# 17-MAVZU:ISHORASI ALMASHINUVCHI QATORLAR

**REJA**

1.Ishorasialmashuvchiqator. Leybnistteoremasi.

2.Absolyutvashartliyaqinlashish.

## 1. Ishorasi navbatlashuvchi qatorlar. Leybnits teoremasi.

Biz hozirgacha musbat hadli qatorlar bilan ish ko'rdik. Bu paragrafda hadlarning ishoralari navbatlashuvchi qatorlarni, ya'ni

$$b_1 - b_2 + b_3 - b_4 + \ldots + (-1)^{k+1} U_k + \ldots = \sum_{k=1}^{\infty} (-1)^{k+1} b_n$$

ko'rinishdagi qatorni tekshiramiz. Bunda: $b_1, b_2, \ldots b_n \ldots$ musbat sonlar.

*Leybnits teoremasi.* Agar ishoralari navbatlashuvchi

$$\sum_{k=0}^{\infty} (-1)^k b_k \qquad (1)$$

qatorning hadlari $b_1 > b_2 > b_3 > \ldots > b_N > \ldots$ (2)

va $\lim_{k \to \infty} b_k = 0$ (3)

bo'lsa, (1) qator yaqinlashadi va uning yig'indisi musbat bo'lib, birinchi hadidan katta bo'lmaydi: $0 < S < b_1$.

**Theorem 1.20 (Leibniz's Alternating Series Test)** *An alternating series* $\sum_{k=0}^{\infty}(-1)^k b_k$ *converges if the following conditions hold*

*i)* $\lim_{k\to\infty} b_k = 0\,;$

*ii) the sequence* $\{b_k\}_{k\geq 0}$ *decreases monotonically.*

*Denoting by* $s$ *its sum, for all* $n \geq 0$ *one has*

$$|r_n| = |s - s_n| \leq b_{n+1} \qquad \textit{and} \qquad s_{2n+1} \leq s \leq s_{2n}\,.$$

3.Canuto, C., Tabacco, A. Mathematical Analysis II, P.16.

Agar ishoralari navbatlashuvchi (1) qator Leybnits teoremasi shartni qanoatlantirsa, u holda uning $n$-qoldig'i

$$R_n=\pm(b_{n+1}-b_{n+2}+b_{n+3}-b_{n+4}+\dots)$$

absolyut qiymat bo'yicha tashlab yuborilgan hadlarning birinchisining modulidan katta bo'lmaydi, ya'ni $|R_n|\leq b_{n+1}$ dan bo'ladi.

Demak, qatorningS yig'indisiniS$_n$xususiyyig'indibilanalmashtirishdaqo'lkeladiganxatoabsolyutqiymati bo'yichatashlabyuborilganhadlarningbirinchisidankattabo'lmaydi.

Misol. Ushbu

$$1-\frac{1}{2!}+\frac{1}{3!}-\frac{1}{4!}+\dots$$

qatoriningyaqinlashishitekshirilsin.

Yechish.Qatorninghadlariabsolyutqiymatibo'yichakamayibboradi:

$$\frac{1}{2!}>\frac{1}{3!}>\frac{1}{4!}>\dots \text{ va } \lim_{k\to\infty} b_k=\lim_{k\to\infty}\frac{1}{k!}=0$$

qatoryaqinlashuvchi.

Agar qatorning hadlari orasida musbatlari ham, manfiylari ham bo'lsa, bunday qator o'zgaruvchan ishorali qator deyiladi.

Bizga o'zgaruvchan ishorali qator:

$$b_1+b_2+b_3+\ldots+b_n+\ldots \qquad (1)$$

berilgan bo'lsin, bunda $b_1, b_2, b_3, \ldots, b_n \ldots$ sonlar musbat ham, manfiy ham bo'lishi mumkin.

Oldin biz ko'rib o'tgan ishoralari navbatlashuvchi qatorlar o'zgaruvchan ishorali qatorlarning xususiy holidir.

O'zgaruvchan ishorali qator yaqinlashishining yetarli shartini ko'ramiz.

*1-teorema.* O'zgaruvchan ishorali qator

$$b_1+b_2+b_3+\ldots+b_n+\ldots \qquad (1)$$

hadlarning absolyut qiymatlaridan tuzilgan:

$$|b_1|+|b_2|+|b_3|+\ldots+|b_n|+\ldots \qquad (2)$$

qator yaqinlashsa, berilgan o'zgaruvchan ishorali qator ham yaqinlashadi.

3.Canuto, C., Tabacco, A. Mathematical Analysis II, P.16.

*1- Misol.* Ushbu

$$\frac{\sin\alpha}{1^2}+\frac{\sin 2\alpha}{2^2}+\frac{\sin 3\alpha}{3^2}+\ldots+\frac{\sin n\alpha}{n^2}+\ldots \qquad (3)$$

o'zgaruvchanishoraliqatorningyaqinlashishitekshirilsin, bunda $\alpha$ istalgan son.

*Yechish.* Berilganqatorgamosbo'lgan

$$\left|\frac{\sin\alpha}{1^2}\right|+\left|\frac{\sin 2\alpha}{2^2}\right|+\left|\frac{\sin 3\alpha}{3^2}\right|+\ldots+\left|\frac{\sin n\alpha}{n^2}\right|+\ldots \qquad (4)$$

qatorniqaraymiz. Bu qatorniyaqinlashuvchibo'lgan:

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}+\ldots \qquad (5)$$

qatorbilansolishtiramiz.

