15-MAVZU:SONLI QATORLARVA XUSUSIY YIG'INDILAR.

REJA.

1.Qator. Qatorningyig'indisi.

2.Sonliqatorlarustidasoddaamallar.

3.Qatoryaqinlashishiningzaruriysharti.

1. Qator. Qatorningyig'indisi.

Elementlarisonlar (haqiqiyyokikompleks) yokifunksiyalarbo'lgan $a_1, a_2, \ldots, a_n, \cdots$ cheksizketma-ketlikniqaraymiz.

1-ta'rif. Ushbu

$$s_n = a_0 + a_1 + \ldots + a_n = \sum_{k=0}^{n} a_k . \qquad (1)$$

ifodacheksizqatordeyiladi.

Kelishibolinganigako'raqatornibelgilashuchun Σ belgisidanfoydalaniladi, ya'ni (1) qatorniqisqacha

$$\sum_{k=0}^{\infty} a_k = \lim_{n\to\infty} \sum_{k=0}^{n} a_k = \lim_{n\to\infty} s_n .$$

ko'rinishdayozishmumkin. $a_1, a_2, \ldots, a_n, \cdots$ ketma-ketlikningelementlariqatorninghadlarideyiladi.

Agar qatorninghadlarisonlar (funksiyalar) daniboratbo'lsa, qatorsonli (funksional) qatordeyiladi. Qatorning n–hadiningifodasigaqatorningumumiyhadideyiladi.

1-misol. Ushbu $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} + \cdots$ qatorsonliqatordir, uningumumiyhadi $\frac{1}{2^n}$ gateng, buqatorniqisqachabundayyozishmumkin: $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

$$s_0 = 1, \qquad s_1 = 1 + \frac{1}{2} = \frac{3}{2}, \qquad s_2 = 1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4},$$

$$\vdots$$

$$s_n = 1 + \frac{1}{2} + \ldots + \frac{1}{2^n} .$$

Biz avvalsonliqatorlarniko'ribchiqamiz.

2-ta'rif. (1) qatorningdastlabki n ta hadiningyig'indisi

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

shuqatorningqismiyyig'indisideyiladi.

Quyidagiqismiyyig'indilarniqaraymiz:

$$S_1 = a_1$$
$$S_2 = a_1 + a_2$$
$$S_3 = a_1 + a_2 + a_3$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$S_n = a_1 + a_2 + \cdots + a_n$$

Agar $\lim\limits_{n\to\infty} S_n = S$ chekli limit mavjudbo'lsa, u (1) qatorningyig'indisi deb ataladivaqatoryaqinlashuvchideyiladi.

Aksholda, ya'ni $\lim\limits_{n\to\infty} S_n$ mavjudbo'lmasa, (1) qatoruzoqlashuvchideyiladivauningyig'indisibo'lmaydi.

3.Canuto, C., Tabacco, A. Mathematical Analysis II, 4-6p.

1. Misol Ushbu

$$1+q+q^2+q^3+\cdots+q^{n-1}+\cdots \qquad (2)$$

qatornitekshiramiz.

Bu qatorbirinchihadi 1vamaxraji q bo'lgangeometrikprogressiyadir. Geometrikprogressiyadastlabki n ta hadiningyig'indisi ($q \neq 1$ bo'lganda):

$$S_n = \frac{1-q^n}{1-q}$$

1) agar $|q|<1$ bo'lsa, u holda $\lim\limits_{n\to\infty} q^n = 0$, demak,

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1-q^n}{1-q} = \frac{1}{1-q}$$

Demak, $|q|<1$ bo'lganda (2) qatoryaqinlashadivauningyig'indisi

$$S = \frac{1}{1-q}$$

2) agar $|q|>1$ bo'lsa, u holda $\lim\limits_{n\to\infty} q^n = \infty$ vashuninguchun

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1-q^n}{1-q} = \infty .$$

Shundayqilib, $|q|>1$ da cheksizgeometrikprogressiyauzoqlashuvchiqatorhosilqiladi.

Demak,

$$\lim_{n\to\infty} q^n = \begin{cases} 0 & |q|<1, \\ 1 & q=1, \\ +\infty & q>1, \end{cases}$$

3. Canuto, C., Tabacco, A. Mathematical Analysis II, P.3-4.

2. Misol

$$\lim_{n\to\infty} \sqrt[n]{p} = \lim_{n\to\infty} p^{1/n} = p^0 = 1\,.$$

Shunday qilib, cheksizgeometrik progressiya $|q|<1$ da yaqinlashuvchi va bo'lganda uzoqlashuvchi qator ekan.

