`qlariga nisbatan simmetrik bo`lgani uchun uning birinchi choragidagi qismini olamiz.

Agar $x>0$ bo`lsa, giperbolaning birinchi chorakdagi qismini aniqlaydi

$$y=\frac{b}{a}\sqrt{x^2-a^2}$$



92-chizma

Giperbolaga tegishli $N_1(x_1,y_1)$ nuqtani va d_1 to`g'ri chiziqqa tegishli $N_2(x_2,y_2)$ nuqtani olaylik.

$$(y_1=\frac{b}{a}\sqrt{x^2-a^2},\ y_2=\frac{b}{a}x)\Rightarrow y_2>y_1$$

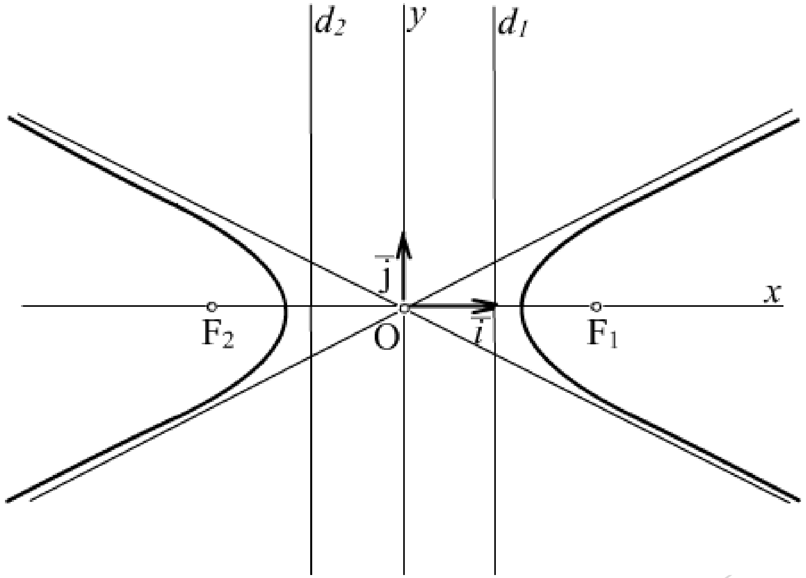
Ta'rif: Giperbolaning berilgan F fokusga mos direktrisasi deb, uning fokal o'qiga perpendikulyar va markazdan shu F fokus yotgan tomonda $\frac{a}{e}$ masofada turuvchi to'g'ri chiziqqa aytiladi.

$F_1(c,0)$ va $F_2(-c,0)$ fokuslarga mos direktrisalarni d_1 va d_2 deb belgilasak, u holda bu direktrisalarning tenglamalari

$$d_1: \quad x - \frac{a}{e} = 0$$
$$d_2: \quad x + \frac{a}{e} = 0 \qquad (47.7)$$

ko'rinishda bo'ladi.

$d_1 \cap (Ox) = D_1,\ d_2 \cap (Ox) = D_2$ deb olsak, Giperbola uchun $e > 1$ bo'lgani uchun $\rho(0, D_1) = \rho(0, D_2) = \frac{a}{e} < a$ bo'ladi.



94-chizma

Parabola tenglamasini sodda ko`rnishga ya’ni kanonik ko`rinishga keltirish uchun (50.4) tenglamani ikkala qismini kvadratga ko`taramiz.

$$x^2-px+\frac{p^2}{4}+y^2=x^2+px+\frac{p^2}{4} \qquad (50.5)$$

yoki
$$y^2=2px \qquad (50.6)$$

(50.6) tenglamani (50.4) ning natijasi sifatida keltirib chiqardik. O’z navbatida (50.4) tenglamani ham (50.6) tenglamaning natijasi sifatida chiqarish mumkinligini ko`rsatish oson. Haqiqatan, (50.6) tenglamadan to’g’ridan-to’g’ri (50.5) tenglama keltirib chiqariladi. So`ngra (50.5) tenglamadan ushbu hosil bo`ladi;

$$\sqrt{(x-\frac{p}{2})^2+y^2}=\pm(x+\frac{p}{2})$$

Agar x,y (50.6) tenglamani qanoatlantirsa, bu yerda faqat musbat ishora olishini ko`rsatish kerak. Ammo bu ravshan chunki, (50.6) tenglamadan $x=\frac{y^2}{2p}$, demak, $x>0$, shu sababli $x+\frac{p}{2}$ musbat sondir. Biz (50.4) tenglamaga kelamiz.

(50.4) va (50.6) tenglamalarning har biri ikkinchisining natijasi bo`lganligidan ular ekvivalentdir.

Bunda (50.6) tenglama parabola tenglamasi bo`ladi degan natijaga kelamiz. Bu tenglamani parabolaning kanonik tenglamasi deyiladi.