(4) qatorninghadlari (5) qatorningmoshadlaridankattaemas, shusabablitaqqoslashning 1-teoremasiga ko'ra (4) qatoryaqinlashuvchi. Bu holdaisbotlanganteoremagako'ra (3) qator ham yaqinlashuvchi.

Yuqorida isbot qilingan yaqinlashish alomati o'zgaruvchan ishorali qatorning yaqinlashishi uchun yetarligina bo'lib, zaruriy emasligini ko'ramiz. Shunday o'zgaruvchan ishorali qatorlar ham borki, ularning o'zlari yaqinlashuvchi bo'lsa ham, hadlarining absolyut qiymatlaridan tuzilgan qatorlar uzoqlashuvchi bo'ladi. Shu munosabat bilan o'zgaruvchan ishorali qatorlarning absolyut va shartli yaqinlashishi haqidagi tushunchani kiritish foydalidir.

## 2.Absolyut va shartli yaqinlashish.

*1-ta'rif.* Ushbu o'zgaruvchan ishorali qator:

$$a_1+a_2+a_3+\ldots+a_n+\ldots \tag{1}$$

hadlarining absolyut qiymatlaridan tuzilgan

$$|a_1|+|a_2|+|a_3|+\ldots+|a_n|+\ldots \tag{2}$$

qator yaqinlashsa, berilgan (1) qator absolyut yaqinlashuvchi deyiladi.

*2-ta'rif.* Agar o'zgaruvchan ishorali (1) qator yaqinlashuvchi bo'lib, bu qatorning hadlari absolyut qiymatlaridan tuzilgan (2) qator uzoqlashuvchi bo'lsa, u holda berilgan o'zgaruvchan ishorali (1) qator shartli yoki absolyut yaqinlashuvchi qator deyiladi.

**Definition 1.22** *The series* $\sum_{k=0}^{\infty} a_k$ **converges absolutely** *if the positive-term series* $\sum_{k=0}^{\infty} |a_k|$ *converges.*

**Theorem 1.24 (Absolute Convergence Test)** *If* $\sum_{k=0}^{\infty} a_k$ *converges absolutely then it also converges, and*

$$\left|\sum_{k=0}^{\infty} a_k\right| \leq \sum_{k=0}^{\infty} |a_k| .$$

3.Canuto, C., Tabacco, A. Mathematical Analysis II, P.17-18.

*1-misol.* Ushbu

$$\frac{\cos\frac{\pi}{4}}{3}+\frac{\cos\frac{3\pi}{4}}{3^2}+\frac{\cos\frac{5\pi}{4}}{3^3}+\ldots+\frac{\cos(2n-1)\frac{\pi}{4}}{3^n}+\ldots \qquad (3)$$

qatorningyaqinlashishitekshirilsin.

*Yechish.* Berilganqatorbilanbirga

$$\left|\frac{\cos\frac{\pi}{4}}{3}\right|+\left|\frac{\cos\frac{3\pi}{4}}{3^2}\right|+\left|\frac{\cos\frac{5\pi}{4}}{3^3}\right|+\ldots+\left|\frac{\cos(2n-1)\frac{\pi}{4}}{3^n}\right|+\ldots \qquad (4)$$

qatorniqaraymiz. Bu qatornigeometrikprogressiyatashkilqiluvchi

$$\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots+\frac{1}{3^n}+\ldots \qquad (5)$$

qatorbilantaqqoslaymiz. (5) qator $q<1$ bo'lganiuchunyaqinlashuvchi.

(4) hadlari (5) qatorningmoshadlaridankattaemas, shusababliqatorlarnitaqqoslashning 1-teoremasiga asosan (3) qator ham yaqinlashuvchi.

*2-misol.* Ushbu

$$-1+\frac{1}{\sqrt{2}}+\ldots+(-1)^n\frac{1}{\sqrt{n}}+\ldots \qquad (6)$$

qatorningyaqinlashishitekshirilsin.

*Yechish.* Berilganqatorbilanbirga

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}+\ldots \qquad (7)$$

qatorniqaraymiz. (6) qatorLeybnitsteoremasigako'rayaqinlashuvchi, ya'ni

$$\lim_{k\to\infty} b_n = \lim_{k\to\infty}\frac{1}{\sqrt{k}} = 0$$

Lekin (6) qatorhadlariningabsolyutqiymatidantuzilgan (7) qatoruzoqlashuvchi, chunki $f(x) = \frac{1}{\sqrt{x}}$, $p = \frac{1}{2} \le 1$ bo'lganqatordir.

Bu holdaberilgan (6) qatorshartliyaqinlashuvchibo'ladi.

Absolyutvashartliyaqinlashuvchiqatorlarningquyidagixossalarini (isbotsiz) keltiramiz:

*2-teorema.* Agar qatorabsolyutyaqinlashuvchibo'lsa, uninghadlariningo'rinlariixtiyoriyravishdaalmashtirilganda ham u absolyutyaqinlashuvchibo'libqolaveradi.Bu holdaqatorningyig'indisiqatorhadlariningyig'indisigabog'liqbo'lmaydi.

Bu hodisashartliyaqinlashuvchiqatoruchuno'zkuchiniyo'qotadi.

*3-teorema.* Agar qatorshartliyaqinlashuvchibo'lsa, u holdabuqatorhadlariningo'rinlarinishundayalmashtiribqo'yishmumkinki, natijadauningyig'indisio'zgaradi, buningustigaalmashtirishdankeyinhosilbo'lganqatoruzoqlashuvchiqatorbo'libqolishimumkin.