2. Sonliqatorlarustidasoddaamallar.

1-teorema. Agar

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots \quad (1)$$

qatoryaqinlashuvchibo'lib, uningyig'indisi S gatengbo'lsa, u holda

$$ca_1 + ca_2 + ca_3 + \cdots + ca_n + \cdots \quad (2)$$

qator ham yaqinlashuvchibo'ladivauningyig'indisi cS gatengbo'ladi, bunda c birorbelgilangano'zgarmas son.

Isboti. (1) va (2) qatorlarning n-qismiyyig'indilarinimosravishda S_n va σ_n bilanbelgilaymiz. U holdaquyidagigaegabo'lamiz:

$$\sigma_n = ca_1 + ca_2 + ca_3 + \cdots + ca_n = c(a_1 + a_2 + a_3 + \cdots + a_n) = cS_n,$$

Bundan $\lim\limits_{n\to\infty} \sigma_n = \lim\limits_{n\to\infty} (cS_n) = c \lim\limits_{n\to\infty} S_n = cS$.

2.

$\lim\limits_{n\to\infty} a_n = \ell$ va $\lim\limits_{n\to\infty} b_n = m$

U holda

$$\lim_{n\to\infty} (a_n \pm b_n) = \ell \pm m$$

$$\lim_{n\to\infty} a_n b_n = \ell m,$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\ell}{m}, \quad b_n \neq 0$$

3.Canuto, C., Tabacco, A. Mathematical Analysis II, p.2.

3. Qatoryaqinlashishiningzaruriysharti.

Qatoryaqinlashishiningzaruriyshartishundayshartki, u bajarilmagandaqatoruzoqlashadi.

Teorema.Agar qatoryaqinlashsa, n cheksizo'sibborgandauning n – hadinolgaintiladi.

Isboti. Farazqilaylik,

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

qatoryaqinlashsin, ya'ni $\lim_{n\to\infty} S_n = S$ tengliko'rinlibo'lsin, bundaSqatorningyig'indisi (cheklison), lekinbuholda $\lim_{n\to\infty} S_{n-1} = S$ tenglikhamo'rinli, chunki $n \to \infty$ da $(n-1) \to \infty$. Oxirgiikkitengliknihadlabayiribquyidaginihosilqilamiz:

$$\lim_{n\to\infty} S_n - \lim_{n\to\infty} S_{n-1} = 0$$

yoki

$$\lim_{n\to\infty}(S_n - S_{n-1}) = 0$$

Lekin

$$S_n - S_{n-1} = a_n$$

Demak,

$$\lim_{n\to\infty} a_n = 0$$

Shuniisbotlashtalabqilinganedi.

Misol.

$$\frac{1}{4}+\frac{2}{7}+\frac{3}{10}+\ldots+\frac{n}{3n+1}+\ldots$$

qatoruzoqlashadi, chunki

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(\frac{n}{3n+1}\right) = \frac{1}{3} \neq 0$$

$$\lim_{n\to\infty} a_n = 0$$

tenglikoʼrinliboʼladiganharqandayqatorhamyaqinlashuvchiboʼlavermaydi. Bu shartningbajarilishiqatoryaqinlashuvchiboʼlishiuchunzaruriyshartboʼlib, ammo yetarlishartemas, yaʼniqatorumumiyhadiningnolgaintilishibilanqatoningyaqinlashuvchiekanligikelib chiqavermaydi, qatoruzoqlashuvchi ham boʼlishimumkin.

Masalan, garmonikqator deb ataluvchiushbu

$$1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}+\ldots$$

qator

$$\lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{1}{n}=0$$

bo'lishigaqaramay, uuzoqlashadi. Buniisbotqilishmaqsadidagarmonikqatornibirnechahadiniquyidagidekguruhlabyozamiz

$$1+\frac{1}{2}+\left(\frac{1}{3}+\frac{1}{4}\right)+\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right)+$$
$$+\left(\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\frac{1}{12}+\ldots+\frac{1}{16}\right)+\left(\frac{1}{17}+\ldots \qquad (1)$$

Endiyordamchiqatortuzamiz, ya'niharqaysiqavsichidagiqo'shiluvchilarniularningkichigibilanalmashtiramiz.Natijada

$$1+\frac{1}{2}+\left(\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)+$$
$$+\left(\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\ldots+\frac{1}{16}\right)+\left(\frac{1}{32}+\ldots \qquad (2)$$

Harqaysiqavsichidagiqo'shiluvchilaryig'indisikichiklashdiva 1/2 gatengbo'ldi, ya'ni

$$S_n = 1 + (n-1)\frac{1}{2}$$

Bundanlimitgao'tsak

$$\lim_{n\to\infty} S_n = \infty$$

Demak, garmonikqatorningyig'indisialbattacheksizlikkaintiladi.Shundayqilib, biz garmonikqatorninguzoqlashuvchiekaniniisbotladik.